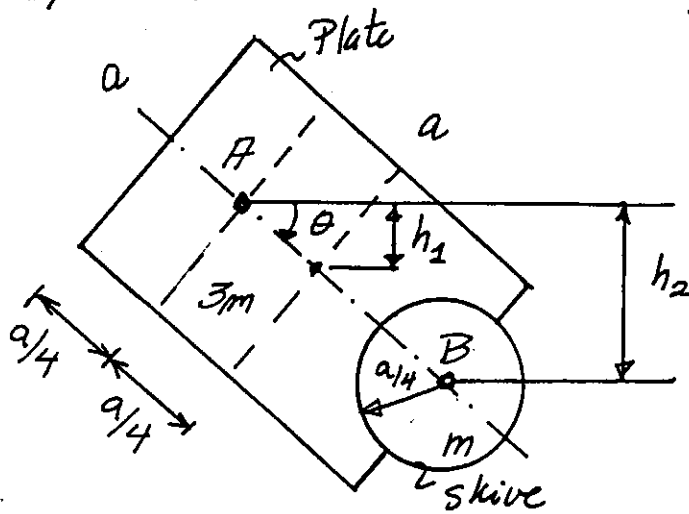


Torsdag 19. desember 1996

side 1 av 4

Oppgave 1



$$I_A^{\text{plate}} = \frac{1}{12} (a^2 + a^2) \cdot 3m + 3m \left(\frac{a}{4}\right)^2 = \frac{11}{16} ma^2$$

Skiva har bare translasjonsenergi.

$$\begin{aligned} U_\theta &= -mgh_2 - 3mgh_1 \\ &= -mg(h_2 + 3h_1) \\ &= -\frac{3}{2} mga \sin \theta. \end{aligned}$$

a) $K_0 + U_0 = K_\theta + U_\theta \Rightarrow$

$$0 + 0 = \frac{1}{2} m v_B^2 + \frac{1}{2} I_A^{\text{plate}} \cdot \dot{\theta}^2 + U_\theta$$

$$= \frac{1}{2} m \left(\frac{3a}{4} \dot{\theta}\right)^2 + \frac{1}{2} \cdot \frac{11}{16} ma^2 \dot{\theta}^2 - \frac{3}{2} mga \sin \theta$$

$$\Rightarrow \frac{5}{8} ma^2 \cdot \dot{\theta}^2 = \frac{3}{2} mga \sin \theta \text{ som}$$

$$\text{gir: } \dot{\theta}^2 = \underline{\underline{\frac{12}{5} \frac{g}{a} \sin \theta}}$$

$$\text{Ved derivasjon: } \ddot{\theta} = \underline{\underline{\frac{6}{5} \frac{g}{a} \cos \theta}}$$

b) $\theta = 0.$

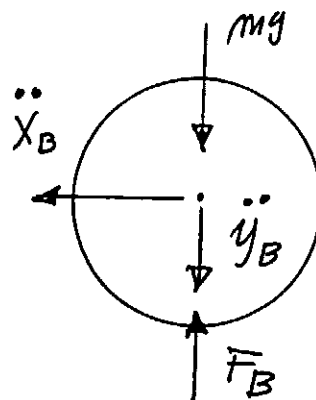
$$\ddot{x}_B = 0 \text{ da } \dot{\theta}(0) = 0$$

$$\ddot{y}_B = \frac{6}{5} \frac{g}{a} \cdot \frac{3}{4} a = \frac{9}{10} g$$

Newtons lov i y-retning:

$$-F_B + mg = m\ddot{y}_B = \frac{9}{10} mg$$

$$\underline{\underline{F_B = \frac{mg}{10}}}$$



Ingen krefter i x-retning.

Oppgave 1 b forts

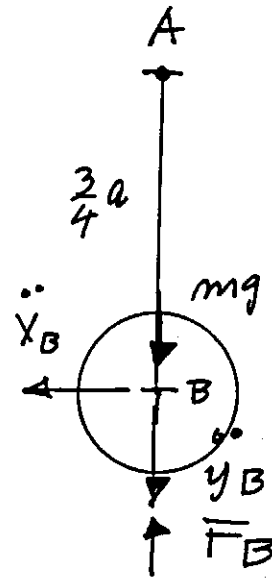
$$\theta = 90^\circ$$

$$\ddot{y}_B = -\frac{3}{4}a \cdot \frac{12}{5}\frac{g}{a}$$

$$= -\frac{9}{5}g$$

$$F_B - mg = -m\ddot{y}_B = \frac{9}{5}mg$$

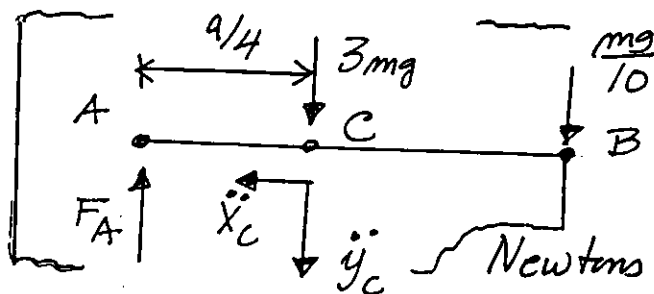
$$F_B = \underline{\underline{\frac{14}{5}mg}}$$



$$\dot{\theta}^2 = \frac{12}{5}\frac{g}{a}$$

$$\ddot{\theta} = 0 \Rightarrow \ddot{x}_B = 0$$

$\therefore$ ingen krefter i x-retning.

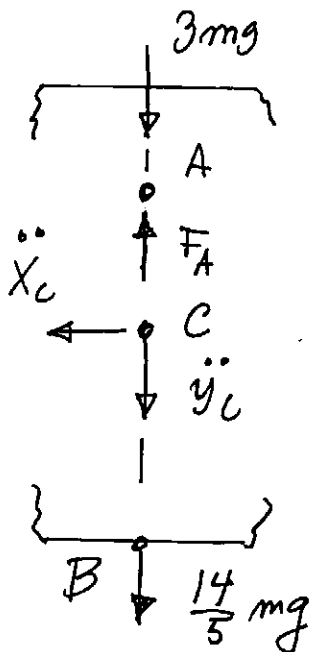
c) Plate. $\theta = 0$ 

Newton's lov:

$$-F_A + 3mg + \frac{mg}{10} = 3m\ddot{y}_C = \frac{9}{10}mg$$

$$\rightarrow F_A = \underline{\underline{\frac{11}{5}mg}}$$

$$\underline{\underline{\theta = 90^\circ}}$$



$\ddot{\theta} = 0 \Rightarrow \ddot{x}_C = 0$ $\therefore$ Ingen krefter i x-retning.

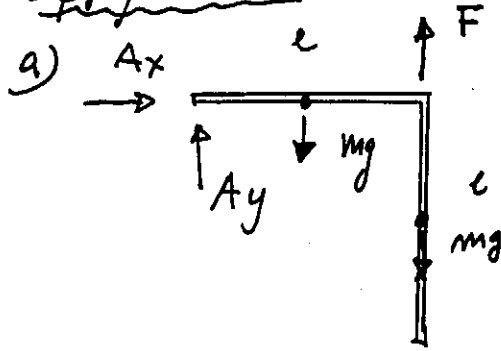
$$\dot{\theta}^2 = \frac{12}{5}\frac{g}{a}$$

$$\ddot{y}_C = -\frac{12}{5}\frac{g}{a} \cdot \frac{a}{4} = -\frac{3}{5}g$$

$$F_A - 3mg - \frac{14}{5}mg = -3m\ddot{y}_C = \frac{9}{5}mg$$

$$\underline{\underline{F_A = \frac{38}{5}mg}}}$$

Oppgave 2

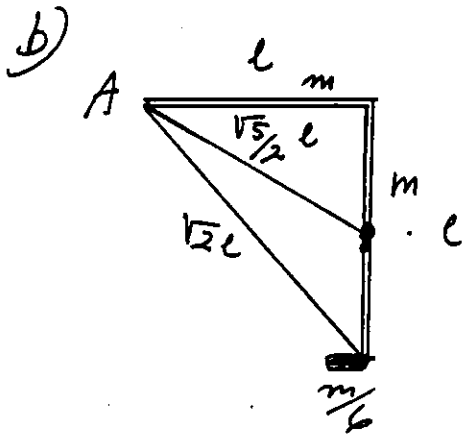


$$\Sigma \tau_A = 0:$$

$$F \cdot l - mg \frac{l}{2} - mg \cdot l = 0$$

$$F = \frac{3mg}{2}$$

$$F = k \cdot \delta_1 \rightarrow \delta_1 = \underline{\underline{\frac{3mg}{2k}}}$$



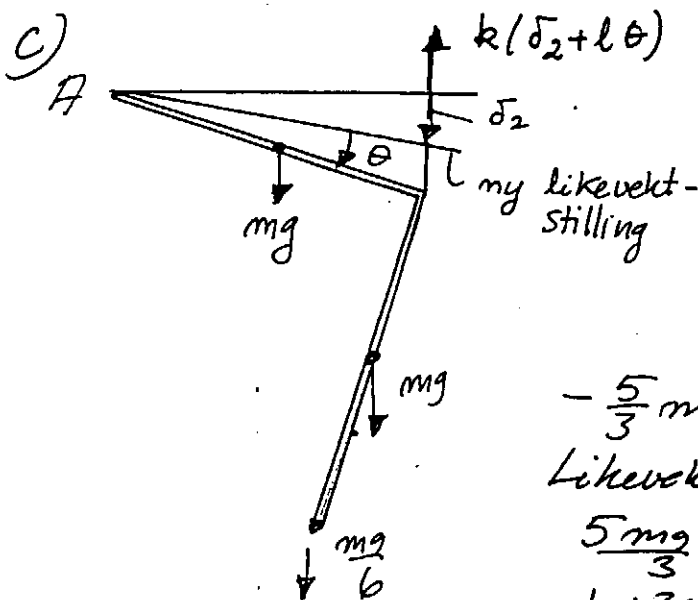
Ingen ytre momentimpuls om A:

$$L_A^{\text{før}} = L_A^{\text{etter}} \rightarrow$$

$$\frac{m}{6} v_0 \cdot l = I_A \cdot \omega_0$$

$$I_A = \frac{1}{3} m l^2 + \frac{1}{12} m l^2 + \frac{5}{4} m l^2 + \frac{m}{6} \cdot 2 l^2 = 2 m l^2$$

$$\frac{m}{6} v_0 l = 2 m l^2 \omega_0 \rightarrow \omega_0 = \underline{\underline{\frac{1}{12} \frac{v_0}{l}}}$$



Antar små utslag.

$$\Sigma \tau_A = I_A \ddot{\theta}$$

$$mg \frac{l}{2} + (mg + \frac{mg}{6}) l$$

$$- k(\delta_2 + l\theta) \cdot l = I_A \ddot{\theta}$$

$$- \frac{5}{3} mg l - k(\delta_2 + l\theta) \cdot l = 2 m l^2 \ddot{\theta}$$

$$\text{Likevekt: } \theta = 0 \text{ og } \ddot{\theta} = 0 \Rightarrow$$

$$\frac{5mg}{3} - k\delta_2 = 0 \rightarrow$$

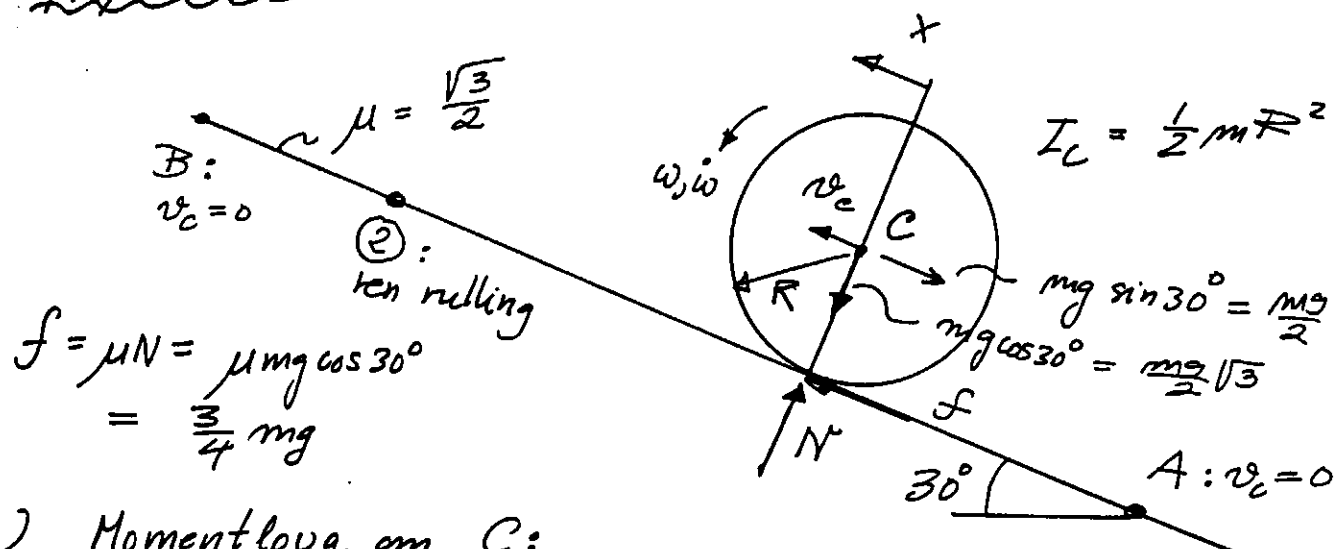
$$- k l^2 \theta = 2 m l^2 \ddot{\theta} \rightarrow$$

$$\ddot{\theta} + \frac{k}{2m} \theta = 0$$

Egen frekvensen $\omega_e = \underline{\underline{\sqrt{\frac{k}{2m}}}}$

Når vi regner fra likevektstillingen, holder tyngdekraft og fjærspenningskraftene hverandre i likevekt og får ingen innflytelse på egenfrekvensen.

Oppgave 3



a) Momentlova om C :

$$-fR = I_C \dot{\omega}, \quad \dot{\omega} = -\frac{\frac{3}{4}mgR}{\frac{1}{2}mR^2} = -\frac{3g}{2R} \rightarrow$$

$$\omega = \omega_0 - \frac{3g}{2R}t \quad \text{da } \omega = \omega_0 \text{ for } t=0.$$

Newtons lov: $f - mg \sin 30^\circ = m\ddot{x}_C$

$$\frac{3}{4}mg - \frac{mg}{2} = m\ddot{x}_C \Rightarrow \ddot{x}_C = \frac{g}{4} \rightarrow$$

$$v_C = \frac{g}{4}t \quad \text{da } v_C = 0 \text{ for } t=0$$

Den rulling i pkt. 2: $v_{C2} = \omega_2 R$ ved tiden $t = t_2$

$$(\omega_0 - \frac{3g}{2R}t_2)R = \frac{g}{4}t_2 \rightarrow t_2 = \frac{4\omega_0 R}{7g}$$

$$0: v_{C2} = \frac{g}{4}t_2 = \underline{\underline{\frac{\omega_0 R}{7}}}$$

b) Mellom ② og B har vi ren rulling.

Newtons lov: $f - \frac{mg}{2} = m\dot{v}_C$

Momentlova om C : $-fR = I_C \dot{\omega} = I_C \frac{\dot{v}_C}{R}$ ved ren rulling.

$$0: f = -\frac{1}{2}m\dot{v}_C$$

$$-\frac{1}{2}m\dot{v}_C - \frac{mg}{2} = m\dot{v}_C \rightarrow \dot{v}_C = -\frac{g}{3}$$

$$v_C = v_{C2} - \frac{g}{3}t, \quad t \text{ regnes fra } 0 \text{ i pkt. 2}$$

$$v_C = 0 \text{ i } B: \frac{g}{3}t_B = v_{C2} = \frac{\omega_0 R}{7} \Rightarrow t_B = \frac{3\omega_0 R}{7g}$$

$$\text{Totalt fra A-B: } t_{AB} = \frac{4\omega_0 R}{7g} + \frac{3\omega_0 R}{7g} = \frac{\omega_0 R}{g}$$