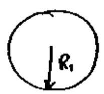
EKSAMEN EL.1104 1 8. aug 96

Løsningsforslag

1

a)



$Q = 20\mu C$

E-felt rundt kule ($r > R_1$): $\underline{E(r) = \frac{Q}{4\pi\varepsilon_0}\cdot\frac{1}{r^2}}$ $\quad \underline{E(r) = 0}$ for $r < R_1$

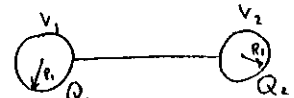
Potensial — " — $\quad V(r) = -\int_\infty^r E(r)\,dr = \frac{Q}{4\pi\varepsilon_0}\cdot\frac{1}{r}$

På kuleoverflaten $\underline{V(R_1)} = \frac{Q}{4\pi\varepsilon_0}\cdot\frac{1}{R_1} = \frac{20\mu C}{4\pi\varepsilon_0}\cdot\frac{1}{0{,}08\,m} = \underline{\underline{2{,}25\ MV}}$

utgår: Energi: $\underline{W_E} = \frac{1}{2}\int V\,dq = \frac{1}{2}\cdot V(R_1)\cdot Q = \frac{1}{2}\cdot 2{,}25\,MV\cdot 20\mu C = \underline{\underline{22{,}5\ J}}$

eller $W_E = \frac{1}{2}\iiint \varepsilon_0 E^2\,d\tau = \frac{1}{2}\int_{R_1}^{\infty}\varepsilon_0 E^2\cdot 4\pi r^2\,dr = \ldots\ldots = \frac{Q^2}{8\pi\varepsilon_0}\cdot\frac{1}{R_1} = \underline{\underline{22{,}5\ J}}$

b)



Potensialet må være likt over hele systemet (ledende materiale):

$V_1 = \frac{Q_1}{4\pi\varepsilon_0}\cdot\frac{1}{R_1} = V_2 = \frac{Q_2}{4\pi\varepsilon_0}\cdot\frac{1}{R_2} \Rightarrow \frac{Q_1}{Q_2} = \frac{R_1}{R_2}$

Innsatt $Q_1 + Q_2 = Q$ $\quad \frac{Q - Q_2}{Q_2} = \frac{R_1}{R_2} \Rightarrow Q_2 = Q\cdot\frac{R_2}{R_1+R_2} = 20\mu C\cdot\frac{6}{14} = \underline{\underline{8{,}6\mu C}}$

$\underline{\underline{Q_1}} = Q - Q_2 = \underline{\underline{11{,}4\mu C}}$

c) $V_1 = V_2 = \frac{Q_2}{4\pi\varepsilon_0}\cdot\frac{1}{R_2} = \frac{8{,}6\mu C}{4\pi\varepsilon_0}\cdot\frac{1}{0{,}06\,m} = \underline{\underline{1{,}28\ MV}}$

Energi*): $W_E = \frac{1}{2}\int V\,dq$ $\quad$ integrert der vi har ladning, dvs kun overflaten

Før: $W_E = \frac{1}{2}V(R_1)\cdot Q = \frac{1}{2}\cdot 2{,}25\,MV\cdot 20\mu C = \underline{\underline{22{,}5\ J}}$

Etter: $W_E = \frac{1}{2}V_1\cdot Q_1 + \frac{1}{2}V_2\cdot Q_2$

$= \frac{1}{2}V_1(Q_1+Q_2) = \frac{1}{2}\cdot V_1\cdot Q$

$= \frac{1}{2}\cdot 1{,}28\,MV\cdot 20\mu C = \underline{\underline{12{,}8\ J}}$

...

9,7 J er tapt i varme i ohmsk motstand i ledning + evt. stråling pga akselererte ladninger.

*) Evt (mer arbeid): $W_E = \int w_E\,d\tau = \frac{1}{2}\int_{R_1}^{\infty}\varepsilon_0 E(r)^2\cdot 4\pi r^2\,dr = $ osv.

2

a) $\underline{Q_{tot}} = \iiint \rho\,d\tau = \frac{\rho_0}{r_0}\int_0^{r_0} r\cdot 4\pi r^2\,dr$ $\quad$ (kuleskall: $d\tau = 4\pi r^2\cdot dr$ = volum kuleskall)

$= \frac{\rho_0}{r_0}\cdot 4\pi\cdot\frac{1}{4}r_0^4 = \underline{\underline{\rho_0\,\pi\,r_0^3}}$

Ladning innenfor $r = r$: $\quad Q(r) = \int_0^r \rho\cdot 4\pi r^2\,dr = \underline{\rho_0\,\pi\cdot\frac{r^4}{r_0}}$

$\Rightarrow \oiint D(r)\,dA \overset{\text{Gauss}}{=} Q(r) \Rightarrow D(r)\cdot 4\pi r^2 = \rho_0\pi\frac{r^4}{r_0} \Rightarrow \underline{\underline{D(r) = \frac{\rho_0}{4}\cdot\frac{r^2}{r_0}}}$ for $r < r_0$

$D(r)$ har retning $\hat{r}$ $\quad$ og $\underline{\underline{D(r) = \frac{Q_{tot}}{4\pi r^2} = \frac{\rho_0}{4}\cdot\frac{r_0^3}{r^2}}}$ for $r > r_0$

b) Nå først bestemme polariseringen:

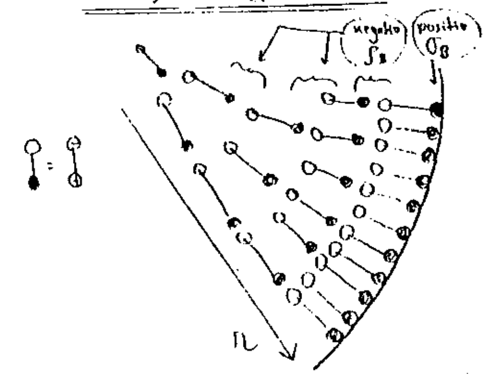
$P = D - \varepsilon_0 E = D - \varepsilon_0\frac{D}{\varepsilon_r\varepsilon_0} = (1 - \frac{1}{\varepsilon_r})D = \begin{cases} 0 & r > r_0 \\ (1-\frac{1}{\varepsilon_r})\cdot\frac{\rho_0 r^2}{4 r_0} & r < r_0 \end{cases}$

Da er $\underline{\underline{\sigma_B(r_0)}} = |P(r_0)| = \underline{\underline{\frac{\varepsilon_r - 1}{\varepsilon_r}\cdot\frac{\rho_0 r_0}{4}}}$ (positiv, fordi $\varepsilon_r > 1$)

c) I følge formler (eller utledning):

$\rho_B(r) = -\vec{\nabla}\cdot\vec{P} \underset{\text{kulekoord}}{=} -\frac{1}{r^2}\frac{\partial}{\partial r}\left(r^2\cdot(1-\frac{1}{\varepsilon_r})\frac{\rho_0 r^2}{4 r_0}\right) = -(1-\frac{1}{\varepsilon_r})\cdot\frac{\rho_0}{4 r_0}\cdot\frac{1}{r^2}\cdot 4r^3$

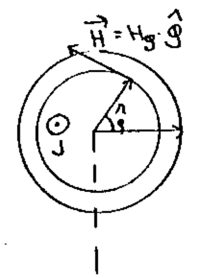
$\Rightarrow \underline{\underline{\rho_B(r) = -\frac{\varepsilon_r - 1}{\varepsilon_r}\cdot\rho_0\cdot\frac{r}{r_0}}}$ for $r < r_0$ $\quad$ Negativ



Polariseringen representert ved el. dipoler. Fordi $P(r) \propto r$ øker tettheten av dipoler utover fra sentrum

3

a)



Strømtetthet $J = \frac{I}{\pi R^2}$

Amperes lov.

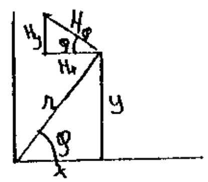
$$\oint \vec{H} \cdot d\vec{s} = I_{inni}$$

$$\oint H_\varphi \cdot ds = \iint J \, dA \qquad (\text{rotasjonssymm.})$$

$$H_\varphi \cdot 2\pi r = J \cdot \pi r^2$$

$$\underline{\underline{H_\varphi(r) = J \cdot \frac{r}{2} = I \frac{r}{2\pi R^2}}} \qquad \vec{H} = H_\varphi \cdot \hat{\varphi}$$

Kartesiske koordinater:



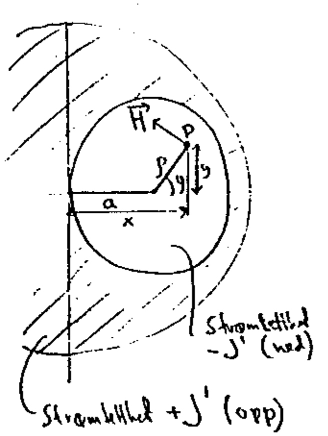
$$H_x = -H_\varphi \cdot \sin\varphi = -\left(J \cdot \frac{r}{2}\right)\left(\frac{y}{r}\right) = -J \cdot \frac{y}{2} = -\frac{I}{2\pi R^2} \cdot y$$

$$H_y = +H_\varphi \cdot \cos\varphi = \left(J \cdot \frac{r}{2}\right)\left(\frac{x}{r}\right) = \underline{\underline{J \cdot \frac{x}{2} = \frac{I}{2\pi R^2} \cdot x}}$$

b) I den gjenværende del av lederen blir strømtettheten $\underline{J' = \frac{I}{\pi R^2 - \pi a^2}}$.

Ved bruk av sup. pos. prinsippet er $\vec{H}'$ sammensatt av felt $\vec{H}^{(a)}$ pga strøm $-J'$ i sylinder med radius a pluss $\vec{H}'$ pga. strøm $+J'$ i sylinder med radius R (se figur). Vi behandler kun inni hulrom.

$$\vec{H}(P) = \vec{H}(x,y) = \left[H_x^{(a)} + H_x', \; H_y^{(a)} + H_y'\right]$$



Pkt P har koordinater: $\begin{cases} x = a + \rho \cdot \cos\varphi \\ y = \rho \cdot \sin\varphi \end{cases}$

Bruker resultatet i a)

$$\underline{H_x^{(a)} = -(-J') \cdot \frac{y}{2} = J' \cdot \frac{y}{2}} \qquad \underline{H_y^{(a)} = (-J') \cdot \frac{x-a}{2}}$$

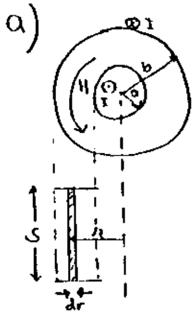
$$\underline{H_x' = -J' \cdot \frac{y}{2}} \qquad \underline{H_y' = J' \cdot \frac{x}{2}}$$

Totalt:

$$\begin{bmatrix} H_x = H_x^{(a)} + H_x' = 0 \\ H_y = H_y^{(a)} + H_y' = J' \cdot \frac{a}{2} = I \cdot \frac{a}{2\pi(R^2 - a^2)} \end{bmatrix}$$

4

a)



$\vec{H}$ og $\vec{B}$ er asimutale, retning mot klokka når I har retning angitt i figuren. Da er også fluksen Φ_m asimutal i samme retning som $\vec{B}$. $dA = s \cdot dr$

$$\Phi_m = \iint \vec{B} \cdot d\vec{A} = s \cdot \int_a^b \mu_0 H(r) dr = s \mu_0 \int_a^b \frac{I}{2\pi r}$$

$$\Rightarrow \underline{\Phi_m = s \frac{\mu_0}{2\pi} \cdot I \cdot \ln\frac{b}{a}} \qquad \left(\Phi_m' = \frac{\Phi_m}{s} = \frac{\mu_0}{2\pi} I \ln\frac{b}{a}\right)$$

Def: $\Phi_m = L \cdot I$ (eller $\mathcal{E} = -\dot{\Phi}_m = -L \cdot \dot{I}$) $\Rightarrow \underline{\underline{L'}} = \frac{\Phi_m'}{I} = \underline{\underline{\frac{\mu_0}{2\pi} \cdot \ln\frac{b}{a}}}$

b) $\underline{\underline{w}} = \frac{1}{2} \vec{B} \cdot \vec{H} = \frac{1}{2} \mu_0 H^2 = \frac{1}{2} \mu_0 \frac{I^2}{(2\pi r)^2} = \underline{\underline{\frac{\mu_0}{8\pi^2} I^2 \cdot \frac{1}{r^2}}}$

Total energi $W = \iiint w \, d\tau = s \cdot \int_a^b w \cdot 2\pi r \cdot dr = s \cdot \frac{\mu_0}{4\pi} I^2 \int_a^b \frac{1}{r} dr = s \underbrace{\frac{\mu_0}{4\pi} I^2 \cdot \ln\frac{b}{a}}_{W'}$ (sylinder)

Dvs $\underline{\underline{W' = \frac{\mu_0}{4\pi} \cdot I^2 \cdot \ln\frac{b}{a}}}$