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Exam in FY3106/FY8303 Phase Transitions and Critical Phenomena

Friday, May 19, 2017
09:00–13:00

Allowed help: Alternativ C

This problem set consists of 2 pages.

Problem 1

When Landau theory is used to calculate the value of critical exponents, one makes the assumption that fluctuations do not contribute to the dominating singularities at the critical point.

- a) Use Landau theory to determine the exponents α and β for the heat capacity and order parameter of an isotropic ferromagnet in d spatial dimensions.
- b) We now include the contribution from the fluctuations assuming that they are Gaussian. The effective Hamiltonian for the spatially fluctuating magnetization is

$$H = \frac{1}{2} \int d\vec{x} \left\{ \left(\vec{\nabla} \vec{m}(\vec{x}) \right)^2 + a\tau \vec{m}(\vec{x})^2 \right\}, \quad (1)$$

where $\tau = (T - T_c)/T_c$ and a is a constant. Determine the thermal exponent y_τ by a suitable renormalization group transformation.

- c) Determine the magnetic exponent y_h and use y_τ and y_h to determine the corrected values for α and β .
- d) What does this calculation say about the fundamental assumption in Landau theory that was quoted in the introduction to the problem?

Given information: $\alpha = 2 - d/y_\tau$ (which we will derive in Problem 3), $\beta = (d - y_h)/y_\tau$ (also to be derived in Problem 3) and $C = -T(\partial^2 \Omega / \partial T^2)_{H=0}$, where Ω is the free energy.

Problem 2

We study the Ising model on the triangular lattice using real-space renormalization theory with triangular cells and the Niemeijer-van Leeuwen majority rule.

- a) Let the original Hamiltonian contain only the nearest neighbor coupling K_1 (with $-1/k_B T$ absorbed into it). Show that the renormalization group transformation in the first cumulant approximation $\mathcal{O}(V^1)$ becomes

$$K'_1 = 2\psi^2(K_1)K_1, \quad (2)$$

where

$$\psi(K_1) = \frac{e^{3K_1} + e^{-K_1}}{e^{3K_1} + 3e^{-K_1}}. \quad (3)$$

- b) Explain the dynamics of this transformation and determine the fixed points.
- c) Add a term including an external magnetic field $h \sum_i s_i$ to the Hamiltonian (with $-1/k_B T$ absorbed). Will the fixed points change? Determine, in the critical fixed point, the magnetic eigenvalue to lowest order $\mathcal{O}(V^0)$.

Problem 3

As a result of a renormalization group transformation, we may write the free energy as

$$\omega(\tau, h) = \omega_0(\tau, h) + \frac{1}{l^d} \omega_s(l^{y_\tau} \tau, l^{y_h} h), \quad (4)$$

where $\tau = (T - T_c)/T_c$, h is the external magnetic field and l the scaling factor.

- a) Use (4) to show that the heat capacity critical exponent $\alpha = 2 - d/y_\tau$.
- b) Derive the hyperscaling relation $2 - \alpha = d\nu$.
- c) Derive $\beta = (d - y_h)/y_\tau$.