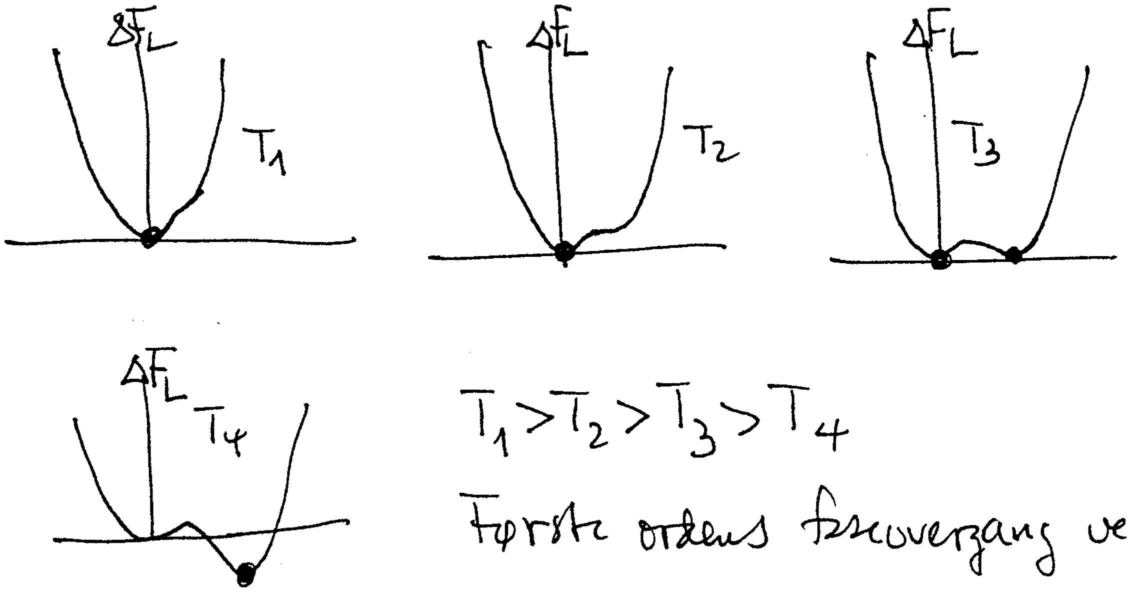
Eksamen Fag 74941 Faseoverg. & krit. fen., 06.06.95. Løsningsforslag

Oppgave I

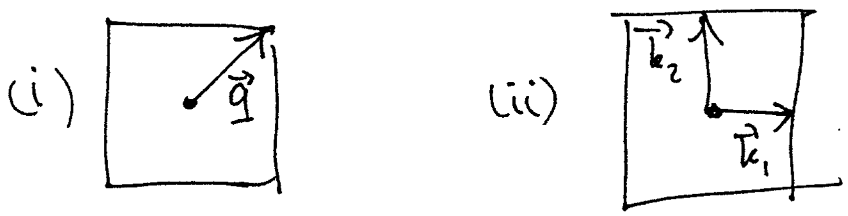
a $\Delta F_L = \frac{r}{2}\psi^2 - \frac{c}{3!}\psi^3 + \frac{u}{4!}\psi^4$



$T_1 > T_2 > T_3 > T_4$

Første ordens faseovergang ved $T = T_3$

b 1. Brillouin sone, kvadratisk gitter



De to mulige stjernene for kontinuerlig faseovergang

Høysymmetripunktene på randa av BZ gir ekstremalpunkter* for alle T

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* I alle funksjoner over BZ, f.eks. ΔF_L.

c Translasjonsinvarians: $T_{\vec{R}} \Delta F_L = \Delta F_L$

$\uparrow$ vektor i direkte gitter

(i) $T_{\vec{R}} \rho_{\vec{q}}^{\,l} = \left(e^{-i\vec{q}\cdot\vec{R}} \rho_{\vec{q}}\right)^l$

$= (-1)^{ml}(-1)^{nl} \rho_{\vec{q}}^{\,l} = \rho_{\vec{q}}^{\,l} \Rightarrow l$ like tall

$\Delta F = \frac{r}{2}\rho_q^2 + \frac{u}{4!}\rho_q^4 + \ldots$ $\boxed{\text{Ising.}}$

$\uparrow = \rho_{-q}\rho_q$

(ii) $\cancel{T_{\vec{R}}\,\rho^l}$ $T_{\vec{R}} \rho_{\vec{k}_1}^{\,l} = \left(e^{-i\vec{k}_1\cdot\vec{R}} \rho_{\vec{k}_1}\right)^l = (-1)^{ml} \rho_{\vec{k}_1}^{\,l}$

$\Rightarrow \boxed{l \text{ like}}$

$T_{\vec{R}} \rho_{\vec{k}_2}^{\,p} = (-1)^{mp} \rho_{\vec{k}_2}^{\,p}$

$\Rightarrow \boxed{p \text{ like}}$

$\rho_{k_1} = \psi_1$

Rotasjon over 90°: $k_1 \rightleftarrows k_2$ $\rho_{k_2} = \psi_2$

$\Delta F = \frac{r}{2}(\psi_1^2 + \psi_2^2) + \frac{u}{4!}(\psi_1^2 + \psi_2^2)^2 + \frac{v}{4!}(\psi_1^4 + \psi_2^4)$

$\boxed{\text{XY med kubisk anisotropi}}$

Oppgave II

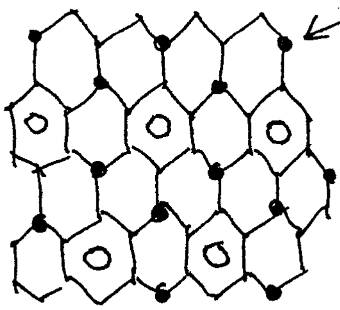
a A: Frastøting som "forbyr" nærmeste nabo, men er tilstrekkelig kortrekkende til å tillate nest nærmeste nabo okkupasjo

B: Frastøting av noe lenger rekkevidde: Nærmeste og 2. nabo forbudt, 3 nabo tillatt.

b A: To ekvivalente undergitre: $\boxed{\text{Ising U-teksse}}$

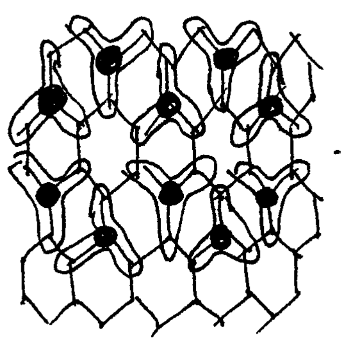
B: 4 ekvivalente undergitre: $\boxed{\text{4 tilst. Potts}}$

Hvorfor 4?

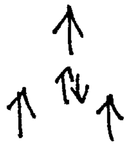


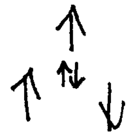
Dette valget er karakterisert ved "gitteret" av O-er. O-ene danner et skjeivt 2×2 gitter, med 4 ekvivalente uavhengige muligheter. qed.

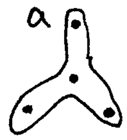
c



• danner åpenbart et bikubegitter, ~~som~~ det underliggende d

 : $e^{3K}+e^{-3K} \equiv x^{3}+x^{-3}$

 : $e^{K}+e^{-K} \equiv x+x^{-1}$



$$\langle S_a \rangle_{s'} = s' \frac{x^{3}+x^{-3}+2(x+x^{-1})-(x+x^{-1})}{x^{3}+x^{-3}+3(x+x^{-1})} \equiv s'\psi(x)$$

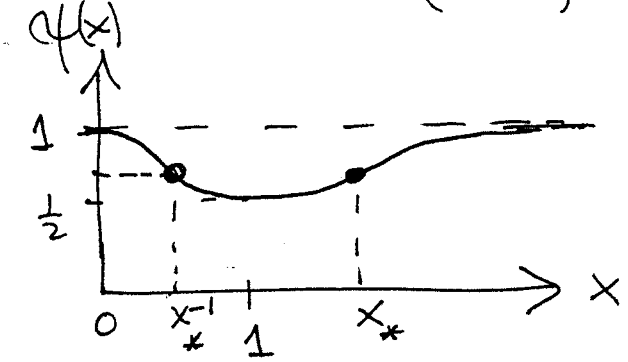
$$x^{3}+x^{-3}+3x+3x^{-1} = (x+x^{-1})^{3}$$

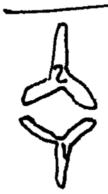
$$(x^{3}+x^{-3}+x+x^{-1}) : (x+x^{-1}) = x^{2}+x^{-2}$$

$$\begin{array}{l} x^{3}+x \\ \hline x^{-3}+x^{-1} \\ x^{-3}+x^{-1} \\ \hline 0 \end{array}$$

$$\supset: \psi(x) = \frac{x^{2}+x^{-2}}{(x+x^{-1})^{2}} \quad \xrightarrow{x\to\infty} 1, \quad \xrightarrow{x\to 0} 1$$

$$\psi(1) = \frac{1}{2}$$





$$\boxed{K' s_1' s_2' = 2K\psi^{2} s_1' s_2' \qquad K' = 2K\psi^{2}}$$

Fikspkt: $K_* = 2K_*\psi^{2}(x_*) \Rightarrow \psi(x_*) = \frac{1}{\sqrt{2}}$

x_* og $\frac{1}{x_*}$ begge fikspunkt

$\boxed{\text{Ising}}$

$\Rightarrow$ K_* og $-K_*$ ferro & antiferrom. fikspunkt

4-tilstands Potts overgangen fanges ikke av denne RG!