

Final exam FY3106/FY8303

Phase Transitions and Critical Phenomena

May 19, 2017

Problem 1

a) Isotropic ferromagnet in
zero external field:

$$\Delta F_L = \frac{\lambda}{2!} (\vec{M})^2 + \frac{\mu}{4!} (\vec{M})^4 + \frac{\nu}{6!} (\vec{M})^6 + \dots$$

$$= \frac{\lambda}{2!} M^2 + \frac{\mu}{4!} M^4 + \dots$$

$$\uparrow |\vec{M}| = M.$$

Dimensionality is irrelevant for
Landau theory.

Minimization:

$$\frac{\partial \Delta F_L}{\partial M} = M \left(\lambda + \frac{\mu}{3!} M^2 \right) = 0$$

$$\uparrow \lambda = a\tau$$

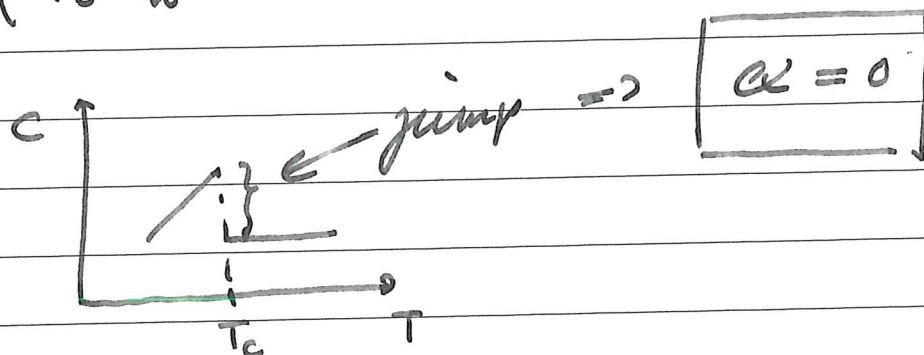
Spontaneous magnetization:

$$M_0 = \begin{cases} 0 & \tau > 0 \\ \sqrt{\frac{6a(-\tau)}{u}} & \tau < 0 \end{cases} \Rightarrow \boxed{\beta = \frac{1}{2}}$$

$$C = \left(\frac{dQ}{dT} \right)_{H=0} = -T \left(\frac{\partial^2 \Omega}{\partial T^2} \right)_{H=0}$$

where $\Omega(T, H) = \min_M \Delta F_L(T, H, M)$

$$C \sim \begin{cases} 0 & \tau > 0 \\ T_c \frac{3a^2}{u} & \tau < 0 \end{cases}$$



b)

$$H = \frac{1}{2} \int d\vec{x} \left\{ (\vec{\nabla} \vec{m}(\vec{x}))^2 + a\tau \vec{m}(\vec{x})^2 \right\}$$

Renormalization group:

$$\vec{x}_e = \vec{x}/e$$

$$\vec{m}_e = c \vec{m}$$

Coefficient in front of $(\vec{\nabla} \cdot \vec{m}(\vec{x}))^2$

rem unchanged.

$$H_c = \frac{1}{2} \int d\vec{x}_c \, l^d \left\{ l^{-2} c^{-2} (\vec{\nabla}_c \cdot \vec{m}_c(\vec{x}_c))^2 + a \tau c^{-2} \vec{m}_c(\vec{x}_c)^2 \right\}$$

$$l^{d-2} c^{-2} = 1 \quad \rightarrow \quad \tau_c = l^d c^{-2} \tau = l^2 \tau$$

$$\Rightarrow \boxed{\gamma_\tau = 2}$$

c)

$$\begin{aligned} \vec{h} \cdot \int d\vec{x} \, \vec{m}(\vec{x}) &\xrightarrow{Rq} \vec{h} \cdot \int d\vec{x}_c \, l^d c^{-1} \vec{m}_c(\vec{x}_c) \\ &= l^{d-\frac{d-2}{2}} \vec{h} \cdot \int d\vec{x}_c \cdot \vec{m}_c(\vec{x}_c) \\ &\rightarrow \vec{h}_c = l^{\frac{d+2}{2}} \vec{h} \end{aligned}$$

$$\Rightarrow \boxed{\gamma_h = \frac{d+2}{2}}$$

Hence,

$$\underline{\underline{d_s = 2 - \frac{d}{\gamma_\tau} = \frac{4-d}{2}}}, \quad \underline{\underline{\beta_s = \frac{d-\gamma_h}{\gamma_\tau} = \frac{d-2}{4}}}$$

d) When $d > 4$, $\alpha_s < \alpha = 0$; $\beta_s > \beta = \frac{1}{2}$.

This means that the Landau contribution dominates over the gaussian fluctuation contribution in both C and M_0 .

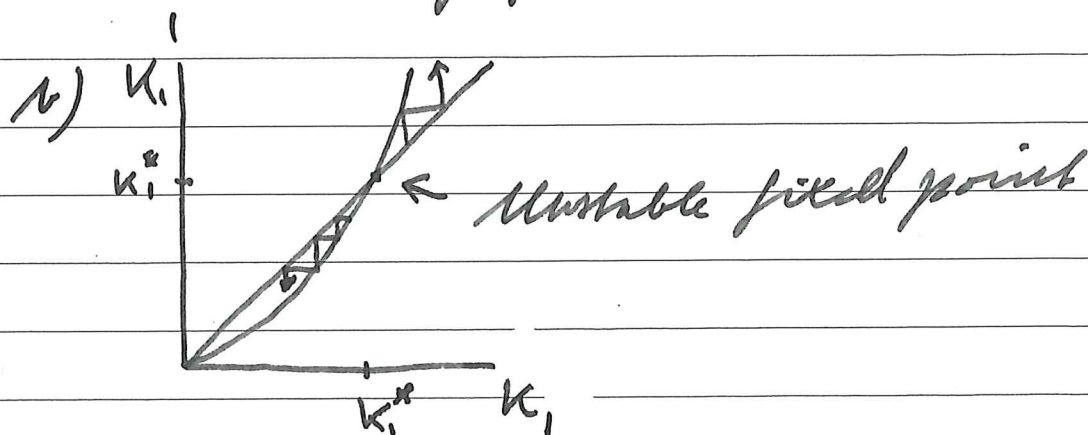
When $d < 4$, $\alpha_s > \alpha = 0$ and $\beta_s < \beta = \frac{1}{2}$

the fluctuation contribution dominates over the Landau contribution.

$\Rightarrow$ Landau ok for $d > d_c = 4$.

Problem 2

a) See derivation in Flange, "60 critical!", pages 135-136.



When $\kappa_1 < \kappa_1^*$, κ_i iterates to zero.

When $\kappa_1 > \kappa_1^*$, κ_i iterates to infinity.

$$\kappa_1^* = 2 \kappa_1^* \psi (\kappa_1^*)^2 \Rightarrow \kappa_1^* = \frac{1}{4} \ln(1 \pm 2\sqrt{2}).$$

c) When $h \neq 0$, the Mig model is not critical. The critical fixed point must therefore be at $h = 0$. Hence, it remains unchanged from the $O(V)$ calculation.

Also the high temperature fixed point will be in $h = 0$ since h as all other coupling constants are reduced with increasing temperature (remember they are scaled with $-1/k_B T$).

The low temperature fixed point is changed, but this fixed point is not interesting.

For g_h , see p. 140 in Flange.

Problem 3

a) $\omega_s(\tau, h=0) \sim \tau^{2-\alpha}$

(Since $C = -T \left(\frac{\partial^2 F}{\partial T^2} \right)_{H=0}$)

$$\omega_s(\tau, 0) = \frac{1}{l^d} \omega_s(l^{y_\tau} \tau, 0)$$

$$\tau^{2-\alpha} \sim l^{-d} (l^{y_\tau} \tau)^{2-\alpha}$$

$$\Rightarrow -d + y_\tau(2-\alpha) = 0$$

$$\Rightarrow \alpha = 2 - \frac{d}{y_\tau}$$

b) Correlation length:

$$\xi(l^{y_\tau} \tau) = \frac{1}{l} \xi(\tau) \Rightarrow \xi(\tau) \sim \tau^{-\nu}$$

$$l^{-y_\tau \nu} \tau^{-\nu} = \frac{1}{l} \tau^{-\nu}$$

$$\Rightarrow \nu = \frac{1}{y_\tau}$$

From a): $\underline{2-\alpha} = \frac{d}{y_\tau} = \underline{d\nu}$

which is the hyperscaling relation.

c) Order parameter $m \sim \tau^\beta \sim \frac{\partial}{\partial h} \psi(\tau, h) \Big|_{h=0}$

$$\omega_s(\tau, h) = \frac{1}{l^d} \omega_s(l^{y_\tau} \tau, l^{y_h} h)$$

$$\tau^\beta \sim l^{-d} l^{y_h} (l^{y_\tau} \tau)^\beta \Rightarrow$$

$$-d + y_h + \beta y_\tau = 0 \Rightarrow$$

$$\underline{\underline{\beta = (d - y_h) / y_\tau}}$$