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Department of Physics, NTNU

Examination paper for **FY3114 / FY8912 Functional materials**

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1 **P1v1-fmat2020**

Optoelectric materials.

Select one alternative

- ☐ QLED and OLED displays are expensive to manufacture
- ☐ Efficiency above 30% is not possible in solar cells
- ☐ Devices based on nanowires are promising in solar cells
- ☐ Quantum effects are well understood in these materials

Topological insulators

Select one alternative:

- ☐ are conducting in the bulk
- ☐ are being used in quantum computation
- ☐ are conducting at the surface
- ☐ already have a wide range of applications

Organic Field Effect Transistors

Select one alternative

- ☐ have by today overcome all performance challenges
- ☐ are expensive to fabricate
- ☐ are being used in display technologies
- ☐ are very stable under ambient conditions

Carbon nanotube transistors

Select one alternative

- ☐ are complicated since the physics is not very well understood
- ☐ may provide fast switching for low electronic currents
- ☐ are hampered by strong electron scattering
- ☐ show very high effective mass

(1 point for each correct answer)

Maximum marks: 4

2 **P2v1-fmat2020**

Materials for optical storage

Select one alternative:

- ☐ can store more data if longer optical wavelengths are employed
- ☐ can be used in long lifetime storage devices
- ☐ must be good thermal conductors
- ☐ can only be used in short lifetime storage devices

Piezoelectric transistors...

Select one alternative

- ☐ represent a mature technology
- ☐ are not suitable for use in touch devices
- ☐ may be made from centrosymmetric materials
- ☐ may be used as strain sensors

Strain effects on transistors

Select one alternative

- ☐ increase leakage currents
- ☐ split degenerate energy levels
- ☐ results in increased phonon scattering
- ☐ reduce mobility of electron and holes

Multiferroic materials for storage devices

Select one alternative

- ☐ may be found in the class of perovskites
- ☐ represents a mature field of engineering
- ☐ cannot be used for Random Access memory (RAM)
- ☐ may be metallic, semiconducting or insulating

(1 point for each correct answer)

Maximum marks: 4

3

P3v1-fmat2020

Spintronic transistors

Select one alternative

- ☐ use less power but are slower than ordinary transistors
- ☐ are already on the marked
- ☐ demonstration devices exists
- ☐ use the spin but not the charge of the electrons

Graphene transistors

Select one alternative

- ☐ utilize high effective mass electrons
- ☐ have electron mobilities almost as high as in GaAs
- ☐ well developed fabrication methods exist
- ☐ are candidates for even smaller electronic components

Ferroelectric memory devices

Select one alternative

- ☐ may be made from centrosymmetric materials
- ☐ may be used in non-volatile memory applications
- ☐ are well suited for low cost production
- ☐ have very high storage densities

Which one of these statements is correct?

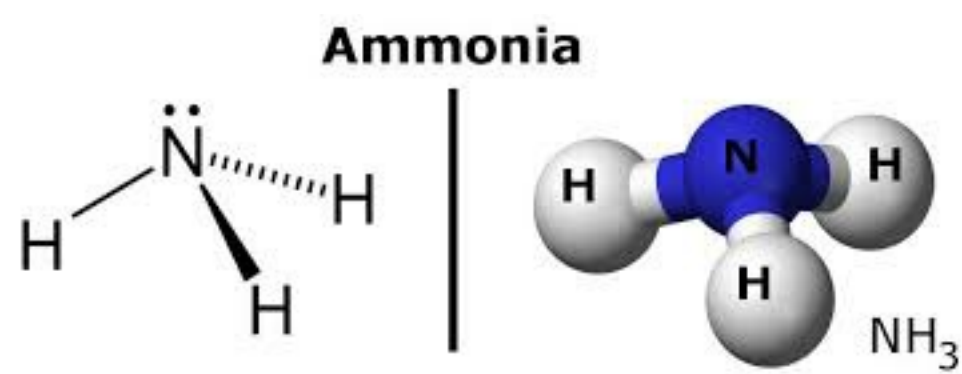
Select one alternative

- ☐ the dielectric constant of a ferroelectric material is low
- ☐ a ferroelectric material is also pyroelectric and piezoelectric
- ☐ a pyroelectric material is also ferroelectric
- ☐ a piezoelectric material is also ferroelectric

(1 point for each correct answer)

Maximum marks: 4

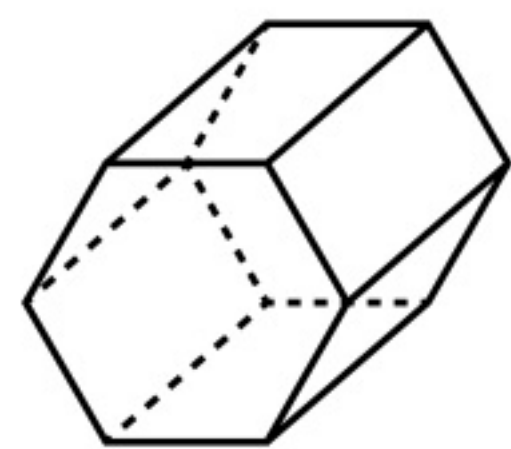
4 **P4v3-fmat2020**



How many symmetry elements are contained in the point group of the NH₃ molecule?

Select one alternative:

- ☐ 8
- ☐ 6
- ☐ 4
- ☐ 2
- ☐ 3



How many symmetry elements are contained in the point group of a regular hexagonal prism as shown in the figure?

Select one alternative

- ☐ 6
- ☐ 24
- ☐ 36
- ☐ 12
- ☐ 18

Maximum marks: 4

5 **P5v1-fmat2020**

A monoclinic crystalline material has dielectric tensor elements $\epsilon_{11}, \epsilon_{12}, \epsilon_{22}, \epsilon_{33} = 2, 1, 4, 3$; respectively. What are the tensor elements for the transformed tensor after an orthogonal transformation that consists of a 45° rotation around the z-axis?

Enter the answers below:

$\epsilon'_{11} =$

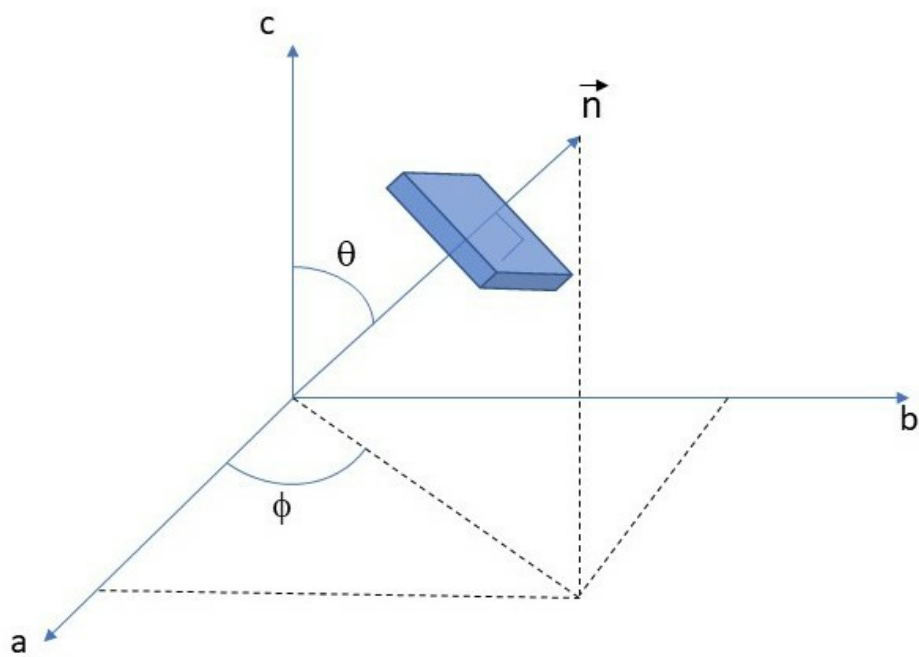
$\epsilon'_{12} =$

$\epsilon'_{22} =$

$\epsilon'_{33} =$

Maximum marks: 4

6 **P6v1-fmat2020**



An orthorhombic crystal is cut in a thin slab where the surface normal is at an angle $\theta = 45^\circ$ with the c-axis, and the angle between the a-axis and the projection of the surface normal in the ab-plane is $\phi = 30^\circ$. There is a temperature gradient across the thin slab. The temperature difference between the big faces of the thin slab is 2 K, and the thickness of the slab is $L = 10\text{ mm}$. The components of the thermal conductivity tensor is: $k_{11}, k_{22}, k_{33} = 8, 4, 2\text{ [W/mK]}$; respectively.

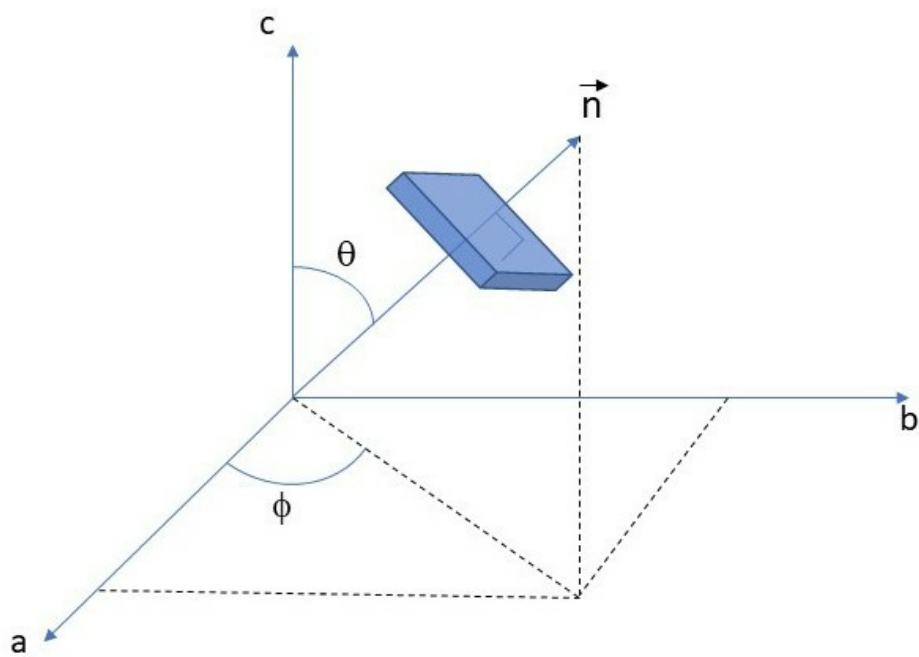
Find the absolute value of the heat flow across the crystal in the direction of the surface normal.

Select one alternative

- ☐ $860W/m^2$
- ☐ $760W/m^2$
- ☐ $1000W/m^2$
- ☐ $900W/m^2$
- ☐ $800W/m^2$

Maximum marks: 4

7 **P6v2-fmat2020**



An orthorhombic crystal is cut in a thin slab where the surface normal is at an angle $\theta = 45^\circ$ with the c-axis, and the angle between the a-axis and the projection of the surface normal in the ab-plane is $\phi = 30^\circ$. There is a temperature gradient across the thin slab. The temperature difference between the big faces of the thin slab is 2 K, and the thickness of the slab is $L = 10\text{ mm}$. The components of the thermal conductivity tensor is: $k_{11}, k_{22}, k_{33} = 6, 5, 1\text{ [W/mK]}$; respectively.

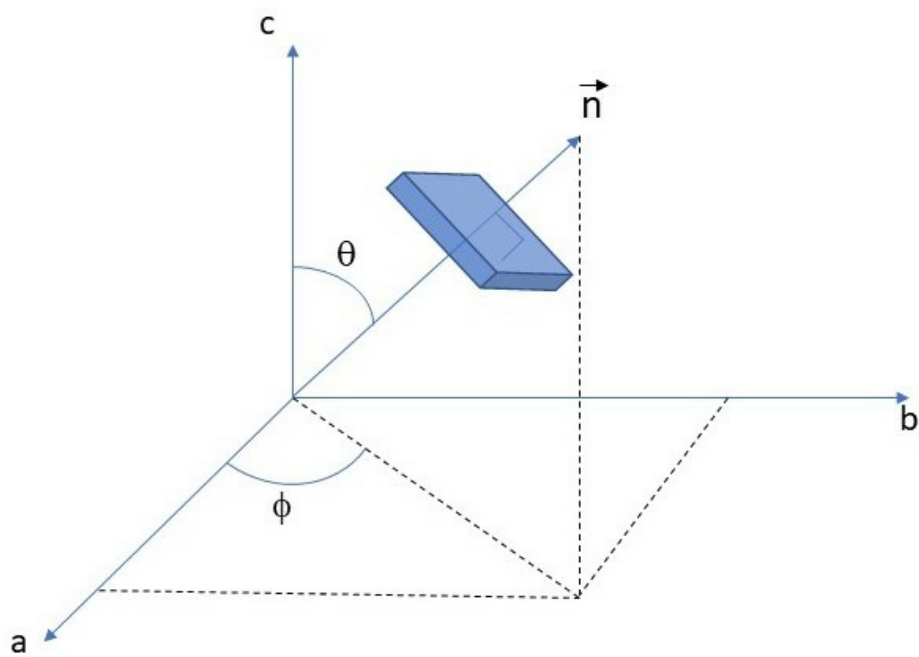
Find the absolute value of the heat flow across the crystal in the direction of the surface normal.

Select one alternative

- ☐ 775W/m^2
- ☐ 800W/m^2
- ☐ 700W/m^2
- ☐ 675W/m^2
- ☐ 900W/m^2

Maximum marks: 4

8 **P6v3-fmat2020**



An orthorhombic crystal is cut in a thin slab where the surface normal is at an angle $\theta = 45^\circ$ with the c-axis, and the angle between the a-axis and the projection of the surface normal in the ab-plane is $\phi = 30^\circ$. There is a temperature gradient across the thin slab. The temperature difference between the big faces of the thin slab is 2 K, and the thickness of the slab is $L = 10\text{ mm}$. The components of the thermal conductivity tensor is: $k_{11}, k_{22}, k_{33} = 4, 2, 4\text{ [W/mK]}$; respectively.

Find the absolute value of the heat flow across the crystal in the direction of the surface normal.

Select one alternative

- ☐ $750W/m^2$
- ☐ $550W/m^2$
- ☐ $675W/m^2$
- ☐ $900W/m^2$
- ☐ $800W/m^2$

Maximum marks: 4

9 **P7v2-fmat2020**

A second rank symmetric tensor for a monoclinic crystal has the following non-zero elements: $A_{11} = 4, A_{12} = 2, A_{22} = 1, A_{33} = 2$. This matrix may be diagonalized by a rotation.

Find the angle of rotation that is required to diagonalize the tensor.

Enter your answer here: $^\circ$.

Maximum marks: 4

10 **P8v1-fmat2020**

An energy band in a semiconductor material is given by $E(k) = ak^2 + bk + c$.

Where $a = 2 \cdot 10^{-37} \text{ Jm}^2$, $b = 3 \cdot 10^{-29} \text{ Jm}$, and $c = 4 \cdot 10^{-25} \text{ J}$.

Find the effective mass of electrons in this energy band in terms of the free electron mass m_0 .

Write the answer here: m_0 .

Maximum marks: 4

11 **P9v1-fmat2020**

The diode equation is given by $I = I_0(e^{eV/k_BT} - 1)$.

Estimate the generating current I_0 in a silicon pn-diode, using the following parameters:

Area of diode $A = 12 \text{ mm}^2$

Donor density $N_D = 2 \cdot 10^{17} \text{ cm}^{-3}$ (assume all states ionized)

Acceptor density $N_A = 4 \cdot 10^{16} \text{ cm}^{-3}$ (assume all states filled)

Electron diffusion coefficient $D_n = 18 \text{ cm}^2/\text{s}$

Hole diffusion coefficient $D_p = 15 \text{ cm}^2/\text{s}$

Electron recombination time $\tau_n = 3 \cdot 10^{-7} \text{ s}$

Hole recombination time $\tau_p = 2 \cdot 10^{-7} \text{ s}$

The generating current is:

Select one alternative:

- ☐ 1,34 nA
- ☐ 0.65 nA
- ☐ 3.53 nA
- ☐ 2.23 nA
- ☐ 1.02 nA

Maximum marks: 4

12 **P9v2-fmat2020**

The diode equation is given by $I = I_0(e^{eV/k_BT} - 1)$.
Estimate the generating current I_0 in a silicon pn-diode, using the following parameters:
Area of diode $A = 12\text{mm}^2$
Donor density $N_D = 2 \cdot 10^{17}\text{cm}^{-3}$ (assume all states ionized)
Acceptor density $N_A = 1 \cdot 10^{16}\text{cm}^{-3}$ (assume all states filled)
Electron diffusion coefficient $D_n = 18\text{cm}^2/\text{s}$
Hole diffusion coefficient $D_p = 15\text{cm}^2/\text{s}$
Electron recombination time $\tau_n = 3 \cdot 10^{-7}\text{s}$
Hole recombination time $\tau_p = 2 \cdot 10^{-7}\text{s}$

The generating current is:

Select one alternative:

- ☐ 2.23 nA
- ☐ 3.53 nA
- ☐ 1,34 nA
- ☐ 0.65 nA
- ☐ 1.02 nA

Maximum marks: 4

13 **P9v3-fmat2020**

The diode equation is given by $I = I_0(e^{eV/k_BT} - 1)$.
Estimate the generating current I_0 in a silicon pn-diode, using the following parameters:
Area of diode $A = 12\text{mm}^2$
Donor density $N_D = 2 \cdot 10^{17}\text{cm}^{-3}$ (assume all states ionized)
Acceptor density $N_A = 8 \cdot 10^{16}\text{cm}^{-3}$ (assume all states filled)
Electron diffusion coefficient $D_n = 18\text{cm}^2/\text{s}$
Hole diffusion coefficient $D_p = 15\text{cm}^2/\text{s}$
Electron recombination time $\tau_n = 3 \cdot 10^{-7}\text{s}$
Hole recombination time $\tau_p = 2 \cdot 10^{-7}\text{s}$

The generating current is:
Select one alternative:

- ☐ 0.61 nA
- ☐ 1,34 nA
- ☐ 2.23 nA
- ☐ 1.02 nA
- ☐ 3.53 nA

Maximum marks: 4

14 **P10v1-fmat2020**

Consider a **GaAs** semiconductor at temperature **300 K**. When the sample is n-doped at $N_d = 2 \cdot 10^{16}\text{cm}^{-3}$ the electron mobility decreases to **6500 cm²/Vs**. Assume that the total scattering rate is the sum of scattering rates of the pure sample and the scattering rate due to impurities.
Calculate the scattering relaxation time for electrons due to impurities.

Enter the answer here: ps.

Maximum marks: 4

15 **P11v1-fmat2020**

Find the energies of electron and hole relative to the respective band edges when a **1.6 eV** photon is adsorbed in **InP** of energy gap **1.34 eV**. The effective masses are given to be $m_e^* = 0.07m_e$ and $m_h^* = 0.4m_e$.

Enter the electron energy here: eV.

Enter the hole energy here: eV.

Maximum marks: 4

16 **P12v1-fmat2020**

By using equilibrium distributions for electron and holes the quasi Fermi levels may be defined:
 $n = N_c e^{(E_{F_n} - E_c)/k_B T}$ and $p = N_v e^{(E_v - E_{F_p})/k_B T}$.

The following parameters for Si at 300 K may be assumed:

Energy gap $E_{gap} = 1.1 eV$

Densities of states at the band edges $N_c = 2.8 \cdot 10^{19} cm^{-3}$ and $N_v = 1.0 \cdot 10^{19} cm^{-3}$.

Find the energy difference between quasi Fermi levels $E_{f_n} - E_{f_p}$ when charge carrier densities $n = 1 \cdot 10^{16} cm^{-3}$ and $p = 3 \cdot 10^{16} cm^{-3}$ are injected into the semiconductor.

Write the energy difference $E_{f_n} - E_{f_p}$ here: eV.

Maximum marks: 4

17 **P13v3-fmat2020**

Calculate the transit time for a hole through a **Si** device of dimension $2 \mu m$ by using the low field approximation. The electric field across the device is $20 kV/cm$. The temperature is 300 K.

Write the transit time here: ps.

Is the low field approximation valid in this case?

Select one alternative

☐ No

☐ Yes

Maximum marks: 4

18 **P14v1-fmat2020**

The piezoelectric tensor of a trigonal crystal of point group 3m is given by:

$$d = \begin{bmatrix} 50 & -50 & 0 & 0 & 40 & 0 \\ 0 & 0 & 0 & 40 & 0 & -100 \\ 30 & 30 & 60 & 0 & 0 & 0 \end{bmatrix}$$
 in units of $10^{-12}C/N$.

Find the polarization when a normal stress $\sigma = 2.3 \cdot 10^4 N/m^2$ is applied in the 1-direction.

Enter P₁ here: 10⁻⁶ C/m²

Enter P₂ here: 10⁻⁶ C/m²

Enter P₃ here: 10⁻⁶ C/m²

Find the polarization when a shear stress $\sigma = 3.1 \cdot 10^4 N/m^2$ is applied around the 3rd axis.

Enter P₁ here: 10⁻⁶ C/m²

Enter P₂ here: 10⁻⁶ C/m²

Enter P₃ here: 10⁻⁶ C/m²

Find the polarization when a shear stress $\sigma = 1.8 \cdot 10^4 N/m^2$ is applied around the 1st axis.

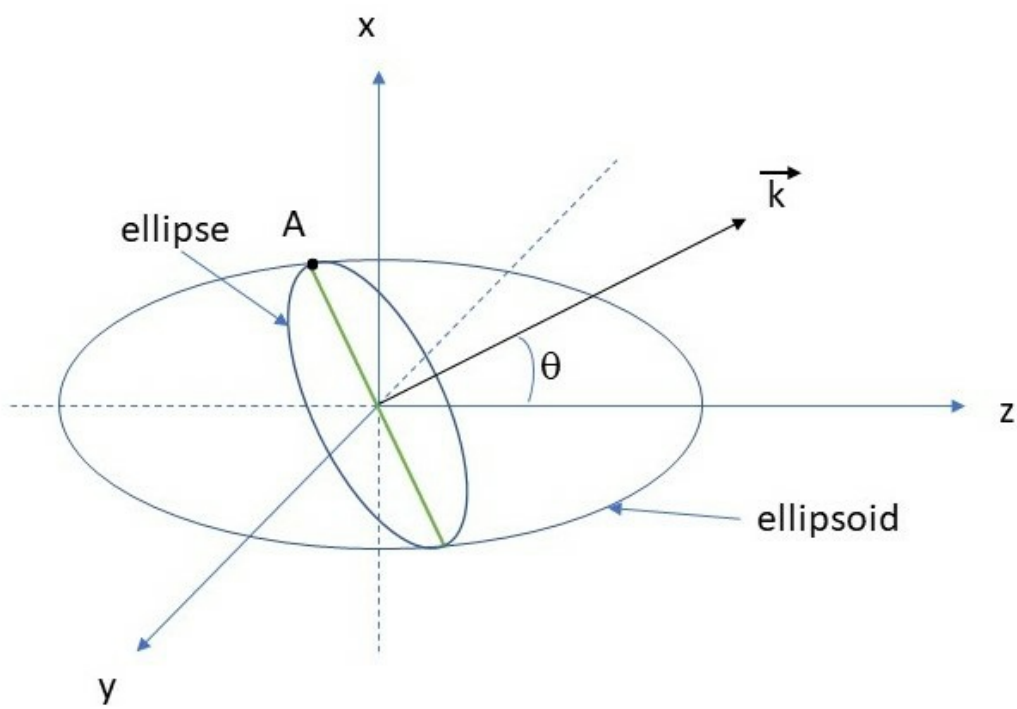
Enter P₁ here: 10⁻⁶ C/m²

Enter P₂ here: 10⁻⁶ C/m²

Enter P₃ here: 10⁻⁶ C/m²

Maximum marks: 4.5

19 **P15v2-fmat2020**



The figure above shows the index ellipsoid of a uniaxial optical active material. The main principal axis is along the z-direction. The propagation direction of the light is perpendicular to an ellipse as shown. The wave vector **k** is in the **xz-plane**. The point **A** is located in the xz-plane and on both the ellipsoid and the ellipse. The angle $\theta = 55^\circ$, and the ordinary and extraordinary indices of refraction are $n_o = 1.53$ and $n_e = 1.17$.

What is the ordinary refractive index $n_o(\theta)$?

Write the answer here:

What is the extraordinary refractive index $n_e(\theta)$?

Write the answer here:

Maximum marks: 4

20 **P16v2-fmat2020**

A Si semiconductor at temperature 300 K is p-doped at a low dopant level.
The acceptor atom density is $N_a = 1 \cdot 10^{13} cm^{-3}$
Assume that 70% of the acceptor states are filled.

What is the conductivity of the semiconductor?

Enter the answer here: $m\Omega^{-1} cm^{-1}$.

What is the conductivity due to electrons?

Enter the answer here: $n\Omega^{-1} cm^{-1}$.

Maximum marks: 4

21 **P17v1-fmat2020**

The **Pockels** tensor elements (contracted notation) for the tetragonal electroactive material BTO are as follows:

$r_{13} = r_{23} = 8 \text{ pm/V}$

$r_{33} = 23 \text{ pm/V}$

$r_{42} = r_{51} = 820 \text{ pm/V}$

other $r_{ij} = 0$

Refractive indices are $n_e = 2.18$ and $n_o = 2.44$

An electric field $E_3 = 5 \cdot 10^7 \text{ V/m}$ is applied in the z-direction.

The impermeability tensor elements are given by $\eta_{ij}(E) = \eta_{ij}(0) + \sum_k r_{ijk} E_k$

Estimate the change in the difference in the refractive indices $\Delta n = \Delta n_o(E) - \Delta n_e(E)$ due to the electric field.

Select one alternative:

- ☐ 0.012
- ☐ 0.015
- ☐ 0.003
- ☐ 0.009
- ☐ 0.006

Maximum marks: 4

22 **P17v2-fmat2020**

The **Pockels** tensor elements (contracted notation) for the tetragonal electroactive material BTO are as follows:

$r_{13} = r_{23} = 8 \text{ pm/V}$

$r_{33} = 23 \text{ pm/V}$

$r_{42} = r_{51} = 820 \text{ pm/V}$

other $r_{ij} = 0$

Refractive indices are $n_e = 2.18$ and $n_o = 2.44$

An electric field $E_3 = 1 \cdot 10^8 \text{ V/m}$ is applied in the z-direction.

The impermeability tensor elements are given by $\eta_{ij}(E) = \eta_{ij}(0) + \sum_k r_{ijk} E_k$

Estimate the change in the difference in the refractive indices $\Delta n = \Delta n_o(E) - \Delta n_e(E)$ due to the electric field.

Select one alternative:

- ☐ 0.012
- ☐ 0.006
- ☐ 0.003
- ☐ 0.015
- ☐ 0.009

Maximum marks: 4

23

P17v3-fmat2020

The **Pockels** tensor elements (contracted notation) for the tetragonal electroactive material BTO are as follows:

$r_{13} = r_{23} = 8 \text{ pm/V}$

$r_{33} = 23 \text{ pm/V}$

$r_{42} = r_{51} = 820 \text{ pm/V}$

other $r_{ij} = 0$

Refractive indices are $n_e = 2.18$ and $n_o = 2.44$

An electric field $E_3 = 2 \cdot 10^8 \text{ V/m}$ is applied in the z-direction.

The impermeability tensor elements are given by $\eta_{ij}(E) = \eta_{ij}(0) + \sum_k r_{ijk} E_k$

Estimate the change in the difference in the refractive indices $\Delta n = \Delta n_o(E) - \Delta n_e(E)$ due to the electric field.

Select one alternative:

- ☐ 0.012
- ☐ 0.009
- ☐ 0.015
- ☐ 0.006
- ☐ 0.003

Maximum marks: 4

24 **P18v2-fmat2020**

A LED based on a GaAs pn-diode has the following parameters (at 300 K):

Donor and acceptor densities $N_d = 5 \cdot 10^{17} \text{ cm}^{-3}$, $N_a = 1 \cdot 10^{16} \text{ cm}^{-3}$

Intrinsic carrier density $n_i^{-3} = 1.84 \cdot 10^6 \text{ cm}^{-3}$. Bandgap $E_{\text{gap}} = 1.41 \text{ eV}$

Electron and hole diffusion coefficients $D_n = 25 \text{ cm}^2/\text{s}$ and $D_p = 12 \text{ cm}^2/\text{s}$

Electron and hole scattering times $\tau_n = \tau_p = 10 \text{ ns}$

LED area $A = 5 \text{ mm}^2$

Injection efficiency is assumed to be 1.0

All donors and acceptors are assumed to be ionized.

The diode equation is given by: $J = J_0(e^{eV/k_B T} - 1)$, where J is the current density.

The external efficiency factor $\eta = 0.1$, and the relation between electron current and number of photons generated per second is given by $I_{\text{photon}} = \eta \cdot I_n / e$ and the optical power $P = \hbar\omega \cdot I_{\text{photon}}$

The LED is to be used in a communication system where binary bits 0 and 1 correspond to power values 1 nW and 50 μW ; respectively.

Calculate the forward bias voltages that are required to send 0 and 1.

Give the answers below:

Forward bias to send 0 is: V

Forward bias to send 1 is: V

Maximum marks: 4

25 **P19v2-fmat2020**

Consider an n-doped Si sample of length $L = 50 \mu\text{m}$ and cross sectional area $A = 20 \mu\text{m}^2$.

The electron density in the conduction band $n = 1 \cdot 10^{17} \text{ cm}^{-3}$. The temperature is 300 K.

The voltage over the length of the sample $V = 2.0 \text{ V}$, and the current $I = 1 \text{ mA}$.

The density of states at the conduction band edge is given to be $N_c = 2.78 \cdot 10^{19} \text{ cm}^{-3}$

What is the position of the Fermi level relative the conduction band edge ($E_F - E_c$) ?

Give the answer here: eV.

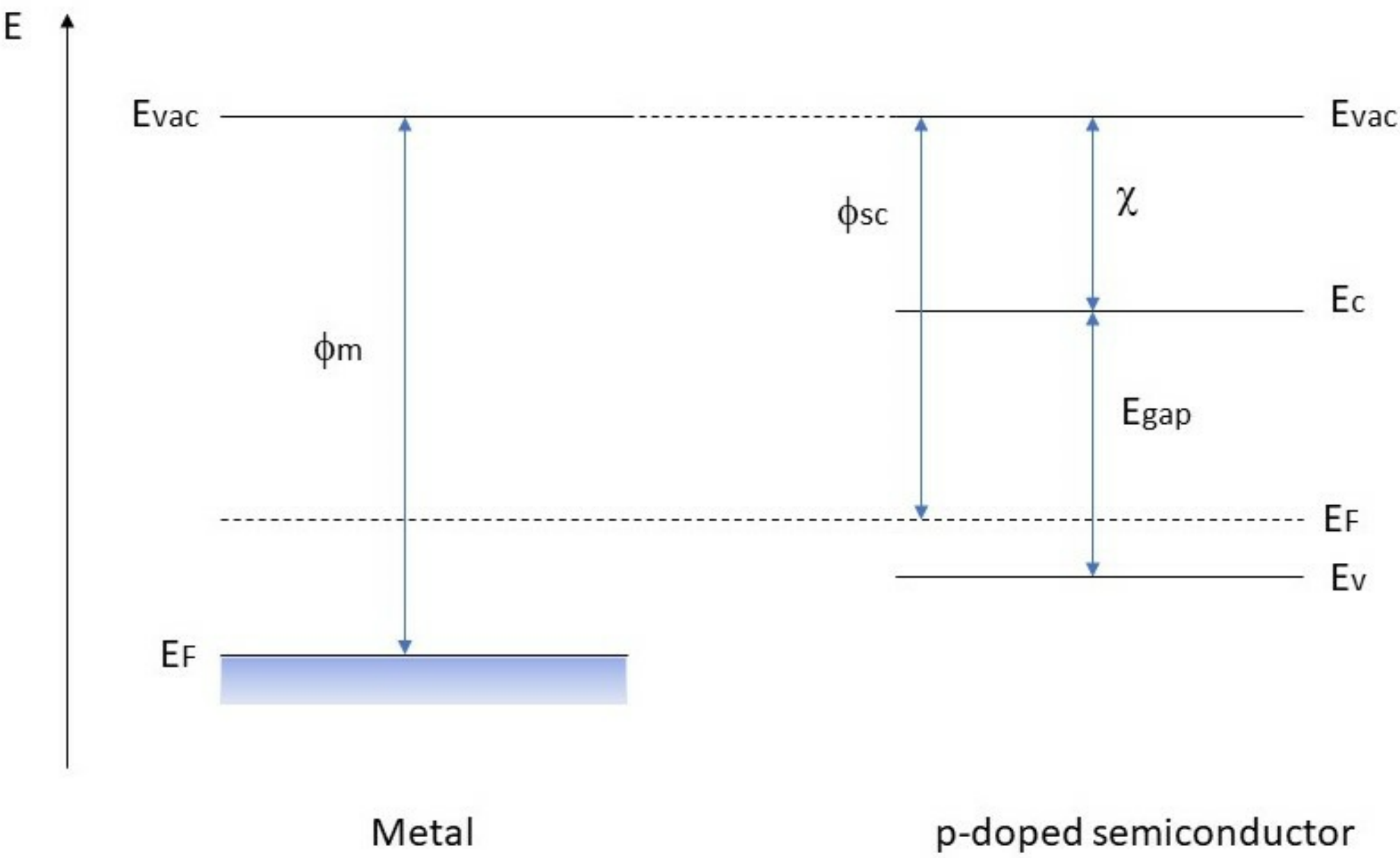
Find the mobility and the average time between collisions for the electrons.

The effective mass of the electrons is $m^* = 0.25 m_e$

The electron mobility is: cm^2/Vs

The average time between scattering is: ps.

Maximum marks: 3



The figure above shows schematic representations of the energy levels in a metal and a p-doped semiconductor. What type of contact is formed when the metal and semiconductor are contacted (e.g. by evaporating the metal onto the semiconductor).

What kind of contact is formed?

Select one alternative:

- ☐ Metallic contact
- ☐ Ohmic contact
- ☐ Schottky contact

Maximum marks: 2

Problem 1-3

Multiple choice

Problem 4Symmetry elements of the NH₃ molecule: 1,3,3²,m,m,m

Regular hexagonal prism has 24 elements in the point group

(2 equivalent points near each corner x 12 corners)

Problem 5Dielectric tensor of a monoclinic material $\epsilon = \begin{bmatrix} \epsilon_{11} & \epsilon_{12} & 0 \\ \epsilon_{12} & \epsilon_{22} & 0 \\ 0 & 0 & \epsilon_{33} \end{bmatrix}$ Rotation of 45° around the z-axis: $x' = \frac{1}{\sqrt{2}}x + \frac{1}{\sqrt{2}}y$, and $y' = -\frac{1}{\sqrt{2}}x + \frac{1}{\sqrt{2}}y$, which gives

$$x'x' = \frac{1}{2}(x^2 + 2xy + y^2) \Rightarrow \epsilon'_{11} = \frac{1}{2}(\epsilon_{11} + 2\epsilon_{12} + \epsilon_{22})$$

$$x'y' = \frac{1}{2}(x+y)(y-x) = \frac{1}{2}(y^2 - x^2) \Rightarrow \epsilon'_{12} = \frac{1}{2}(\epsilon_{22} - \epsilon_{11})$$

$$y'y' = \frac{1}{2}(x^2 - 2xy + y^2) \Rightarrow \epsilon'_{22} = \frac{1}{2}(\epsilon_{11} - 2\epsilon_{12} + \epsilon_{22})$$

$$V1: \epsilon_{11}', \epsilon_{12}', \epsilon_{22}', \epsilon_{33}' = 4, 1, 2, 3$$

$$V2: \epsilon_{11}', \epsilon_{12}', \epsilon_{22}', \epsilon_{33}' = 7, -1, -1, 1$$

$$V3: \epsilon_{11}', \epsilon_{12}', \epsilon_{22}', \epsilon_{33}' = 4, -1, 0, 5$$

Problem 6.

The temperature gradient has components along the a-, b-, and c- axes given by

$$\left(\frac{\delta T}{\delta x}\right) = \frac{\Delta T}{L} \sin\theta \cos\varphi, \quad \left(\frac{\delta T}{\delta y}\right) = \frac{\Delta T}{L} \sin\theta \sin\varphi, \quad \left(\frac{\delta T}{\delta z}\right) = \frac{\Delta T}{L} \cos\theta$$

The heat flow is given by $\vec{h} = -k\nabla T$, or $h_i = -\sum_j k_{ij} \left(\frac{\delta T}{\delta x_j}\right)$ which gives $h_1 = -k_{11}\left(\frac{\delta T}{\delta x}\right)$, $h_2 = -k_{22}\left(\frac{\delta T}{\delta y}\right)$, $h_3 = -k_{33}\left(\frac{\delta T}{\delta z}\right)$ (since other $k_{ij} = 0$).Since $\vec{h} = -k\nabla T$ the magnitude of h along the surface normal is given by

$$h_n = \frac{\vec{h} \cdot \nabla T}{|\nabla T|} = \sum_i h_i \left(\frac{\delta T}{\delta x_i}\right) / |\nabla T| = \sum_i h_i \left(\frac{\delta T}{\delta x_i}\right) \frac{L}{\Delta T} = -\sum_{ij} k_{ij} \left(\frac{\delta T}{\delta x_j}\right) \left(\frac{\delta T}{\delta x_i}\right) \frac{L}{\Delta T}$$

$$h_n = -k_{11} \left(\frac{\delta T}{\delta x_1}\right)^2 \frac{L}{\Delta T} - k_{22} \left(\frac{\delta T}{\delta x_2}\right)^2 \frac{L}{\Delta T} - k_{33} \left(\frac{\delta T}{\delta x_3}\right)^2 \frac{L}{\Delta T}$$

$$h_n = -k_{11} \left(\frac{\Delta T}{L} \sin\theta \cos\varphi\right)^2 \frac{L}{\Delta T} - k_{22} \left(\frac{\Delta T}{L} \sin\theta \sin\varphi\right)^2 \frac{L}{\Delta T} - k_{33} \left(\frac{\Delta T}{L} \cos\theta\right)^2 \frac{L}{\Delta T}$$

$$h_n = -k_{11} \frac{\Delta T}{L} \sin^2 \theta \cos^2 \varphi - k_{22} \frac{\Delta T}{L} \sin^2 \theta \sin^2 \varphi - k_{33} \frac{\Delta T}{L} \cos^2 \theta$$

$$h_n = -k_{11} \frac{2}{0.01} \frac{1}{2} \frac{3}{4} - k_{22} \frac{2}{0.01} \frac{1}{2} \frac{1}{4} - k_{33} \frac{2}{0.01} \frac{1}{2}$$

$$V1: h_n = 900 \text{ W/m}^2$$

$$V2: h_n = 675 \text{ W/m}^2$$

$$V3: h_n = 750 \text{ W/m}^2$$

Problem 7

Rotation around the z-axis gives

$$A'_{12} = (A_{22} - A_{11}) \frac{1}{2} \sin 2\theta + A_{12} \cos 2\theta \text{ therefore } \tan 2\theta = \frac{2A_{12}}{A_{11} - A_{22}} = -2$$

$$V1: \theta = -31.7^\circ$$

$$V2: \theta = 26.6^\circ$$

$$V3: \theta = 42.2^\circ$$

Problem 8

The effective mass is $m^* = \left(\frac{1}{\hbar^2} \frac{\partial^2 E}{\partial k^2} \right)^{-1}$ and $E(k) = ak^2 + bk + c$

$$\text{which gives } m^* = \frac{\hbar^2}{2a}$$

$$V1: m^* = 0.031 m_e$$

$$V2: m^* = 0.020 m_e$$

$$V3: m^* = 0.012 m_e$$

Problem 9

The generating current is given by $I_0 = A \left(\frac{eD_n n_p}{L_n} + \frac{eD_p p_n}{L_p} \right)$ where $L_n = \sqrt{D_n \tau_n}$ and $L_p = \sqrt{D_p \tau_p}$

Using $n_n p_n = n_p p_p = n_i^2 = N_a p_n = n_p N_a$ we get

$$I_0 = A e n_i^2 \left(\frac{D_n}{L_n N_a} + \frac{D_p}{L_p N_d} \right)$$

$$V1: I_0 = 1.02 \text{ nA}$$

$$V2: I_0 = 3.53 \text{ nA}$$

$$V3: I_0 = 0.605 \text{ nA}$$

Problem 10

The scattering relaxation time is given by $\tau_{sc} = \frac{m^* \mu_n}{e}$

Scattering rates are related as $r_{doped} = r_{intrinsic} + r_{impurity} = \frac{1}{\tau_{intrinsic}} + \frac{1}{\tau_{impurity}} = \frac{1}{\tau_{doped}}$

Therefor $\tau_{impurity} = \left(\frac{1}{\tau_{doped}} - \frac{1}{\tau_{intrinsic}} \right)^{-1} = \frac{m^*}{e} \left(\frac{1}{\mu_{doped}} - \frac{1}{\mu_{intrinsic}} \right)^{-1} = 1.32 \cdot 10^{-12} s$

V1: $t = 1.321$ ps

V2: $\tau = 0,915$ ps

V3: $\tau = 0.671$ ps

Problem 11

We have $\hbar\omega = E_e - E_h = E_{gap} + \frac{\hbar^2 k^2}{2m_r^*}$ which gives $\frac{\hbar^2 k^2}{2m_r^*} = \hbar\omega - E_{gap}$

Therefore $E_e - E_c = \frac{m_r^*}{m_e^*} (\hbar\omega - E_{gap})$ and $E_h - E_v = -\frac{m_r^*}{m_h^*} (\hbar\omega - E_{gap})$

V1: $E_e = 0.22$ eV, and $E_h = -0.039$ eV

V2: $E_e = 0.39$ eV, and $E_h = -0.069$ eV

V3: $E_e = 0.56$ eV, and $E_h = -0.098$ eV

Problem 12

We have $n = N_c e^{(E_{fn} - E_c)/k_B T}$ and $p = N_v e^{(E_v - E_{fp})/k_B T}$

This gives $E_{fn} - E_{fp} = E_c - E_v + k_B T (\ln \left(\frac{n}{N_c} \right) - \ln \left(\frac{p}{N_v} \right))$

V1: $E_{fn} - E_{fp} = 0.74$ eV

V2: $E_{fn} - E_{fp} = 0.79$ eV

V3: $E_{fn} - E_{fp} = 0.84$ eV

Problem 13

Transit time $t = \frac{L}{v_d}$ and the low field approximation $v_d = \mu_p E$ gives $t = \frac{L}{\mu_p E}$

If $v_d < 10^7$ cm/s the low field approximation is assumed to be valid.

V1: $t = 114$ ps, $v_d = 0.18e7$ cm/s, low-field approx. is valid

V2: $t = 57$ ps, $v_d = 0.35e7$ cm/s, low-field approx. is valid

V3: $t = 28$ ps, $v_d = 0.70e7$ cm/s, low-field approx. is valid

Problem 14

Piezoelectric tensor

1) $P_1 = d_{11}\sigma_1 = 1.15e-6$, $P_2 = d_{21}\sigma_1 = 0$, $P_3 = d_{31}\sigma_1 = 0.69e-6$

2) $P_1 = d_{16}\sigma_6 = 0$, $P_2 = d_{26}\sigma_6 = -3.1e-6$, $P_3 = d_{36}\sigma_6 = 0$

3) $P_1 = d_{14}\sigma_4 = 0$, $P_2 = d_{24}\sigma_4 = 0.72e-6$, $P_3 = d_{34}\sigma_4 = 0$

V1=V2=V3:

1) $P_1 = 1.15e-6 \text{ C/m}^2$, $P_2 = 0$, $P_3 = 0.69e-6 \text{ C/m}^2$

2) $P_1 = 0$, $P_2 = -3.1e-6 \text{ C/m}^2$, $P_3 = 0$

3) $P_1 = 0$, $P_2 = 0.72e-6 \text{ C/m}^2$, $P_3 = 0$

Problem 15

Uniaxial optic active material $\frac{1}{n_e(\theta)^2} = \frac{\cos^2\theta}{n_o^2} + \frac{\sin^2\theta}{n_e^2}$

We get $n_e(\theta) = 1/\sqrt{\frac{\cos^2\theta}{n_o^2} + \frac{\sin^2\theta}{n_e^2}} = 1.44$ ($n_o = 1.53$ and $n_e = 1.17$)

V1: $n_e(\theta) = 1.38$, and $n_o(\theta) = 1.53$

V2: $n_e(\theta) = 1.26$, and $n_o(\theta) = 1.53$

V3: $n_e(\theta) = 1.19$, and $n_o(\theta) = 1.53$

Problem 16

Conductivity of p-doped Si

$$\sigma = \sigma_n + \sigma_p = n_p e \mu_n + p_p e \mu_p = \frac{n_i^2}{(0.7N_a)} e \mu_n + (0.7N_a) e \mu_p, \text{ using } n_p p_p = n_i^2 = n_p (0.7N_a)$$

$$\sigma_n = \frac{n_i^2}{(0.7N_a)} e \mu_n$$

V1: $\rho = 3.9e-5$, $\rho_e = 5.1e-8 \text{ } \Omega^{-1}\text{cm}^{-1}$

V2: $\rho = 7.8e-4$, $\rho_e = 2.6e-9 \text{ } \Omega^{-1}\text{cm}^{-1}$

V3: $\rho = 1.2e-2$, $\rho_e = 1.7e-10 \text{ } \Omega^{-1}\text{cm}^{-1}$

Problem 17

Electroactive optic material BTO

Impermeability tensor elements $\eta_{ij} = \eta_{ij}^0 + r_{ij3}E_3$

Index ellipsoid given by $\sum_{ij} \eta_{ij} x_i x_j = 1$

This gives $\eta_{11}^0 x^2 + \eta_{22}^0 y^2 + \eta_{33}^0 z^2 + r_{113}E_3 x^2 + r_{223}E_3 y^2 + r_{333}E_3 z^2 = 1$

Therefore $\left(\frac{1}{n_o^2} + r_{13}E_3\right)x^2 + \left(\frac{1}{n_o^2} + r_{13}E_3\right)y^2 + \left(\frac{1}{n_e^2} + r_{33}E_3\right)z^2 = 1$

And change in refractive index in the E-field is $\Delta\left(\frac{1}{n_o^2}\right) = r_{13}E_3$ and $\Delta\left(\frac{1}{n_e^2}\right) = r_{33}E_3$

Which gives $\Delta n_o = -\frac{1}{2}n_o^3r_{13}E_3$ and $\Delta n_e = -\frac{1}{2}n_e^3r_{33}E_3$

And $\Delta n = \Delta n_o - \Delta n_e = -\frac{1}{2}E_3(n_o^3r_{13} + n_e^3r_{33})$

V1: $\Delta n = 0.003$

V2: $\Delta n = 0.006$

V3: $\Delta n = 0.012$

Problem 18

GaAs pn diode for LED

$$I_n = AeD_n \frac{n_p}{L_n} (e^{\frac{eV}{k_B T}} - 1) \quad \text{where} \quad n_p = \frac{n_i^2}{N_a} \quad \text{and} \quad L_n = \sqrt{D_n \tau_n}$$

Furthermore, for the optical power $P = \hbar\omega \cdot I_{\text{photon}} = \eta \hbar\omega I_n / e$

We get $V = \frac{k_B T}{e} \ln (PL_n / AD_n n_p \eta \hbar\omega)$

V1: $V(0) = 0.68 \text{ V}$, $V(1) = 0.96 \text{ V}$

V2: $V(0) = 0.64 \text{ V}$, $V(1) = 0.92 \text{ V}$

V3: $V(0) = 0.60 \text{ V}$, $V(1) = 0.88 \text{ V}$

Problem 19

Fermi level in n-doped Si

$$n = N_c e^{(E_c - E_F)/k_B T} \quad \text{gives} \quad E_F - E_c = k_B T \ln\left(\frac{n}{N_c}\right)$$

Resistance $R = \frac{V}{I} = \frac{1}{\sigma A} \frac{L}{A}$ and $\sigma = ne\mu$ gives $\mu = \frac{\sigma}{ne} = \frac{LI}{V A ne}$

Time between collisions $\tau = \frac{m^* \mu}{e}$

V1=V2=V3: $E_F - E_c = -0.15 \text{ eV}$, $m = 781 \text{ cm}^2/\text{Vs}$, $t = 1.1 \times 10^{-13} \text{ s}$

Problem 20

Ohmic contact is formed when $\phi_m > \phi_{sc}$ for p-type SC

Schottky contact is formed when $\phi_m > \phi_{sc}$ for n-type SC