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Department of physics

Examination paper for FY3114 / FY8912 Functional materials

Examination date: December 19, 2023

Examination time (from-to): 15:00 - 19:00

Permitted examination support material:

C: Specified printed support material is allowed. A specific basic calculator is allowed.
Allowed: Mathematical formulas (Rottmann or equivalent). English dictionary.

Academic contact during examination: Steinar Raaen, **Phone:** 482 96 758

Academic contact present at the exam location: No

OTHER INFORMATION

Get an overview of the question set before you start answering the questions.

Read the questions carefully and make your own assumptions. If a question is unclear/vague, make your own assumptions and specify them in your answer. The academic person is only contacted in case of errors or insufficiencies in the question set. Address an invigilator if you suspect errors or insufficiencies. Write down the question in advance.

No hand drawings: This exam does not include hand drawings. If you receive hand drawing sheets, this is by mistake. **You will not be able to submit the sheets, and they will not be graded.**

Notifications: If there is a need to send a message to the candidates during the exam (e.g. if there is an error in the question set), this will be done by sending a notification in Inspira. A dialogue box will appear. You can re-read the notification by clicking the bell icon in the top right-hand corner of the screen.

Withdrawing from the exam: If you become ill or wish to submit a blank test/withdraw from the exam for another reason, go to the menu in the top right-hand corner and click "Submit blank". This cannot be undone, even if the test is still open.

Access to your answers: After the exam, you can find your answers in the archive in Inspira. Be aware that it may take a working day until any hand-written material is available in the archive.

1 P1-FY3114h2023

Materials for phase change memories (PCMs)

Select one alternative

- ☐ dopants have been proved to decrease the electrical resistivity of the crystalline state
- ☐ a detailed model for the amorphous state is missing
- ☐ PCMs may not be used for non-volatile memory devices
- ☐ the problem of volume change between the crystalline and amorphous phases has been solved

Topological insulators

Select one alternative:

- ☐ are conducting in the bulk
- ☐ are being used in quantum computation
- ☐ already have a wide range of applications
- ☐ are conducting at the surface

Spintronic transistors

Select one alternative

- ☐ demonstration devices exist
- ☐ are already on the market
- ☐ use the spin but not the charge of the electron
- ☐ use less power but are slower than ordinary transistors

Ferroelectric memory devices

Select one alternative

- ☐ may be used in non-volatile memory devices
- ☐ are well suited for low cost production
- ☐ have very high storage densities
- ☐ may be made from centrosymmetric materials

(1 point for each correct answer)

Maximum marks: 4

2 P2-FY3114h2023

Materials for optical storage

Select one alternative:

- ☐ can store more data if longer optical wavelengths are employed
- ☐ can be used in long lifetime storage devices
- ☐ must be good thermal conductors
- ☐ can only be used in short lifetime storage devices

Piezoelectric transistors...

Select one alternative

- ☐ represent a mature technology
- ☐ may be used as strain sensors
- ☐ are not suitable for use in touch devices
- ☐ may be made from centrosymmetric materials

Graphene transistors

Select one alternative

- ☐ utilize high effective mass electrons
- ☐ have electron mobilities almost as high as in Si
- ☐ are easy to functionalize by chemical means
- ☐ are candidates for high speed computing applications

Which statement is correct?

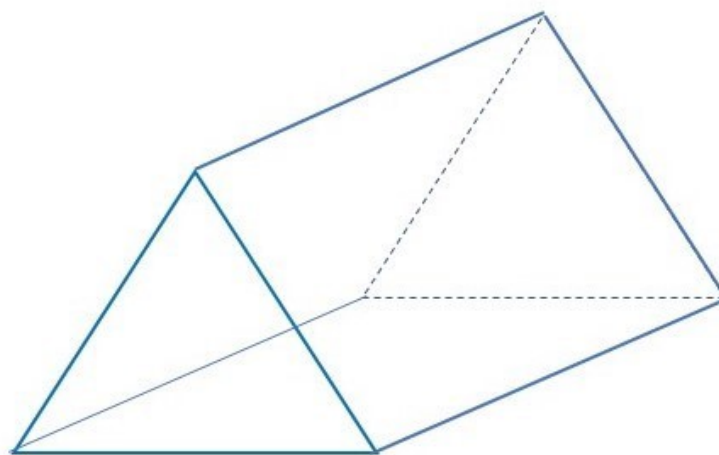
Select one alternative

- ☐ a piezoelectric material is also ferroelectric
- ☐ a pyroelectric material is also ferroelectric
- ☐ the dielectric constant of a ferroelectric material is low
- ☐ a ferroelectric material is also pyroelectric and piezoelectric

(1 point for each correct answer)

Maximum marks: 4

3 P3-FY3114h2023



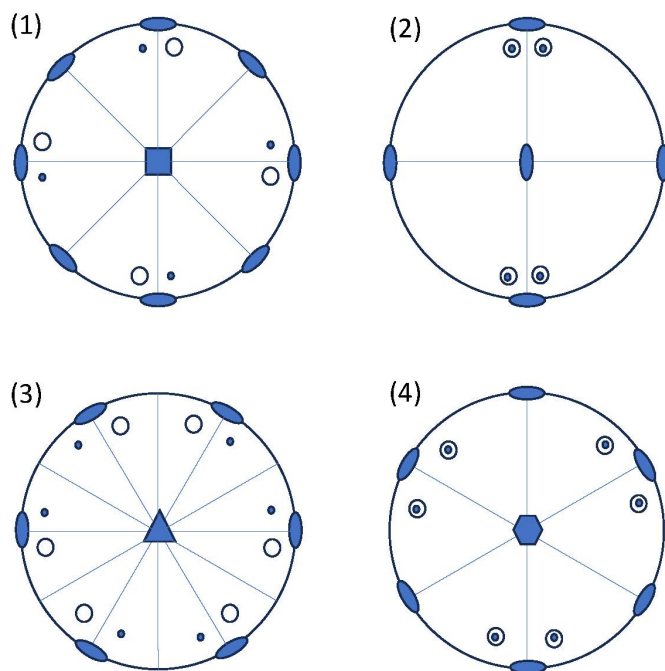
Regular triangular prism

List the complete set of symmetry elements in the point group of the regular triangular prism as shown in the figure above. (Show rotation-inversion symmetry elements as $-n$, where n indicates the rotation, and show multiple operations by n^2 , n^3 , etc).

Fill in your answer here

Maximum marks: 4

4 P4-FY3114h2023

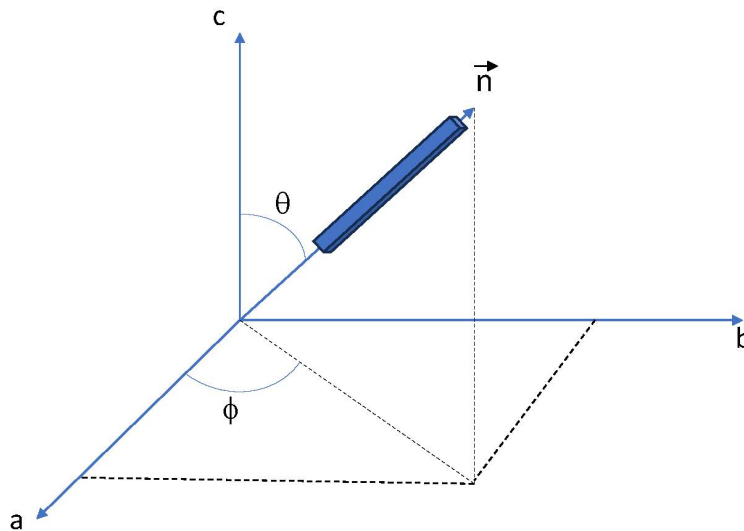


Identify the point group that corresponds to each of the stereograms (2D projections) shown above.

Fill in your answer here

Maximum marks: 4

5 P5-FY3114h2023



A hexagonal crystal is cut in a thin rod with a direction at an angle $\theta = 54^\circ$ with the c-axis, and the angle between the a-axis and the projection of the direction of the rod in the ab-plane is $\phi = 48^\circ$.

A current is flowing along the rod. Find the electrical resistivity of the rod when the components of the resistivity tensor are $\rho_{11} = 5 \cdot 10^{-8} \Omega \text{cm}$ and $\rho_{33} = 3 \cdot 10^{-8} \Omega \text{cm}$

Select one alternative

- ☐ $4.45 \cdot 10^{-8} \Omega \text{cm}$
- ☐ $4.05 \cdot 10^{-8} \Omega \text{cm}$
- ☐ $4.31 \cdot 10^{-8} \Omega \text{cm}$
- ☐ $3.98 \cdot 10^{-8} \Omega \text{cm}$
- ☐ $3.27 \cdot 10^{-8} \Omega \text{cm}$

Maximum marks: 4

6 P6-FY3114h2023

The dielectric tensor of a monoclinic crystal is given by:

$$\epsilon = \begin{bmatrix} 4 & 3 & 0 \\ 3 & 2 & 0 \\ 0 & 0 & 1 \end{bmatrix}, \text{ in units of } 10^{-11} \text{ F/m}$$

This tensor can be diagonalized by rotation by an angle θ around the z-axis.
Find the angle θ .

The angle is:

Select one alternative:

☐ 56°

☐ 36°

☐ 72°

☐ 48°

☐ 28°

Maximum marks: 4

7 P7-FY3114h2023

Ge has a bandgap of 0.7 eV. Find the energy of the electron and the hole relative to the respective band edges that result when a 1.2 eV photon is absorbed.

The energy of the electron is: eV

The energy of the hole is: eV

Maximum marks: 4

8 P8-FY3114h2023

A metal-semiconductor junction is made between Mn and n-doped GaAs. The work function of Mn is 3.8 eV, the work function of the n-doped GaAs is 4.2 eV, and the electron affinity of GaAs is 4.1 eV. What is the size of the barrier height at the metal-semiconductor junction?

Select one alternative:

- ☐ + 0.4 eV
- ☐ + 0.3 eV
- ☐ - 0.4 eV
- ☐ - 0.1 eV
- ☐ + 0.1 eV
- ☐ - 0.3 eV

Which type of contact is formed between Mn and n-doped GaAs?

Select one alternative

- ☐ Ohmic contact
- ☐ Schottky contact

Maximum marks: 4

9 P9-FY3114h2023

An InP semiconductor is n-doped with an electron concentration of $N_D = 3 \cdot 10^{17} \text{ cm}^{-3}$. Find the conductivities σ_n (electrons) and σ_p (holes) of the sample in units $(\Omega \text{ cm})^{-1}$.

The $\ln$ (conductivity due to the electrons) $\ln(\sigma_n) =$.

The $\ln$ (conductivity due to holes) $\ln(\sigma_p) =$.

Maximum marks: 4

10 P10-FY3114h2023

Mention 3 applications in which soft ferromagnetic materials are used.

Fill in your answer here

List 3 ferromagnetic elements.

Fill in your answer here

Maximum marks: 4

11 P11-FY3114h2023

A cubic electro-optic active material has inversion symmetry. An electric field $E = 1.5 \cdot 10^6 \text{ V/m}$ is applied in the z-direction. The refractive index in the absence of an electric field is $n = 2.3$. The influence of the electric field on the impermeability tensor is given by the Kerr effect.

$$\eta_{ij}(E) = \eta_{ij}^0 + \sum_{kl} s_{ijkl} E_k E_l, \text{ where } \eta_{11}^0 = \eta_{22}^0 = \eta_{33}^0 = 1/n^2, \text{ and others } \eta_{ij}^0 = 0$$

The only non-zero elements of the fourth rank tensor are:

$$s_{11} = s_{22} = s_{33} = 2.0 \cdot 10^{-15} (\text{m/V})^2$$

$$s_{12} = s_{13} = s_{23} = 3.5 \cdot 10^{-15} (\text{m/V})^2$$

$$s_{44} = s_{55} = s_{66} = 1.5 \cdot 10^{-15} (\text{m/V})^2$$

(using contracted notation i.e. $s_{11} = s_{1111}$ and $s_{12} = s_{1122}$, etc).

What is the difference of the extraordinary and ordinary index of refraction $\Delta n = n_e - n_o$?

Select one alternative:

- ☐ 0.09
- ☐ 0.003
- ☐ 0.02
- ☐ 0.04
- ☐ 0.008

(4 points for correct answer)

Maximum marks: 4

12 P12-FY3114h2023

The inverse piezoelectric effect is given by $\epsilon_{ij} = d_{ijk}E_k$ where ϵ_{ij} is the strain tensor, d_{ijk} is the inverse piezoelectric tensor, and E_k is the electric field.

We consider an α -quartz single crystal (trigonal crystal, point group 32).

The inverse piezoelectric tensor elements are given by:

(using contracted notation i.e. $d_{111} = d_{11}, d_{41} = d_{231}$, etc):

$$d_{11} = -d_{21} = 2.27 \cdot 10^{-12} \text{ m/V}$$

$$d_{41} = -d_{52} = -0.67 \cdot 10^{-12} \text{ m/V}$$

$$d_{62} = -2d_{11}$$

$$\text{others } d_{ij} = 0$$

An electric field $E = 5 \cdot 10^7 \text{ V/m}$ is applied in the x-direction (along the a-axis of the crystal).

Find the resulting strain in the x, y and z-directions.

The strain in the x-direction is: $\cdot 10^{-6}$

The strain in the z-direction is: $\cdot 10^{-6}$

Maximum marks: 4

13 **P13-FY3114h2023**

A solar cell consists of a Ge pn-diode of area $A = 2\text{cm}^2$. The temperature is 300 K. The total current when the diode is connected to an external load is given by

$$I = I_L - I_0(e^{eV/k_B T} - 1)$$

where I_L is the photocurrent and I_0 is the diffusion current of electrons and holes. Find the open circuit ($I = 0$) voltage V_{OC} for the solar cell.

Parameter values are:

Density of electrons and holes: $n_n = 5 \cdot 10^{18}\text{cm}^{-3}$ and $p_p = 3 \cdot 10^{18}\text{cm}^{-3}$

Electron diffusion coefficients: $D_n = 7\text{cm}^2/\text{s}$ and $D_p = 4\text{cm}^2/\text{s}$

Electron and hole recombination times: $\tau_n = 9 \cdot 10^{-7}\text{s}$ and $\tau_p = 7 \cdot 10^{-7}\text{s}$

Photocurrent: $I_L = 45\text{mA}$

The open circuit voltage V_{OC} is:

Select one alternative:

- ☐ 0.97 V
- ☐ 0.31 V
- ☐ 0.53 V
- ☐ 0.22 V
- ☐ 1.03 V

Maximum marks: 4

14 P14-FY3114h2023

Which crystal systems are bi-axial?

Fill in your answer here

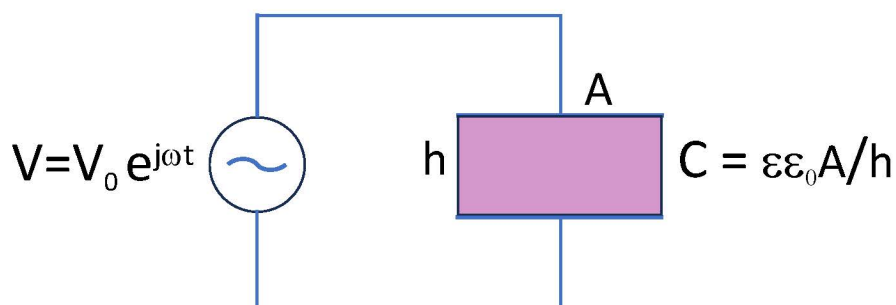
How should a **pn-diode** be biased to be used as a light detector?

Indicate the sign of the voltage between the p-side and n-side of the diode.

Fill in your answer here

Maximum marks: 4

15 P15-FY3114h2023



A dielectric solid has a homogeneous complex relative dielectric constant $\epsilon = \epsilon'_r - j\epsilon''_r$. Consider the circuit as shown in the figure above. Numerical values are as follows: The voltage amplitude $V_0 = 25\text{V}$, the voltage angular frequency $\omega = 10000\text{ s}^{-1}$, the capacitor area $A = 10\text{ cm}^2$, the separation of the capacitor plates $h = 1\text{ mm}$, and the relative dielectric constant $\epsilon = 2 + j0.5$.

Find the dissipated power in the capacitor.

The dissipated power is: μW .

Maximum marks: 4

16 P16-FY3114h2023

Find the difference in quasi-Fermi levels $\Delta E = E_{F_n} - E_{F_p}$ in an **InP** semiconductor at $T=300\text{K}$ that is injected by electron and hole densities of $n = 1 \cdot 10^{17} \text{cm}^{-3}$ and $p = 1 \cdot 10^{17} \text{cm}^{-3}$.

Select one alternative:

- ☐ 1.20 eV
- ☐ 1.42 eV
- ☐ 0.82 eV
- ☐ 1.38 eV
- ☐ 0.94 eV

Maximum marks: 4

17 P17-FY3114h2023

Consider a GaAs semiconductor at $T = 300\text{K}$ in which a low concentration of holes p are injected. The recombination rate may be written as

$$R_{\text{spont}} = \frac{1}{\tau_0} N_{CV} \left(\frac{n}{N_C} \right) \left(\frac{p}{N_V} \right)$$

where $\tau_0 = 0.6 \cdot 10^{-9} \text{s}$ is the recombination time for an electron if a hole is available at the same $\mathbf{k}$ vector,

N_{CV} is the joint density of states, N_C and N_V are the densities of electrons and holes at the band edges; respectively. Assume that $p = 1 \cdot 10^{16} \text{cm}^{-3}$.

What is the average lifetime of a single electron in the semiconductor.

Write the answer here: ns .

Maximum marks: 4

Constants and parameters (at 300 K)

$m_e = 9.1 \cdot 10^{-31} \text{ kg}$	$e = 1.6 \cdot 10^{-19} \text{ C}$	$k_B = 1.38 \cdot 10^{-23} \frac{\text{J}}{\text{K}}$	$h = 6.63 \cdot 10^{-34} \frac{\text{J}}{\text{K}}$
$\epsilon_0 = 8.85 \cdot 10^{-12} \text{ s}^4 \text{ A}^2 / \text{kg m}^3$			
$n_i(\text{Si}) = 1.5 \cdot 10^{10} \frac{1}{\text{cm}^3}$	$\mu_n(\text{Si}) = 1000 \frac{\text{cm}^2}{\text{Vs}}$	$\mu_p(\text{Si}) = 350 \frac{\text{cm}^2}{\text{Vs}}$	(low field values)
$n_i(\text{GaAs}) = 1.84 \cdot 10^6 \frac{1}{\text{cm}^3}$	$\mu_n(\text{GaAs}) = 8000 \frac{\text{cm}^2}{\text{Vs}}$	$\mu_p(\text{GaAs}) = 400 \frac{\text{cm}^2}{\text{Vs}}$	(low field values)
$n_i(\text{InP}) = 1.3 \cdot 10^7 \frac{1}{\text{cm}^3}$	$\mu_n(\text{InP}) = 4600 \frac{\text{cm}^2}{\text{Vs}}$	$\mu_p(\text{InP}) = 150 \frac{\text{cm}^2}{\text{Vs}}$	(low field values)
$n_i(\text{Ge}) = 2.33 \cdot 10^{13} \frac{1}{\text{cm}^3}$	$\mu_n(\text{Ge}) = 3900 \frac{\text{cm}^2}{\text{Vs}}$	$\mu_p(\text{Ge}) = 1900 \frac{\text{cm}^2}{\text{Vs}}$	(low field values)
$m_e^*(\text{GaAs}) = 0.067 m_e$	$m_h^*(\text{GaAs}) = 0.45 m_e$	$m_e^*(\text{Ge}) = 0.56 m_e$	$m_h^*(\text{Ge}) = 0.29 m_e$
$m_e^*(\text{Si}) = 0.26 m_e$	$m_h^*(\text{Si}) = 0.5 m_e$	$m_e^*(\text{InP}) = 0.07 m_e$	$m_h^*(\text{InP}) = 0.4 m_e$
$E_{\text{gap}}(\text{Si}) = 1.1 \text{ eV}$	$E_{\text{gap}}(\text{GaAs}) = 1.41 \text{ eV}$	$E_{\text{gap}}(\text{InP}) = 1.35 \text{ eV}$	$E_{\text{gap}}(\text{Ge}) = 0.7 \text{ eV}$
Refractive indices:	$n_r(\text{Si}) = 3.98$	$n_r(\text{GaAs}) = 3.95$	$n_r(\text{Ge}) = 5.70$

Formulas

$$\text{Rotation} \begin{bmatrix} x' \\ y' \\ z' \end{bmatrix} = \begin{bmatrix} \cos\theta & \sin\theta & 0 \\ -\sin\theta & \cos\theta & 0 \\ 0 & 0 & 1 \end{bmatrix} \begin{bmatrix} x \\ y \\ z \end{bmatrix} \quad \text{and} \quad \begin{bmatrix} x \\ y \\ z \end{bmatrix} = \begin{bmatrix} \cos\theta & -\sin\theta & 0 \\ \sin\theta & \cos\theta & 0 \\ 0 & 0 & 1 \end{bmatrix} \begin{bmatrix} x' \\ y' \\ z' \end{bmatrix}$$

Orthogonal transformations

$$T'_{ij} = \sum_{kl} R_{ik} R_{jl} T_{kl} \quad \text{and} \quad T'_{ijk} = \sum_{lmn} R_{il} R_{jm} R_{kn} T_{lmn}$$

$$x'_i x'_j = \sum_{kl} R_{ik} R_{jl} x_k x_l \quad \text{and} \quad x'_i x'_j x'_k = \sum_{lmn} R_{il} R_{jm} R_{kn} x_l x_m x_n$$

Dielectric permittivity tensor

$$\epsilon_{\text{mono}} = \begin{bmatrix} \epsilon_{11} & \epsilon_{12} & 0 \\ \epsilon_{12} & \epsilon_{22} & 0 \\ 0 & 0 & \epsilon_{33} \end{bmatrix} \quad \epsilon_{\text{ortho}} = \begin{bmatrix} \epsilon_{11} & 0 & 0 \\ 0 & \epsilon_{22} & 0 \\ 0 & 0 & \epsilon_{33} \end{bmatrix} \quad \epsilon_{\text{tetra,tri,hex}} = \begin{bmatrix} \epsilon_{11} & 0 & 0 \\ 0 & \epsilon_{11} & 0 \\ 0 & 0 & \epsilon_{33} \end{bmatrix}$$

Conductivity, drift velocity, mobility, diffusion

$$\sigma = ne^2 \tau_{sc} / m^* \quad v_d = \mu E \quad \mu = e \tau_{sc} / m^* \quad D_n = \mu_n k_B T / e \quad J_{\text{diff}} = e D_n \frac{dn}{dx}$$

Electrons and holes in semiconductors

$$n = N_c e^{(E_c - E_F) / k_B T} \quad p = N_v e^{(E_F - E_v) / k_B T} \quad np = N_c N_v e^{-E_{\text{gap}} / k_B T} \quad N_c = 2 \left(\frac{m_e^* k_B T}{2\pi \hbar^2} \right)^{3/2}$$

$$E_e = E_C + \hbar^2 k^2 / 2m_e^* \quad \text{and} \quad E_h = E_V - \hbar^2 k^2 / 2m_h^*$$

Law of mass action

$$n_n p_n = n_p p_p = n_i^2$$

Diode equation (ideal diode)

$$I = I_0 (e^{eV / k_B T} - 1)$$

Diffusion length

$$L_n = \sqrt{D_n \tau_n}$$

Emission

$$W_{em}^{st}(\hbar\omega) = \frac{e^2 n_r \hbar \omega}{3\pi \epsilon_0 m_e^2 c^3 \hbar^2} |p_{cv}|^2 \cdot n_{ph}(\hbar\omega) \quad \frac{2|p_{cv}|^2}{m_e} = 23 \text{ eV (GaAs)}$$

Impermeability tensor

$$\sum_{ij} \eta_{ij} x_i x_j = 1 \quad \eta = \frac{1}{\epsilon_r} = \frac{1}{n_r^2}$$

Index ellipsoid

$$\left(\frac{x_1}{n_1}\right)^2 + \left(\frac{x_2}{n_2}\right)^2 + \left(\frac{x_3}{n_3}\right)^2 = 1$$

Paramagnetic susceptibility

$$\chi_P = \frac{\mu_0 M}{B} = \frac{C}{T}$$

Magnetization in ferromagnet

$$\mu_0 M = \chi_P (B_a + B_E)$$

Solution ExamFY3114h2023 - Dec19,2023

Problems 1-2

Multiple choice questions from projects

Problem 3

Regular triangular prism. Point group elements: $1\ m\ m\ m\ 1/m\ 2\ 2\ 2\ 3\ 3^2\ \bar{6}\ \bar{6}^5$

Problem 4

(1) : $4\ 2\ 2$; (2) : $2/m\ 2/m\ 2/m$; (3) : $\bar{3}\ 2/m$; (4) : $\bar{6}\ m\ 2$

Problem 5

We have $E_J = \frac{\vec{E} \cdot \vec{J}}{J} = \sum_i E_i J_i / J = \sum_{ij} \rho_{ij} J_j J_i / J$

therefore $\rho_E = \frac{E_J}{J} = \sum_{ij} \rho_{ij} (\frac{J_j}{J})(\frac{J_i}{J})$ and $J_1 = J \cos \phi \sin \theta$, $J_2 = J \sin \phi \sin \theta$, $J_3 = J \cos \theta$
hexagonal crystal gives that only $\rho_{11}, \rho_{22}, \rho_{33}$ are non zero, and that $\rho_{11} = \rho_{22}$.

Therefore: $\rho_J = \rho_{11}(\sin \theta \cos \phi)^2 + \rho_{11}(\sin \theta \sin \phi)^2 + \rho_{33} \cos^2 \theta = \rho_{11} \sin^2 \theta + \rho_{33} \cos^2 \theta$

Inserting numbers: $\rho_J = 4.31 \cdot 10^{-8} \Omega \text{cm}$

Problem 6

Diagonalizing monoclinic tensor. Rotation matrix $\begin{bmatrix} x' \\ y' \\ z' \end{bmatrix} = \begin{bmatrix} \cos \theta & \sin \theta & 0 \\ -\sin \theta & \cos \theta & 0 \\ 0 & 0 & 1 \end{bmatrix} \begin{bmatrix} x \\ y \\ z \end{bmatrix}$

$x'y' = (yy - xx) \sin \theta \cos \theta + xy(\cos^2 \theta - \sin^2 \theta) = (yy - xx) \frac{1}{2} \sin 2\theta + xy \cos 2\theta$

therefore $\epsilon'_{12} = (\epsilon_{22} - \epsilon_{11}) \frac{1}{2} \sin 2\theta + \epsilon_{12} \cos 2\theta$

If $\epsilon'_{12} = 0$ then $\tan 2\theta = 2\epsilon_{12}/(\epsilon_{11} - \epsilon_{22})$. Inserting numbers: $\tan 2\theta = 3$ and $\theta = 35.8^\circ$

Problem 7

$\hbar \omega = E_e - E_h = E_C + \frac{\hbar^2 k^2}{2m_e^*} - (E_V - \frac{\hbar^2 k^2}{2m_h^*}) = E_{gap} + \frac{\hbar^2 k^2}{2} (\frac{1}{m_e^*} + \frac{1}{m_h^*}) = E_{gap} + \frac{\hbar^2 k^2}{2m_r^*}$

therefore $E_e - E_C = \frac{m_r^*}{m_e^*} (\hbar \omega - E_{gap}) = 0.17 \text{eV}$

and $E_h - E_V = -\frac{m_r^*}{m_h^*} (\hbar \omega - E_{gap}) = -0.33 \text{eV}$

Problem 8

$V_B = \phi_M - \chi = -0.3 \text{eV}$, an ohmic contact is formed at the junction

Problem 9

The conductivity is given by $\sigma = ne\mu_e + pe\mu_p$

The law of mass action is: $n_n p_n = n_p p_p = n_i^2$, and $n_n = N_D$

$\sigma_n = N_D e \mu_e = 220.8 \frac{1}{\Omega \text{cm}}$, and $\sigma_p = \frac{n_i^2}{N_D} e \mu_p = 1.352 \cdot 10^{-20} \frac{1}{\Omega \text{cm}}$

$\ln \sigma_n = 5.4$, and $\ln \sigma_p = -45.75$

Problem 10

(1) transformers, flux guides, magnetic shielding

(2) Fe, Co, Ni, rare earth elements

Problem 11

$$\begin{aligned}\eta_{11}(E) &= \eta_{11}^0 + s_{1133}E^2 = 1/n^2 + s_{13}E^2, \\ \eta_{22}(E) &= \eta_{22}^0 + s_{2233}E^2 = 1/n^2 + s_{13}E^2, \\ \eta_{33}(E) &= \eta_{33}^0 + s_{3333}E^2 = 1/n^2 + s_{33}E^2, \\ \text{others } \eta_{ij} &= 0\end{aligned}$$

The index ellipsoid is

$$\Sigma_{ij} \eta_{ij}(E) x_i x_j = 1 = x^2 \left(\frac{1}{n^2} + s_{13}E^2 \right) + y^2 \left(\frac{1}{n^2} + s_{13}E^2 \right) + z^2 \left(\frac{1}{n^2} + s_{33}E^2 \right)$$

The change in the ordinary index of refraction is

$$\Delta \left(\frac{1}{n^2} \right) = s_{13}E^2 \Rightarrow \Delta n = -\frac{1}{2}n^3 s_{13}E^2$$

The change in the extraordinary index of refraction is

$$\Delta \left(\frac{1}{n^2} \right) = s_{33}E^2 \Rightarrow \Delta n = -\frac{1}{2}n^3 s_{33}E^2$$

$$\text{Therefore } \Delta n = n_e - n_o = \frac{1}{2}n^3 E^2 (s_{13} - s_{33}) = 0.02$$

Problem 12

Inverse piezoelectric effect.

$$\begin{bmatrix} \epsilon_{11} \\ \epsilon_{22} \\ \epsilon_{33} \\ \epsilon_{23} \\ \epsilon_{13} \\ \epsilon_{12} \end{bmatrix} = \begin{bmatrix} \epsilon_1 \\ \epsilon_2 \\ \epsilon_3 \\ \epsilon_4 \\ \epsilon_5 \\ \epsilon_6 \end{bmatrix} = \begin{bmatrix} d_{11} & 0 & 0 \\ d_{21} & 0 & 0 \\ 0 & 0 & 0 \\ d_{41} & 0 & 0 \\ 0 & d_{52} & 0 \\ 0 & d_{62} & 0 \end{bmatrix} \begin{bmatrix} E_1 \\ E_2 \\ E_3 \end{bmatrix}, \text{ where } d_{11} = -d_{21}, d_{41} = -d_{52}, d_{62} = -2d_{11}$$

The electric field E is applied in the x-direction.

This gives $\epsilon_{11} = d_{11}E = 113 \cdot 10^{-6}$, and $\epsilon_{33} = 0$

Problem 13

Open circuit voltage for solar cell $I = 0 = I_L - I_0(e^{eV/k_B T} - 1)$

Diffusion currents $J_n = eD_n \frac{dn}{dx} = eD_n \frac{n_p}{L_n}$ and $J_p = eD_p \frac{dp}{dx} = eD_p \frac{p_n}{L_p}$

Law of mass action $n_n p_n = n_p p_p = n_i^2 \Rightarrow n_p = n_i^2 / p_p, p_n = n_i^2 / n_n$

Diffusion lengths $L_n = \sqrt{D_n \tau_n}$ and $L_p = \sqrt{D_p \tau_p}$

Total diffusion current $I_0 = A(J_n + J_p) = Ae \left(\sqrt{\frac{D_n}{\tau_n}} \frac{n_i^2}{p_p} + \sqrt{\frac{D_p}{\tau_p}} \frac{n_i^2}{n_n} \right) = 2.44 \cdot 10^{-7} A$

Thus $V_{oc} = \frac{k_B T}{e} \ln(1 + \frac{I_L}{I_0}) = 0.31V$

Problem 14

Biaxial crystals: triclinic, orthorhombic, monoclinic.

A pn diode used as light detector should be reversed biased,

i.e. -V on p-side, +V on n-side.

Problem 15

$V = V_0 e^{j\omega t}$, $I = C \frac{dV}{dt} = j\omega CV$, $C = \epsilon_r \epsilon_0 A/h = (\epsilon'_r - j\epsilon''_r) \epsilon_0 A/h$

Power dissipated $P = \overline{VI} \sim \frac{1}{T} \int_0^T (\sin \omega t \cos \omega t + \cos^2 \omega t) dt = 0 + \frac{1}{2}$

Which means that only the imaginary part of the dielectric constant contributes to the dissipated power in the capacitor.

We then get $P = \frac{1}{2} \omega \epsilon''_r V_0^2 \epsilon_0 A/h = 13.8 \cdot 10^{-6} W$

Problem 16

Quasi Fermi levels $n = N_C e^{(E_{F_n} - E_C)/k_B T}$ and $p = N_V e^{(E_V - E_{F_p})/k_B T}$

Which gives $E_{F_n} = E_C + k_B T \ln(n/N_C) = E_C - 0.04 \text{ eV}$

and $E_{F_p} = E_V - k_B T \ln(n/N_V) = E_V + 0.11 \text{ eV}$, and $E_{F_n} - E_{F_p} = E_{gap} - 0.15 \text{ eV} = 1.2 \text{ eV}$
when $E_{gap} = E_C - E_V = 1.35 \text{ eV}$

Problem 17

The spontaneous recombination rate is $R_{spon} = \frac{1}{\tau_0} N_{CV} (\frac{n}{N_C}) (\frac{p}{N_V})$ and the densities of states are $N_C = 2 (\frac{m_e^* k_B T}{2\pi \hbar^2})^{3/2}$ and $N_V = 2 (\frac{m_h^* k_B T}{2\pi \hbar^2})^{3/2}$ and $N_C = 2 (\frac{m_r^* k_B T}{2\pi \hbar^2})^{3/2}$

The lifetime of a single electron is

$$\tau_r = \frac{n}{R_{spon}} = \frac{\tau_0}{p} N_C N_V / N_{CV} = 2 \frac{\tau_0}{p} (\frac{k_B T m_e^* m_h^*}{2\pi \hbar^2 m_r^*})^{3/2} = 2 \frac{\tau_0}{p} (\frac{k_B T (m_e^* + m_h^*)}{2\pi \hbar^2})^{3/2}$$

$$\tau_r = 5.57 \cdot 10^{-7} \text{ s} = 557 \text{ ns}$$