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EXAMINATION IN FY3201 ATMOSPHERIC PHYSICS AND CLIMATE CHANGE
Faculty for Natural Sciences and Technology
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Permitted help sources: 1 side of an A5 sheet with printed or handwritten formulas permitted
Bi-lingual dictionary permitted
Approved calculators are permitted

You may take:

Molar mass of water vapour ~18 kg/kmole — $g=9.8$ m s^{-2} and constant in z
Molar mass of dry air ~29 kg/kmole — 1 hPa = 10^2 Pa = 10^2 N m^{-2}
273 K = 0 °C — Scale Height, $H=R\cdot T/g$
Values for dry air: $C_p=1004$ J·K^{-1}·kg^{-1} $C_v=718$ J·K^{-1}·kg^{-1} $R_d=287$ J·K^{-1}·kg^{-1}
$\gamma = C_p / C_v$ $\kappa = R_d / C_p$ $R_d=C_p - C_v$ $\Gamma_{da}=9.8$ K/km

Answer all 5 questions (and good luck!):

SOLUTIONS

1) Atmospheric structure (20 %):

a) A meteorological station is located on land 50 m below sea level. If this station measures the surface pressure to be 1020 hPa, the mean temperature for the layer between the surface (1020 hPa) and 1000 hPa to be 15 C, and the mean temperature for the 1000 to 500 hPa layer to be 0 C, compute the height of the 500 hPa pressure level above sea level. Assume the air is dry. (15%)

b) Assume that these temperatures are the temperatures at the centre of each layer. What is the stability of the atmosphere with regard to vertical motions and why is it stable or unstable? Again assume the air is dry. (5%)

a) Here we start with the hydrostatic equation:

$$\frac{\partial}{\partial z} p = -\rho g$$

And the perfect gas law:

$$\rho := \frac{p}{R\,T}$$

To get a relation between p and T, the hypsometric equation:

$$\frac{dp}{p} = -\frac{g\,dz}{R\,T}$$

If T, normally a function of z, is taken as the average temperature in the layer between altitudes z_1 and z_2, we can integrate this over the layer and get the hypsometric equation:

$$\ln\left(\frac{p2}{p1}\right) = -\frac{g\,(z2 - z1)}{R\,T}$$

For the first layer between the surface at z_1=-50m, p_1=1020 hPa and p2=1000 hPa with the average temperature of 15C=(273+15)K=288K, we can solve for z2=117m above sea level (since we have taken z_1=-50m).

Now, we can repeat the process for the layer between z_2=117m, p_2=1000 hPa, and p_3=500 hPa with a mean temperature T=0C=273K. This gives z_3=5658 m above z_2, which we took to be -50m in the calculation. Thus, this is the height above sea level. If one solves the thickness of the lower layer as dZ_1, and the thickness of the upper layer as dZ_2 and adds them, this is the height above the ground. We then have to subtract 50 m from dZ_2 to get the height above sea level.

b) The middle of the top layer is ½(5658-117)+117=2888m, middle of the bottom is ½[117- (-50)]+(-50)=34 m, giving an altitude difference of 2.85km. The temperature lapse rate is therefore the temperature difference divided by the altitude difference, or -(288-273K)/2.85 km = 5.2 K/km.*

Since the dry adiabatic lapse rate is 9.8 K/km, it means that an air parcel moving upward would cool 9.8 K every km, or about 30 K as it went from the middle of the bottom layer to the middle of the top. Thus it would be 260 K, or -13 C and be surrounded by warmer air at -0C. It would therefore be less buoyant than the surrounding air and sink back towards its starting point and return the atmosphere to the same state it started. Thus, the atmosphere is stable.

2) Radiation and atmospheric structure (20 %)

A parallel light beam from the sun is incident on an atmosphere which consists of a gas with a constant absorption coefficient, k_λ. If the atmosphere is isothermal, use the definition of optical depth and the fundamental atmospheric structure equations to show that the optical depth has a linear relationship to pressure.

Here we just use the definition of optical depth:

$$\tau(z) = \int k\, \rho(z)\, dz$$

And then take into account the fact that k_ν is independent of altitude, z, to write:

$$\tau(z) = k \int \rho(z)\, dz$$

Now, we can use the hydrostatic equation:

$$\frac{\partial}{\partial z} p = -\rho\, g$$

To solve for ρ. This gives:

$$\rho(z) := -\frac{\frac{\partial}{\partial z} p(z)}{g}$$

Which, when substituted in for τ gives:

$$\tau = -\frac{k\, p(z)}{g}$$

Thus, τ is linearly dependent upon p

3) Radiation transfer (20 %)

The Schwarzschild radiation transfer equation may be integrated to give the radiance at any altitude, z, as follows:

$$L_\nu^{\uparrow}(z) = L_{\nu s}^{\uparrow} \cdot e^{-(\tau_s - \tau_z)/\mu} - \int_{\tau_s}^{\tau_z} J_\nu(\tau') \cdot e^{-(\tau' - \tau_z)/\mu} \cdot \frac{d\tau'}{\mu}$$

$$L_\nu^{\downarrow}(z) = L_{\nu\infty}^{\downarrow} \cdot e^{\tau_z/\mu} - \int_{\tau_z}^{\tau_z} J_\nu(\tau') \cdot e^{-(\tau' - \tau_z)/\mu} \cdot \frac{d\tau'}{\mu}$$

A short wavelength parallel beam of solar radiation of radiance, $L^{\downarrow}_{\nu\infty}$ is incident at the top of an isothermal atmosphere at a zenith angle of 0° (μ=-1). It is absorbed by the gas which has a density, $\rho(z)$, and an absorption coefficient, k_ν, that is independent of altitude. Select the terms of the equations you would need to describe the radiance as a function of altitude (assume that there is no scatter or albedo and a surface temperature ~300K) (5%).

Using this, show that the absorption per unit volume (i.e., $dL^{\downarrow}_\nu/dz$) reaches a maximum at the level where the optical depth is unity. (15%)

For short wavelengths, the source terms $J_\nu(\tau')$ in the integrated Schwarzschild equations are ~0. Similarly, the amount of short wavelength radiation emitted from the surface at a temperature of 288K is negligible. Since there is no albedo or scattering, the upward radiation terms, $L^{\uparrow}_\nu$, can be neglected. This leaves only the term:

$$L_\nu^{\downarrow}(z) = L_{\nu\infty}^{\downarrow} \cdot e^{\tau_z/\mu}$$

Now, differentiating this to give the absorption per unit volume yields:

$$\frac{Absorption}{Unit\_Volume} = \frac{Lo\left(\frac{\partial}{\partial z}\tau(z)\right) e^{\left(\frac{\tau(z)}{\mu}\right)}}{\mu}$$

Now we can put in the definition of τ:

$$\tau(z) = \int k\,\rho(z)\,dz$$

And the fact that k_ν=constant and for an isothermal atmosphere $\rho(z) := \rho 0\, e^{\left(-\frac{z}{H}\right)}$

To give:

$$\tau(z) := -k\,\rho 0\, H\, e^{\left(-\frac{z}{H}\right)}$$

And therefore:

$$\frac{Absorption}{Unit\_Volume} = \frac{Lo\,k\,\rho 0\, e^{\left(-\frac{z}{H}\right)} e^{\left(-\frac{k\,\rho 0\, H\, e^{\left(-\frac{z}{H}\right)}}{\mu}\right)}}{\mu}$$

We can identify the terms $k_\nu \cdot \rho_o e^{-z/H}$ as $\tau(z)$, and re-write this as:

$$\frac{Absorption}{Unit\_Volume} = -\frac{Lo\,\tau\,e^{\left(\frac{\tau}{\mu}\right)}}{H\,\mu}$$

Differentiating this with respect to $\tau(z)$ *gives:*

$$\frac{\partial}{\partial\tau}\frac{Absorption}{unit\_volume} = -\frac{Lo\,e^{\left(\frac{\tau}{\mu}\right)}}{H\,\mu} - \frac{Lo\,\tau\,e^{\left(\frac{\tau}{\mu}\right)}}{\mu^2\,H}$$

We can set this equal to zero and solve for τ *to find* $\tau(z_{max})$

$$\tau(z_{max})=-\mu=-(-1)=1$$

Therefore, the maximum absorption/unit volume is maximum where $\tau=1$

4) Climate and modelling (20 %)

a) Briefly describe the difference between weather and climate? (5%)
b) What is meant by a radiative equilibrium temperature? (5 %)
c) In atmospheric models (for example climate models), how does one treat physical or chemical processes that have a scale size smaller than the model grid size? (5%)
d) In a radiative transfer model, what is the "two-stream" approximation? (5%)

a) Weather occurs on short time scales compared to the phenomenon we are studying. Climate occurs on time scales much longer than the phenomenon we are studying. One could say that weather is what we have today, climate is what we expected to have today based on the average weather over many years.

b) When the radiative energy absorbed = radiative energy emitted. If we assume that the atmosphere radiates as a grey body, the Planck formula may be used to determine the temperature at which one must be to radiate that much energy. This temperature is the radiative equilibrium temperature.

c) We actually have to create a sub-model, called a parameterization, of the small-scale process that is in terms of the grid variables. For example, the gravity wave forcing is parameterized as the difference between the wind speed and the average

d) In a radiative transfer model, the interaction of the light with the atmosphere through absorption and emission of radiation must be integrated over zenith angle (or path length) and frequency. However, frequencies with high probability for interaction will not reach the next layer if the path is too long (ie, far off the zenith). These have a different average angle (closer to the vertical, shortest path) than those with a low probability for interaction which require the long path (large zenith angles) to interact. Thus, the integrals are not separable and cannot be approximated.

The two stream approximation makes use of the fact that only frequencies with where the change in optical depth between the two layers is close to 1 will have any significant interaction. Thus, by restricting $\Delta\tau=1$ we can define an effective zenith angle, μ, where the interaction integrals maximize. This then says the integral over all zenith angles has the same effect as a parallel beam at this effective zenith angle, and the entire integral is replaced by two streams, one upward and one downward, at this effective zenith angle.

5) Atmospheric Structure and Thermodynamics (20 %)

An air parcel's entropy, S, is constant for an adiabatic (isentropic) process ($\delta q=0$). However, during a cloud-free evening, long wavelength heat transfer causes an air parcel to descend from 900 to 910 hPa, and its entropy to decrease by 15 $J \cdot K^{-1} \cdot kg^{-1}$. If its initial temperature is 280 K, determine:

a) The parcel's final temperature (10%), and

b) The parcel's final potential temperature, θ (10%).

Here the solution relies on the relationship between entropy and potential temperature. We can take P_o to be anything as long as we keep it fixed. Here I have taken it to be 1000hPa. At the initial position at P_1=900hPa:

$$S1 = Cp \ln(\theta 1) + So$$

Where

$$\theta 1 := T1 \left(\frac{Po}{P1}\right)^{\kappa}$$

Similarly, at position 2, where P_2=910hPa:

$$S2 := Cp \ln(\theta 2) + So$$

The difference in entropy, $dS = S_2 - S_1$, which is given as $-15\ J \cdot K^{-1} \cdot kg^{-1}$, is therefore:

$$dS = Cp \ln(\theta 2) - Cp \ln\left(T1 \left(\frac{Po}{P1}\right)^{\kappa}\right)$$

Given that $\kappa = R/C_p$, we can solve for θ_2 as:

$$\theta 2 := e^{\left(\frac{dS + \ln(T1)\,Cp + R \ln\left(\frac{Po}{P1}\right)}{Cp}\right)}$$

Just substitute in:

T_1=280K, $C_p = 1004\ J \cdot K^{-1} \cdot kg^{-1}$, $R = 287\ J \cdot K^{-1} \cdot kg^{-1}$, $dS = -15\ J \cdot K^{-1} \cdot kg^{-1}$, P_1=900hPa to get

$$\theta_2 = 284.3K$$

From the definition of potential temperature:

$$\theta 2 = T2 \left(\frac{Po}{P2}\right)^{\kappa}$$

And with P_2=910 hPa, we get a temperature T_2=276.7 K