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EXAMINATION IN FY3201 ATMOSPHERIC PHYSICS AND CLIMATE CHANGE

Faculty for Natural Sciences and Technology

03 June 2011

Time: 15:00-19:00

Number of pages: 9

Permitted help sources: 1 side of an A5 sheet with printed or handwritten formulas permitted
Bi-lingual dictionary permitted
Calculators meeting NTNU examination criteria are permitted

You may take:

Molar mass of water vapour ~18 kg/kmole $g=9.8\ m\ s^{-2}$ and constant in z
Molar mass of dry air ~29 kg/kmole   1 hPa = 10^2 Pa = 10^2 N m^{-2}
273 K = 0 ^oC Latent heat of vaporization water = $L_v=2.6\times10^6\ J\cdot kg^{-1}$
Scale Height, $H=R\cdot T/g$
Values for dry air: $C_p=1004\ J\cdot K^{-1}\cdot kg^{-1}$ $C_v=718\ J\cdot K^{-1}\cdot kg^{-1}$ $R_d=287\ J\cdot K^{-1}\cdot kg^{-1}$
$\gamma = C_p / C_v$ $\kappa = R_d / C_p$ $R_d=C_p - C_v$ $\Gamma_{da}=9.8$ K/km

Answer all questions (and good luck!):

Solutions

1) (25%) Weather balloons typically burst when they reach an atmospheric pressure of 100 hPa. Meteorologists release balloons at the Equator, where the atmosphere has a uniform, isothermal temperature of 35 °C, and at Svalbard, where the atmosphere has a uniform temperature of -35 °C. Assume dry air, and that the identical balloons start at 0 m where the surface air pressure at both sites is 1000 hPa.

a) At what height above the surface does each balloon burst? (10 %)

This is a relatively easy on to get them started. Here, as we have done many times in class, we start with the hydrostatic equation:

$$\frac{\partial}{\partial z} p = -\rho\, g$$

And the perfect gas law:

$$\rho := \frac{p}{R\, T}$$

To get a relation between p and T, the hypsometric equation:

$$\frac{dp}{p} = -\frac{g\, dz}{R\, T}$$

Since T, normally a function of z, is taken as isothermal in this case, we can integrate this to get the hypsometric equation:

$$\ln\left(\frac{p2}{p1}\right) = -\frac{g\,(z2 - z1)}{R\, T}$$

Then we just need to solve for the burst altitude, z2 and substitute in the values:

$$Z2 := Z1 + \frac{Rd\, T \ln\left(\frac{P1}{P2}\right)}{g}$$

And `Z1:=0; Rd:=287; P1:=1000; P2:=100; g:= 9.8;`
Gives for Svalbard, Ts=238 K, that Z2 = 16.049 km
And for the Equator, Te=308 K, that Z2 = 20.729 km
This, and the fact that balloon material becomes more brittle in cold temperatures, explains why Antarctic stations seldom report radiosonde data above 15 km,

b) If the only wind was due to the temperature gradient between the Pole and the Equator, in which directions would the balloons drift? Why? (10 %)

The above calculations show that at any altitude, there is a pressure gradient with higher pressures at the equator than at the pole. This will cause the balloons to drift poleward. However, over long distances the Coriolis force will cause them to turn towards the east. They get ½ credit for poleward, and full credit if they get the Coriolis force term correct.

c) Assuming the dry He inside each of the balloons does not exchange heat with its surroundings, what is the temperature of the gas in each balloon when it bursts? For Helium, take R_{He}= 2077 J·K^{-1}·kg^{-1} and C_p=5190 J·K^{-1}·kg^{-1} (5 %)

Here we can treat the balloon as an air packet that rises adiabatically (ho heat exchange, including radiative). The potential temperature of the He inside the balloon is the same as the ground temperature if we take the reference to be 1000 hPa. At the burst pressure altitude pressure, 100 hPa, the temperature of the helium, Tp, is:

$$Tp := \theta 0 \left(\frac{P}{Po}\right)^{\kappa}$$

The catch is that $\kappa=R/C_p$ is determined by the values for helium, so κ=0.40. Substituting in for P=100 hPa, Po=1000 hPa, we get:
Equator: $\theta_{equator}$=308 K, Tp = 122 K. Pole: θ_{pole}=238K and Tp=95K.

Some might be tempted to use the fact that the adiabatic lapse rate $\Gamma_a = g/C_p$ can be used with a linear temperature profile of the parcel:

$$Tp = To - \frac{g\,Z}{Cp}$$

*However, this is only true if the parcel is in hydrostatic equilibrium at every point (the adiabatic lapse rate, $\Gamma_a = g/C_p$, was derived using hydrostatic equilibrium). Thus, this is only true if the environmental lapse rate is adiabatic as well. Here we have the environmental temperature profile is T=To=constant. That yields that P=Po*exp(-Z*g/(Rd*To). If we put this into our expression for Tp derived from the fact that its potential temperature is constant:*

$$Tp = \Theta 0 \left(\frac{P}{Po}\right)^{\kappa}$$

With Θ_o=To, we get:

$$Tp := To\,\mathbf{e}^{\left(-\frac{Z\,g}{Cp\,To}\right)}$$

And if we do a Taylor's series expansion about small Z, we re recover to first order:

$$Tp = To - \frac{g}{Cp}Z + \mathrm{O}(Z^2)$$

Not very accurate over 16 to 20 km!

2) (25%) Volcanic ash of 0.1μ (1μ=10^{-6}m) radius is distributed with a constant mass mixing ratio of 3% in the lowest 10 km of an isothermal atmosphere of temperature 270 K. The atmospheric density at the surface is 1.29 kg·m^{-3}, and the scale height for the atmosphere is 8 km. For 500 nm light, take the attenuation coefficient of clear air to be 0, and for ash to be 0.01 m^2·kg^{-1}.

a) At ground level, what is the optical depth of 500 nm light for the sun directly overhead? (8%)

Since it is an isothermal atmosphere with a given scale height, the density of the ash is given by μ, the ash's mass mixing ration, times the atmospheric density, or:

$$\rho := \mu\,\rho0\,\mathbf{e}^{\left(-\frac{z}{H}\right)}$$

This can be derived from substituting the perfect gas law into the hydrostatic equation and integrating to get P as a function of z, and then using perfect gas law and the definition of H to solve for ρ as above. However, they should be able to just write down the above expression. Next, they just need to use the definition of optical depth, and realize they only need to integrate to 10 km.

$$\tau := \int_0^{10000} \mu\,\rho0\,\mathbf{e}^{\left(-\frac{z}{H}\right)} k\,dz$$

Or $$\tau := -\mu\,\rho0\,k\,H\,\mathbf{e}^{\left(-10000\,\frac{1}{H}\right)} + \mu\,\rho0\,k\,H$$

Substituting in the values:

$$\mu := .03$$

$$\rho0 := 1.29$$

$$H := 8000$$

$$k := .01$$

Gives τ = 2.21

b) What is the atmospheric transmission in this case? (3%)

The transmission is given by:

$$T := \mathbf{e}^{(-\tau)}$$

Or, here, with τ = 2.21, the atmospheric transmission is 11% (89% absorption). That is why the pictures from Iceland look like it is the middle of the night!

c) At what altitude does the optical depth = 1? (8%)

This one comes from the definition of the optical depth as above. Integrating from Z1 to the top of the ask cloud, the point where τ = 1 = τ1:

$$\tau1 := -\mu\,\rho0\,k\,H\,\mathbf{e}^{\left(-\frac{ZT}{H}\right)} + \mu\,\rho0\,k\,H\,\mathbf{e}^{\left(-\frac{Z1}{H}\right)}$$

Solving for Z1 gives:

$$Z1 := -\ln\left(\frac{\tau 1 + \mu\,\rho 0\,k\,H\,e^{\left(-\frac{ZT}{H}\right)}}{\mu\,\rho 0\,k\,H}\right)H$$

And substituting:

$$\mu := .03$$

$$\rho 0 := 1.29$$

$$H := 8000$$

$$\tau 1 := 1$$

$$k := .01$$

$$ZT := 10000$$

Gives Z1= 3.96 km.
Though not requested, the transmission at this altitude would be 37% (63% absorption), and much brighter!

d) If the particle radius increased to 3 µ with the same mass mixing ratio, how would the extinction coefficient, transmission and asymmetry factor (ratio of forward to backward scatter) change for 500 nm light? (6%)

Here they should realize that they have been dealing with Rayleigh scatter since the wavelength λ=500nm = 0.5μ, is much larger than the particle radius, R=0.1μ. This is characterized by a scattering cross section much smaller than the geometric cross sectional area of the particle as well as equal forward and backward scatter, giving an asymmetry factor of 1.

If the radius increases to larger than the wavelength, then the scattering shifts to the Mie regime. In doing so, the scattering cross section will increase faster than πR^2 to a level larger than the geometric cross sectional area of the particle. In fact, over most of the Rayleigh range the cross section goes as the 6th power of the radius. If the mass density of particles remains constant, which it does here, one will find 1) a larger extinction coefficient (which grows as R^3), 2) a correspondingly lower transmission, and 3) a preference for forward scattering so that the asymmetry factor becomes greater than one

Unfortunately there was a typo here in that 0.1μ should have been 0.01μ. At 0.1μ the scattering is at the limit of the Rayleigh range, and the growth of the cross section is slower than R^6. In fact, for the numbers given, the growth will not compensate for the fact that the mass per particle grows as R^3, and the extinction coefficient will actually decrease slightly (leading to an increase of transmission)! Students who managed to write down the Rayleigh scattering curve on their notes might get this result. However, they must still mention that the cross section grows faster than πR^2 as one goes from Rayleigh to Mie scatter.

3) (25 %) A dry air parcel is raised adiabatically 2 km from a pressure, P_o=1000 hPa and a temperature of 285 K, to a pressure, P=783 hPa.

a) What is the temperature of the <u>air parcel</u> at 783 hPa? (5%)

Since all motion is stated to be adiabatic, then the potential temperature that the parcel has at the starting point is maintained as it ascends. The potential temperature is:

$$\theta := T\left(\frac{Po}{P}\right)^{\kappa}$$

And at P=Po and T = To, θo = To, or 285 K. At P = 785 hPa, T may be found from:

$$Tp := \theta 0\left(\frac{P}{Po}\right)^{\kappa}$$

And with $\kappa = Rd/Cp = 0.286$, Po = 1000 hPa, P= 785 hPa and θo=285K, that gives Tp = 266 K.

b) If the atmospheric pressure as a function of altitude is given by the expression:

$P = P_o\cdot(1 - \Sigma\cdot z)^{\alpha}$, where Σ= 0.02 km^{-1}, and α=6.

What is the temperature of the air surrounding the parcel at 783 hPa? (9%)

Here one must use the hydrostatic equation to find the density as a function of altitude, z. This is:

$$\frac{\partial}{\partial z}P = -g\,\rho$$

Performing the differentiation and solving for ρ gives:

$$\rho := \frac{Po\,(1-\Sigma\,z)^{\alpha}\,\alpha\,\Sigma}{g\,(1-\Sigma\,z)}$$

From the perfect gas law

$$P = \rho\,Rd\,Ta$$

Given the expressions for P and ρ, we can solve for the atmospheric temperature, Ta:

$$Ta = \frac{P}{\rho\,Rd}$$

Or

$$Ta := \frac{g\,(1-\Sigma\,z)}{\alpha\,\Sigma\,Rd}$$

Remembering to convert Σ into m^{-1} (since the units must be compatible with g), the values given are:

$$Rd := 287$$

$$g := 9.8$$

$$Po := 1000$$

$$z := 2000$$

$$\alpha := 6$$

$$\Sigma := .00002$$

Which gives Ta = 273 K. Note that this gives the temperature at 0 m as 285 K, as it was given. A quick check like that would catch any unit problems and check your maths on the calculation of Ta.

c) Are these atmospheric conditions stable or unstable with respect to vertical motion? Why? (3%)

This should be easy. We have seen that a parcel raised 2 km has a temperature of 266 K, but the air surrounding it has a temperature of 273 K. So the parcel is colder, and therefore denser, than the surrounding air, and it will sink back to its starting point. If a student has not been able to calculate the temperatures correctly, then if they interpret the results they have correctly, and give the correct explanation, they should receive full credit for this answer.

d) If the air parcel contains moisture that condenses as it ascends, will the air parcel be warmer or colder than a corresponding dry parcel? Why? (3%)

Again, this should be easy. The condensing water will release latent heat of vaporization which will heat the parcel. Thus, a moist air parcel with condensing water will be warmer than a corresponding dry parcel.

e) If the air parcel contains water, what mass mixing ratio of water must condense during its ascent in order to change the parcel air temperature by 10 K? (Assume the atmospheric mass is the mass of dry air.) (5%)

This should also be relatively easy. The condensing water gives off latent heat, which will be the energy source heating the parcel. The units of latent heat for water, $L_v = 2.6x10^6\ J \cdot kg^{-1}$*, tell you how many Joules of heat will be released for every kg of water condensed. Similarly, the units of heat capacity for air,* $C_p = 1004\ J \cdot K^{-1} \cdot kg^{-1}$*, tell you how many Joules of heat will raise each kg of air one K. Putting these together, we have:*

$$L_v \cdot m_{water} = C_p \cdot M_{air} \cdot \Delta T$$

Note, we use C_p *since the condensation is taking place with no walls, hence the volume can change but the pressure will be constant. We are told to take* M_{air} *as the mass of dry air,* M_d*. Given the temperature increase of 10 K, we can solve for* $m_{water}/M_{air} = m_{water}/M_d = \Delta T \cdot C_p / L_v$*, and get* $\mu = m_{water}/M_d = 0.004$*. This can also be expressed as 4 g/kg.*

Note, if this much water condensed, the parcel would be warmer than the air mass around it, and the atmosphere would be unstable with respect to vertical motions.

4) Short Answers (25 %)

a) What are the most important optical properties of a gas that allow it to create a greenhouse effect? (4 %)

Very concisely: It must transmit at the short, visible, wavelengths where the Sun is putting the heat into the planet, and it must absorb at long wavelengths where the planet is radiating away energy.

b) With the addition of a greenhouse gas to the atmosphere, briefly describe the process by which the lower atmosphere warms. (4%)

The Earth's long-wavelength radiation is absorbedby the greenhouse gas which re-radiates the energy. Approximately ½ of it is returned back to the surface to create additional surface heating which warms the lower atmosphere via conduction and convection

c) What is meant by a radiative equilibrium temperature? (4 %)

The radiative equilibrium temperature is the temperature a body will maintain at the point where the energy it radiates away is equal to radiative energy it receives.

d) If the greenhouse effect produces a warming in the troposphere, why is there a net 2 K/day radiative cooling in the upper troposphere? (5%)

This follows from the definition of radiative equilibrium temperature. The radiative heating or cooling rate is just the difference between these numbers, input-output, which is 0 when it is in radiative equilibrium. If it receives additional, non-radiative energy, for example from latent heat driven by convection, then its temperature will increase to radiate away this extra energy. Thus, since it is radiating more energy than it receives <u>from radiation</u>, there will be a net difference in the "radiative energy in" – "radiative energy out", and the negative difference is the cooling rate. Hence, a 2K/day cooling indicates that there is another, non-radiative process occurring in the upper troposphere.
Again, the better student should be able to say that the radiative temperature gradient in an atmosphere is greater than the adiabatic temperature gradient. Thus, convective overturning would be expected and the upper troposphere would have heat from below advected into it. This would also cause condensation and release of latent heat, providing another, non-radiative heat source for the upper troposphere that would increase its temperature above that of radiative equilibrium. The better student would also hasten to point out that the total energy balances, and that the displacement from radiative equilibrium is like a spring, and there is a characteristic raditive relaxation time constant.

e) In a radiative transfer model, what is the "two-stream" approximation? (4%)

In a radiative transfer model, the interaction of the light with the atmosphere through absorption and emission of radiation must be integrated over zenith angle (or path length) and frequency. However, frequencies with high probability for interaction will not reach the next layer if the path is too long (ie, far off the zenith). These have a different average angle (closer to the vertical, shortest path) than those with a low probability for interaction which require the long path (large zenith angles) to interact. Thus, the integrals are not separable and cannot be approximated.
The two stream approximation makes use of the fact that only frequencies with where the change in optical depth between the two layers is close to 1 will have any

significant interaction. Thus, by restricting $\Delta\tau=1$ *we can define an effective zenith angle,* μ*, where the interaction integrals maximize. This then says the integral over all zenith angles has the same effect as a parallel beam at this effective zenith angle, and the entire integral is replaced by two streams, one upward and one downward, at this effective zenith angle.*

f) Describe briefly the differences between the two most common numerical grid models. (4 %)

The two main types of numerical grid models are Finite Difference Models, and Spectral Models. In the first, the spatial derivatives in the differential equations of motion, continuity and energy are solved iteratively using a Taylor series expansion, and then using the finite difference between the grid variables as an approximation to the differentials. The time differentials are solved again using a Taylor series expansion where the parameter at a future time is solved as a function of its current value and the value of its spatial derivatives (which are, in turn, solved as described above).

In a spectral model, a Fourier or Spherical expansion is fit in a least/squares sense to the grid variables. This then gives a closed form expression as a function of latitude, longitude and sometimes altitude that approximates each variable, and these closed form expressions may be differentiated to yield the differentials used in the equations of motion, continuity and energy. Frequently even in spectral models the vertical components are solved using finite differences. The time evolution is also performed using a finite difference method, but now the spatial differentials are from the differentiation of the closed form expressions.