Norwegian University of Science and Technology
Department of Physics

EXAMINATION IN FY3201/FY8902 ATMOSPHERIC PHYSICS AND CLIMATE CHANGE
Faculty for Natural Sciences and Technology
31 May 2013
Time: 09:00-13:00

Number of pages: 8

Permitted help sources: 1 side of an A5 sheet with printed or handwritten formulas permitted
Bi-lingual dictionary permitted
All calculators permitted

You may take:

Molar mass of water vapour: ~18 kg/kmole $g=9.8$ m s^{-2} and constant in z
Molar mass of dry air: ~29 kg/kmole   1 hPa = 10^2 Pa =10^2 N m^{-2}
273.15 K = 0 ºC   Scale Height: $H=R\cdot T/g$
Stefan–Boltzmann constant: $\sigma = 5.67\times10^{-8}$ W·m^{-2}·K^{-4}
Solar constant at Earth's orbit (1 au) = 1367 W·m^{-2}
Latent heat of vaporization water: $L_v=2.5\times10^6$ J· kg^{-1}
Gas constant for water vapour: $R_v=461$ J·K^{-1}·kg^{-1}
Values for dry air: $C_p=1004$ J·K^{-1}·kg^{-1} $C_v=718$ J·K^{-1}·kg^{-1} $R_d=287$ J·K^{-1}·kg^{-1}
$\gamma = C_p / C_v$ $\kappa = R_d / C_p$ $R_d=C_p - C_v$ $\Gamma_{da}=9.8$ K/km

Clausius–Clapeyron relation: $e_s = 6.112\,hPa\cdot\exp\left[\dfrac{L_v}{R_v}\left(\dfrac{1}{273K} - \dfrac{1}{T}\right)\right]$

Answer all questions (and good luck!):

1)   **(20%)** The atmospheric pressure as a function of altitude is given by the expression:

$P = P_o\cdot(1-z/L)$,   where L = 8.5 km and $P_o=1000$ hPa .

a) At P=950 hPa, what is the atmospheric temperature and density (8%)

So, we are given $P(z) = P_o\cdot(1-z/L)$, and using the hydrostatic equation, we can easily differentiate P to get the density $\rho(z)$: $\dfrac{\partial}{\partial z}P(z) = -g\,\rho$, to give: $\rho := \dfrac{Po}{g\,L}$

To get the temperature one needs to use the perfect gas law: $P(z)=\rho(z)\cdot R\cdot T(z)$ to get: $T := \dfrac{(L-z)\,g}{R}$. This is all we need to calculate z, ρ and T at any pressure level.

*For L=8500 m, Po=1000 hPa = 1x10⁵ **Pa** and g=9.8 m/s², we find **$\rho = 1.2$ kg/m³***

(remember we have to use Pa, not hPa. If this was not done, I deducted 2 points).

We can find that P=950 hPa occurs at z=425 m. This gives us the temperature,

*using R=287 J·K⁻¹·kg⁻¹, of **T=275.7 K = 2.6 ℃**. Note, one could also skip solving*

for z and just use the fact that T=P/(R·ρ), and use P = 950x10² Pa and

ρ = 1.2 kg/m³ from above.

b) A hot-air tourist balloon is at hydrostatic equilibrium at an altitude of 950 hPa.
   The balloon equipment (but not the air inside the balloon) weighs 600 kg. If the
   balloon volume is 3000 m³, what is the temperature of the air inside the balloon?
   (If you have not solved part A, assume the temperature at 950 hPa is 15 ºC) (12%)

This is directly from the homework. We have to know that hydrostatic equilibrium implies:

*Mass of (balloon + air inside balloon)   =   Mass of environmental air at
                                                 the balloon altitude that occupies
                                                 the same volume as the balloon*

That is: $m + \rho_{in} \cdot V = \rho_e \cdot V$

Where m = mass of the balloon, ρ_{in} is the mass of air inside the balloon, V is the volume of the balloon, and ρ_e the mass of the environmental air (the air outside of the balloon) at the balloon altitude. We are given pressure, volume and temperature, and we know that the pressure inside the balloon is equal to the pressure outside. So we should convert the densities into pressures and temperatures using the perfect gas law:

$$m + p \cdot V/(R \cdot T_{in}) = p \cdot V/(R \cdot T_e)$$

Now, we just need to solve for our unknown T_{in}: $\dfrac{1}{Tin} = \dfrac{1}{Te} - \dfrac{M\,R}{p\,V}$

Which can be written:

$$Tin := \dfrac{1}{\dfrac{1}{Te} - \dfrac{m\,R}{p\,V}}$$

*Given m = 600 kg, V = 3000 m³ and T_e = 275.7 K from part a), we find that the temperature inside the balloon is **T_{in} = 330.9 K = 57.7 ℃.***

*If part a) was not done, and T_e = 15 ℃ = 288.1 K has been used, the air inside of the balloon would be **T_{in} = 348.9 K = 75.5 ℃***

2) **(20%)** The Voyager I spacecraft observed Titan, a moon of Saturn. It discovered a hydrocarbon aerosol layer at a pressure of 1000 hPa, where the atmospheric temperature was measured to be 88 K. Even though the surface temperature could not be measured, the surface pressure was measured to be 1500 hPa. For the dry Titan atmosphere, which is 80% nitrogen, the gas constant is $R_T=290$ J·K^{-1}·kg^{-1}, $C_{PT}=1044$ J·K^{-1}·kg^{-1}, and $g_T \ll g_{Earth}$.

a) Estimate the maximum surface temperature assuming the atmosphere is stable with respect to vertical motions. (7%)

The maximum temperature at the surface for a stable atmosphere would be the adiabatic lapse rate (if it were higher, convective motions would develop and rapidly redistribute heat vertically until the lapse rate would adjust to the adiabatic lapse rate). Since we do not have P(z), we have to use the potential temperature equation. Easiest is to use the reference pressure, P_o = 1000 hPa where the temperature is 88 K. That means the potential temperature of a parcel at 1000 hPa is θ = 88 K. Then at a surface pressure P=1500 hPa, we have:

$$T = \Theta \left(\frac{P}{Po} \right)^{\kappa}$$

*Where $\kappa = R_T / C_{pT}$. For the values of R_T and C_{pT} given we calculate $\kappa = 0.278$, and at P = 1500 hPa we get a temperature of **T_s = 98.5 K**.*

b) The true surface temperature was measured later by Voyager II to be 94 K. Do you expect convection in Titan's atmosphere? Why or why not? (6%)

We would not expect convective motions on Titan. *From part a) we have shown that a parcel starting at the surface at a pressure of 98.5 K would cool to 88 K when it reached 1000 hPa. However, **the air at the surface is only 94 K, so if it ascends to 1000 hPa it will be proportionally cooler than 88 K. Since it will be cooler and heavier than the surrounding air, it will tend to fall back to its starting position, and convection will cease**. In fact, a parcel of air at the surface (T=94 K and Po=1500 hPa) has a potential temperature, θ = 94 K (taking the reference pressure to be 1500 hPa). It will keep this θ as it ascends to P = 1000 hPa, and its temperature there will decrease to T_p = 84 K, cooler than the surrounding 88 K air. This latter calculation is not required, but won't be penalized. The real key is to realize the atmospheric temperature gradient is constant, and the parcel changes its temperature as it moves vertically. So if you bring a parcel at T = 88 K down to the surface and it heats to 98.5 K, then if you start at the surface and bring a parcel of cooler surface air up, it will be cooler than 88 K. Since it is cooler than the surrounding air we have stability.*

c) Assuming isothermal conditions, how many scale heights above the surface was the hydrocarbon aerosol layer? (7%)

Under isothermal conditions, we know that the scale height, $H = R \cdot T/g$, is constant. Therefore the hydrostatic equation can be integrated to give:

$$P(z) = Po \; e^{\left(-\frac{z}{H} \right)}$$

Solving this for Z_d/H in terms of the pressure of the dust layer, P_d = 1000 hPa and the surface pressure, P_o = 1500 hPa, we get: $\frac{Zd}{H} = -\ln\left(\frac{Pd}{Po} \right)$, *or **$Z_d/H = 0.41$**.*

*That is, **Z_d = 0.41 scale heights**.*

3) **(20 %)** An air parcel at a pressure of 1000 hPa has a temperature of 15°C and a relative humidity of 58%.

a) What is the partial pressure of water vapour in the parcel, and what is the water vapour mass mixing ratio in the parcel? (5%)

Since the relative humidity (in %) is $RH = 100*e_w/e_s(T)$, where e_w is the partial pressure of water vapour and $e_s(T)$ is the saturation water vapour partial pressure at temperature, T, one needs to use the Clausius–Clapeyron relation given in the formulae. That relation for a temperature of 15 °C = 288.15 K, yields $e_s = 17.4$ hPa. For a relative humidity of 58%, this gives the partial pressure of water vapour as $\underline{e_w = 0.58*17.4\ hPa = 10.1\ hPa}$.
(if $T(K) = 273+T(C)$ instead of 273.15 is used, get 9.97 hPa)

The mass mixing ratio of water vapour at pressure P is given by $\mu = \varepsilon \cdot e/P$, where ε is the ratio of the molar mass of H_2O vapour to the molar mass of air. Since there is so little water vapour in air, one can approximate the molar mass of air as the molar mass of dry air, so $\varepsilon \approx M_v/M_d$, which from the values given in the formulae yields $\varepsilon \approx 0.622$. For 10.1 hPa of water vapour at a pressure of 1000 hPa, this gives $\underline{\mu \approx 0.00622, or\ 6.22\ g/kg}$. (again, with 273K=0C get 6.02 g/kg)

b) What is the dew-point temperature of the parcel? (3%)

No trick here; the dew point temperature is the temperature at which the water vapour in the parcel would just be saturated. That is, $e_w = e_s$. One needs to invert the Clausius-Clapeyron relation for T_d, giving:

$$Td = \cfrac{1}{\cfrac{1}{273} - \cfrac{Rv \ln\left(\cfrac{ew}{6.112}\right)}{Lv}}$$

For a water vapour partial pressure $e_w = 10.1$ hPa, we get $\underline{Td = 280\ K.}$

c) If the parcel is blown up a mountain side, at what pressure level can clouds begin to form? (Note, as stated the problem is too difficult to solve. Information was given at the exam that they should assume that the dew point temperature when condensation began was the same as at the parcel's starting point) (4%)

As the parcel rises, the pressure drops, the parcel expands and the parcel temperature will drop. When the temperature reaches the dew-point temperature, then clouds will begin to form. However, up to the point where condensation occurs, the lifting condensation level (LcL) this will be an adiabatic process. Thus, the pressure at which the temperature reaches the dew-point can be determined from the potential temperature equation (see how useful potential temperature is!). So, $\theta = T\left(\dfrac{Po}{P}\right)^\kappa$, and if we take as our reference $P = P_o = 1000$ hPa, where $T = 288.15$ K, the potential temperature of the parcel is $\theta = 288.15$ K. This will be constant as long as the water does not condense. The pressure at which $T = T_d = 280$ K is then given by $P = Po\left(\dfrac{Td}{\theta}\right)^\kappa$, which yields $\underline{P_{LCL} = 905\ hPa}$. (if Po and P are reversed in the calculation, I deducted 2 points)

d) If it blows higher than this and it snows and rains such that the parcel loses ¼ of its water mass, what temperature does the parcel have when it comes back down to 1000 hPa? (5%)

The amount of latent heat released upon condensation of a mass of water, M_w, is $L_v \cdot M_w$ (look at the units of the latent heat $\Rightarrow J \cdot kg^{-1}$). For an amount of heat deposited in the air in the parcel, the air will raise its temperature by $C_p \cdot M_{air} \cdot \Delta T$ (again, the clue is in the units of the heat capacity $\Rightarrow J \cdot K^{-1} \cdot kg^{-1}$).

Now, if some condensed water is lost, it cannot gather this heat up from the parcel when the water vapour re-vaporizes. Thus, the heat remaining in the parcel will be: $L_v \cdot M_{W-Lost}$, and this amount of energy will heat the air by $C_p \cdot M_{air} \cdot \Delta T$. Equating these and solving for the temperature difference, we get: $\Delta T = L_v/C_p \cdot (M_{W-Lost}/ M_{air})$.

The part in the bracket is just the mass mixing ratio of the water lost, $\mu_{lost} = \frac{1}{4} \cdot \mu$. We calculated μ in part a) to be 6.22×10^{-3}, which gives a $\underline{\boldsymbol{\Delta T = 3.9\ K}}$. and the parcel will be the original temperature plus this amount when it returns to the starting pressure. That is, $\underline{\boldsymbol{T_{new} = T_{start} + \Delta T = 292.1\ K = 18.9\ °C}}$. If ΔT is calculated but the student fails to calculate the new temperature or indicate clearly that the parcel is warmer by this amount, I will deduct one point.

e) If it did not snow and rain, and the parcel kept all of its water vapour, what temperature would the parcel have had when it came back down to 1000 hPa? Why? (3%)

Since all the latent heat that went into the parcel when the water condensed went back into the internal energy of the water vapour when it re-vaporized, the parcel will have the exact same temperature as when it started. If an explanation was not given I deducted 2 points

4) **(20 %)** Jupiter, with a radius of 71490 km, has an albedo of 0.34 and is 5.203 AU (Astronomical Units) from the Sun (the Earth is 1 AU from the Sun).

a) What is the radiative equilibrium temperature of Jupiter. (12%)

So to calculate the radiative equilibrium temperature of Jupiter, we need to know the solar flux (W/m^2) incident on Jupiter's surface. However, all we know is the solar flux at the radius of the earth's orbit, R_{eo} (1 au, given in the formulae), and the radius of Jupiter's orbit, R_{jo}. So, if the Sun has an intensity of L_o Watts, then the solar constant $S_o = L_o/(4 \cdot \pi \cdot R_{eo}^2)\ W/m^2$. At Jupiter, the solar constant $S_j = L_o/(4 \cdot \pi \cdot R_{jo}^2)\ W/m^2$. That is, $S_j = S_o \cdot (R_{eo}/R_{jo})^2$. In astronomical units, this is just $\underline{\boldsymbol{S_j = S_o \cdot (1/5.203)^2 = 1367 \cdot (1/5.203)^2\ W/m^2 = 50.5\ W/m^2}}$.

Now, it is just the radiative equilibrium problem of balancing the Power Absorbed by the Power Radiated, and realizing that the amount reflected, α (the albedo), does not contribute to the heating, this is:
$$S_j\ (1 - \alpha)\ \pi\ R_j^2 = 4\ \pi\ R_j^2\ \sigma\ T_j^4$$

Where R_j is the planet's radius and σ is the Stefan-Boltzmann constant (from the formulae). Of course The planetary radius drops out and we have the familiar form:

$$\frac{Sj\,(1-\alpha)}{4} = \sigma\, Tj^4$$

Solving for T_j given the solar constant at Jupiter and the albedo yields a temperature of __$T_j = 110.1$ K__.

b) The observed blackbody temperature of Jupiter is 124 K. Compare with your radiative equilibrium temperature from part A, and estimate the power emitted from the top of the atmosphere that is generated internally by processes on the planet. (8%)

The internally generated power must equal the difference between the radiative equilibrium power (which is proportional to the temperature calculated above, T_j) and the emitted power (which is proportional to the observed blackbody temperature, T_{bb}). That is, if it is radiating more power than it is receiving, that power has to come from somewhere, and hence it is generated in the planet's atmosphere. Thus:

$$Pint = 4\,\pi\,Rj^2\,\sigma\,(Tbb^4 - Tj^4)$$

Now we need the planet's radius, and can calculate the internally generated power to be, upon substituting for T_j from above: __$P_{int} = 3.25 \times 10^{17}$ Watts__.

5) **(20 %)** In class we calculated the radiative equilibrium temperature of the Earth using a single layer to represent the atmosphere and observed a greenhouse effect. To simulate both a boundary layer and the free tropopause, we can expand the model by taking two layers at 0.5 km and 2 km height. Each layer absorbs all infrared radiation (Transmission$_{L\lambda}$=0) and is transparent to solar radiation (Transmission$_{S\lambda}$=1).

a) Calculate the surface temperature and the temperature of the two layers assuming a surface albedo of 0.3 and radiative equilibrium. (12%)

Here we have to realize that a perfect absorber is also a perfect radiator (Kirchoff's Law). Thus, the IR radiation absorbed by a layer of the atmosphere will be re-radiated in both directions. When we look at the flux to the surface, the flux from the surface to the lower level (it will not pass through the lower level to the upper level), and the flux from the atmospheric layers themselves, we have:
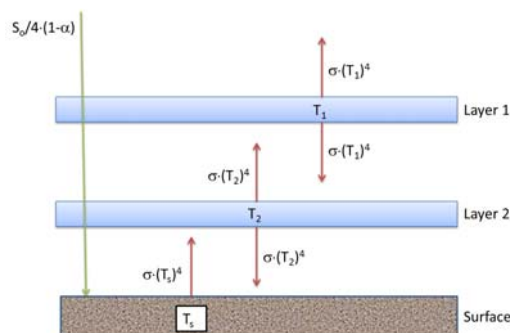
Where Layer 1 only receives radiation from layer 2, and layer 2 receives radiation from both the surface and Layer 1. Since the atmospheric transmission for visible light is 1, only the surface absorbs the incoming solar radiation.

The explanation for the So/4·(1-α) is the same as in problem 4. That is, solar power absorbed by the earth = $\pi \cdot R_e^2 \cdot S_o \cdot (1-\alpha)$, and the power radiated from all levels is $4 \cdot \pi \cdot R_e^2 \cdot (T_{level})^4$.

We need to balance the power in vs the power out at each level, so we have at Layer 1:

$$\sigma T2^4 = 2\, \sigma\, T1^4$$

At Layer 2:

$$\sigma\, T1^4 + \sigma\, Ts^4 = 2\, \sigma\, T2^4$$

And at the Surface

$$\frac{So\,(1-\alpha)}{4} + \sigma\, T2^4 = \sigma\, Ts^4$$

If we Start with Layer 1 and get T_1 in terms of T_2, we can eliminate T_1 in the expression for Layer 2, and then get and expression for $\sigma(T_2)^4$ in terms of $\sigma(T_s)^4$. This lets us go down to the surface and get an expression for $\sigma(T_s)^4$ in terms of the incoming solar radiation. These steps give:

$$\sigma\, T1^4 = \frac{\sigma\, T2^4}{2}$$

$$\sigma\, T2^4 = \frac{2\, \sigma\, Ts^4}{3}$$

$$\sigma\, Ts^4 = 3\, \frac{So\,(1-\alpha)}{4}$$

In fact, we can show that for N layers, $\quad \sigma\, Ts^4 = \dfrac{(N-1)\, So\,(1-\alpha)}{4}$

Putting in S_o= 1367 W/m² and α= 30%, we get:

$\underline{T_s = 335.4\ K = 62.3\ °C}$
$\underline{T_2 = 303.1\ K = 29.9\ °C}$
$\underline{T_1 = 254.9\ K = -18.3\ °C}$

b) If the dry adiabatic lapse rate is taken to be 9.8 K/km, is the air between layer 1 and layer 2 stable? Why or why not? (4%)

Here we can calculate the environmental lapse rate to be:
$\Gamma_e = -(\Delta T/\Delta z) = 48.2\ K/1.5\ km = 32\ K/km$.

*So that any **air displaced upward from the lower layer would find its temperature dropping only 9.8 K/km·1.5 km = 14.7 K, and it would be much warmer than the surrounding air. Thus, it will continue to rise and be unstable.***

c) What will happen to the air between layer 2 and layer 1?
How will this affect the power radiated to space from the uppermost layer? (4%)

*Since the air is unstable, **convective motions will develop** and **redistribute heat vertically** until the lapse rate adjusts to the dry adiabatic lapse rate.*

*This will **raise the temperature of Layer 1 above that of radiative equilibrium** (convectively transporting heat up to that layer) which will **cause it to radiate more power into space in an attempt to return to its radiative equilibrium temperature** (the Newtonian radiative spring). It will appear to have a hotter blackbody temperature.*

VEDLEGG

Bokmål oversettelse (tilnærmet) av eksamen hvis det er spørsmål om terminologi.

1) Relasjonen mellom det atmosfæriske lufttrykket og høyden, z, er gitt som:
$$P = P_o \cdot (1-z/L), \quad \text{der } L=8{,}5 \text{ km og } P_o=1000 \text{ hPa}.$$
   a) Hva er atmosfærens temperatur og tetthet ved høyden der lufttrykket er 950 hPa?
   b) En varmluft turistballong er i hydrostatisk likevekt ved høyden der lufttrykket er 950 hPa. Ballongens utstyr (men ikke luften inne i ballongen) veier 600 kg. Dersom ballongens volum er 3000 m^3, hva er temperaturen i luften inne i ballongen? (Hvis du ikke har løst del a), anta at temperaturen er 15 ^{o}C)

2) Romfartøyet Voyager-I observerte Titan, en av Saturns måner. Det oppdaget et sjikt med hydrokarbon aerosoler ved et lufttrykk på 1000 hPa, hvor den atmosfæriske temperaturen ble målt til 88 K. Selv om overflatetemperaturen ikke kunne måles, ble overflatetrykket målt til 1500 hPa. For Titans tørre atmosfære, som består av 80% nitrogen, er gasskonstanten R_T=290 J$\cdot$K$^{-1}\cdot$kg^{-1}, C_{PT}=1044 J$\cdot$K$^{-1}\cdot$kg^{-1}, og $g_T \ll g_{Earth}$.
   a) Beregn den maksimale overflatetemperaturen forutsatt at atmosfæren er stabil med hensyn til vertikale bevegelser.
   b) Senere ble den sanne overflatetemperaturen målt til 94 K av Voyager-II. Forventer du konveksjon i Titans atmosfære? Begrunn svaret.
   c) Forutsatt isoterme forhold, hvor mange skalahøyder over overflaten finnes hydrokarbon-aerosolsjiktet?

3) En luftpakke har trykk p= 1000 hPa, temperatur T = 15°C og relativ fuktighet på 58%.
   a) Hva er luftpakkens partialtrykk for vanndamp og blandingsforhold for vanndamp?
   b) Hva er luftpakkens duggpunktstemperatur?
   c) Hvis luftpakken blir ført langs fjellsiden, ved hvilket lufttrykknivå kan skyer dannes?
   d) Luftpakken i del (c) stiger opp over fjelltoppen der det snør og regner slik at pakken mister ¼ av sin vannmasse. Hvilken temperatur vil pakken ha når den kommer ned igjen høyden der lufttrykket er 1000 hPa?
   e) Dersom det ikke snødde eller regnet, og luftpakken i del (c) beholdt all sin vanndamp, hvilken temperatur ville luftpakken ha hatt da den kom ned igjen til høyden der lufttrykket var 1000 hPa?

4) Jupiter har en radius på 71.490 km, en albedo på 0,34 og ligger 5,203 AE (astronomiske enheter) fra Solen (Jorden ligger 1 AE fra Solen).
   a) Hva er Jupiters strålingsbalansetemperatur?
   b) Den observerte luminanstemperatur (et sort legemes temperatur) fra Jupiter er 124 K. Sammenlign med strålingsbalansetemperaturen fra del A, og anslå effekten som slippes ut fra toppen av atmosfæren, generert av interne prosesser på planeten?

5) I løpet av kurset beregnet vi jordens strålingsbalansetemperatur ved å la et enkelt sjikt representer atmosfæren og ut fra det bestemte en drivhuseffekt. For å simulere både et atmosfærisk grensesjikt og troposfæren kan vi utvide modellen til to lag, ett på 0.5 km og ett på 2 km. Hvert lag slipper gjennom synlig lys fra sola (Transmisjon$_{S\lambda}$=1) men absorberer all varmestråling (Transmisjon$_{L\lambda}$=0)
   a) Beregn overflatetemperaturen og temperaturen av de to lagene. Anta strålingsbalanse og at overflatealbedoen er 0,3.
   b) Hvis vi antar at den tørradiabatiske temperaturendringen er 9,9 K/km, er luften mellom lag 1 og lag 2 stabilt? Begrunn svaret.
   c) Hva vil skje med luften mellom lag 1 og lag 2? Hvordan vil dette påvirke utstrålt effekt fra det øverste laget mot verdensrom?