



**NTNU – Trondheim**  
Norwegian University of  
Science and Technology

Department of Physics

## **Examination paper for FY3201 Atmospheric Physics and Climate Change**

**Academic contact during examination: Patrick Espy**

**Phone: +47 41 38 65 78**

**Examination date: 4 June 2018**

**Examination time (from-to): 09:00 – 13:00**

**Permitted examination support material:**

Single or Bi-lingual dictionary permitted

All calculators permitted

1 side of an A5 sheet with printed or handwritten formulas permitted

**Other information:**

**Language: English**

**Number of pages: 5 + cover**

**Number of pages enclosed:**

**Checked by:**

---

Date

Signature

Norwegian University of Science and Technology  
Department of Physics

# EXAMINATION IN FY3201 ATMOSPHERIC PHYSICS AND CLIMATE CHANGE

Faculty for Natural Sciences and Technology

27 may 2019

Time: 09:00-13:00

Number of pages: 14

Permitted help sources: 1 side of an A5 sheet with printed or handwritten formulas permitted  
Single or Bi-lingual dictionary permitted  
All calculators permitted

You may take:

Molar mass of dry air:  $\sim 29 \text{ kg/kmole}$

Molar mass of helium:  $\sim 4 \text{ kg/kmole}$

Molar mass of  $\text{H}_2\text{O}$ :  $\sim 18 \text{ kg/kmole}$

$N_A = 6.02 \times 10^{23}$  molecules/mole

Boltzmann's constant  $k = 1.38 \times 10^{-23} \text{ J/K}$

$273.15 \text{ K} = 0^\circ \text{C}$

$1 \text{ hPa} = 10^2 \text{ Pa} = 10^2 \text{ N m}^{-2}$

$g = 9.8 \text{ m s}^{-2}$  and constant in  $z$

Stefan–Boltzmann constant:  $\sigma = 5.67 \times 10^{-8} \text{ W} \cdot \text{m}^{-2} \cdot \text{K}^{-4}$

Solar photospheric temperature,  $T_s = 5786 \text{ K}$

Radius of the Sun =  $695800 \text{ km}$

Radius of the Earth =  $6370 \text{ km}$

1 AU (Earth-Sun distance) =  $150 \times 10^6 \text{ km}$

Latent heat of vaporization water:  $L_v = 2.5 \times 10^6 \text{ J} \cdot \text{kg}^{-1}$

Latent heat of sublimation ice:  $L_i = 2.8 \times 10^6 \text{ J} \cdot \text{kg}^{-1}$

Gas constant for water vapour:  $R_v = 461 \text{ J} \cdot \text{K}^{-1} \cdot \text{kg}^{-1}$

Values for dry air:  $C_p = 1004 \text{ J} \cdot \text{K}^{-1} \cdot \text{kg}^{-1}$   $C_v = 718 \text{ J} \cdot \text{K}^{-1} \cdot \text{kg}^{-1}$   $R_d = 287 \text{ J} \cdot \text{K}^{-1} \cdot \text{kg}^{-1}$

$\gamma = C_p / C_v$   $\kappa = R_d / C_p$   $R_d = C_p - C_v$   $\Gamma_{da} = 9.8 \text{ K/km}$

Clausius–Clapeyron relation:  $e_s = 6.112 \text{ hPa} \cdot \exp \left[ \frac{L_v}{R_v} \left( \frac{1}{273 \text{ K}} - \frac{1}{T} \right) \right]$

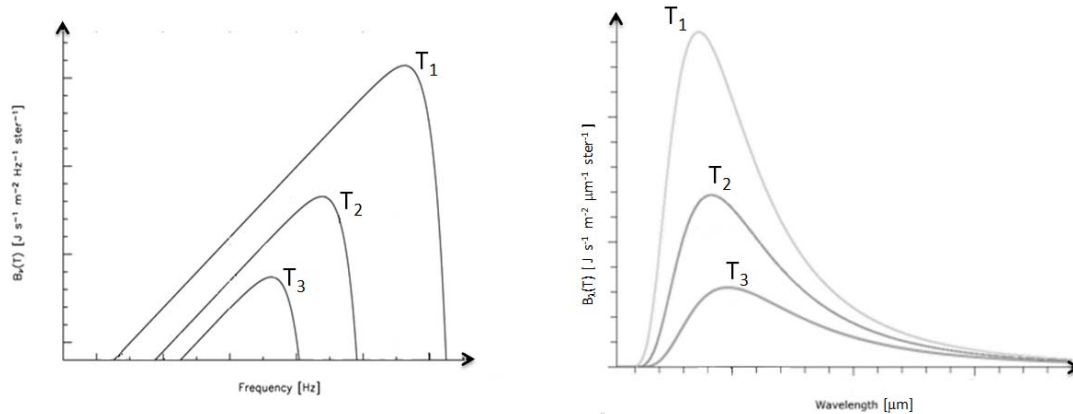
***Answer all questions (English, Norwegian, or Swedish).***

***State all assumptions.***

***Good Luck!***

### 1) (5%) Black body radiation

Sketch the spectral radiance (with intensity on the y-axis and wavelength on the x-axis) for three blackbodies at temperatures  $T_1 > T_2 > T_3$ . Label the curves with their temperatures and give the units used for the axes.

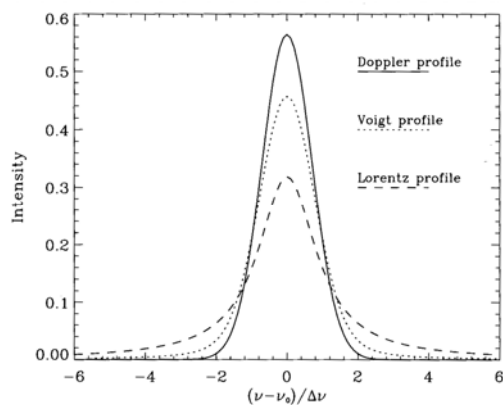


*Radiance units should be correct, although I would accept either wavelength or wavenumber units (either the left or right plot). I do not need the actual temperatures on the curves or the numbers on the axes. Curves should not cross, and the hottest temperature,  $T_1$ , should peak at shorter wavelengths (higher frequencies).*

### 2) (5%) Atmospheric Spectroscopy

Greenhouse gases absorb photons at specific wavelengths corresponding to the energy differences of the internal energy states of the molecule. However, there is a range of wavelengths about the line centre,  $\nu_0$ , which can also absorb. Sketch the relative absorption coefficients,  $k_\nu$ , for an absorption line at temperature  $T$  for both high and low pressures. For the y-axis, use  $(\nu - \nu_0)/\alpha$ , the distance from the line centre measured in line-widths,  $\alpha$ . Label the two curves with the line shape type.

*By definition, a greenhouse gas is transparent in the visible and absorbs in the infrared. Since we are looking at infrared absorption, the line shape would vary between Doppler and Lorentz-collision broadened. Since the pressure might not be low enough to be pure Doppler, I would also accept Voigt for the low pressure result.*



### 3) (20%) Atmospheric Stability

- a) Assuming dry air explain what is meant by (1) a stable, (2) an unstable and (3) a neutrally stable atmosphere. (4%)

Stable: the atmospheric (environmental) temperature profile drops more slowly than the dry adiabatic rate ( $\Gamma_{\text{dalr}} > \Gamma_{\text{env}}$ ). Any air parcel that is lifted will cool to a temperature lower than the surrounding gas at the new altitude, be heavier and sink back toward its original location.

Unstable: The atmospheric (environmental) temperature profile drops more rapidly than the dry adiabatic lapse rate ( $\Gamma_{\text{dalr}} < \Gamma_{\text{env}}$ ). Any air parcel that is lifted will cool to a temperature but be warmer than the surrounding gas at the new altitude, be lighter and continue to rise.

Neutrally stable: The atmospheric (environmental) temperature profile drops at the same rate as the dry adiabatic lapse rate ( $\Gamma_{\text{dalr}} = \Gamma_{\text{env}}$ ). Any air parcel that is lifted will cool to a temperature and be the same temperature as the surrounding gas at the new altitude. Thus it will remain at this position.

- b) A meteorological balloon is required to take measurements up to an altitude where the pressure and temperature are 10 hPa and 225 K respectively. The balloon is constructed of a non-stretch material that remains slack (that is, not stretched tightly) until the balloon reaches its peak altitude and has expanded to its full spherical shape. If the balloon contains helium (molecular weight  $4 \text{ g}\cdot\text{mol}^{-1}$ ), the payload weighs 100 kg and the fabric is of thickness  $25 \text{ }\mu\text{m}$  and density  $1000 \text{ kg}\cdot\text{m}^{-3}$ , what approximate radius of balloon is needed? (8%)

The bottom line is that the downward force due to gravity must be balanced by the buoyancy force. Thus, we must solve:

$$g \cdot (m_{\text{payload}} + m_{\text{fabric}}) = g \cdot (m_{\text{air}_10\text{hPa}} - m_{\text{He}_10\text{hPa}})$$

$$\text{or } (m_{\text{payload}} + \rho_{\text{fabric}} \cdot V_{\text{fabric}}) = (\rho_{\text{air}_10\text{hPa}} - \rho_{\text{He}_10\text{hPa}}) \cdot V_{\text{balloon}}$$

The way to look at this is that if I replace a mass of air that is sitting at 10 hPa with a balloon whose total mass (payload, balloon material and helium) is equal to that mass of the air, it, too, will just sit at 10 hPa.

Assuming dry air, we have that at the balloon altitude, the conditions outside are  
 $p = 10 \text{ hPa} = 1000 \text{ Pa}$

$T = 225 \text{ K}$

$$\rho_d = p / (R_d \cdot T) = 1000 \text{ Pa} / (287 \text{ J/kg}\cdot\text{K} \cdot 225 \text{ K}) = 0.015 \text{ kg/m}^3$$

Inside the balloon, we have

$$p' = p = 1000 \text{ Pa}$$

$$T' = T_{\text{He}}$$

$$\rho' = \rho_{\text{He}} = p / (R_{\text{He}} \cdot T') = p / (R_{\text{He}} \cdot T_{\text{He}})$$

To get  $T_{\text{He}}$ , I don't have any information on the ground temperature, and even if I did, I don't know the specific heats for helium to calculate its temperature at float. I must assume that the balloon has been at float altitude long enough so that the helium is at the same temperature as the surrounding air,

$$T_{\text{He}} = 225 \text{ K}.$$

Since the specific gas constant is just  $R^*/M_{\text{gas}}$ , I can calculate

$$R_{\text{He}} = R_d \cdot M_d / M_{\text{He}} = 287 \cdot 29 / 4 \text{ J/K}\cdot\text{kg} = 2081 \text{ J/K}\cdot\text{kg}, \text{ giving } \rho_{\text{He}} = 0.0021 \text{ kg/m}^3$$

At the float altitude, the balloon is spherical with radius  $R$ . The approximate volume of balloon material is  $4\pi R^2 t$ , where  $t$  is the thickness of the material. Therefore:

$$(m_{\text{payload}} + m_{\text{fabric}}) = 100 \text{ kg} + 25 \times 10^{-6} \text{ m} \cdot 4\pi R^2 \cdot 1000 \text{ kg/m}^3$$

Here we have approximated the volume of the balloon material as the balloon area \* thickness. One could also say that it is the difference between the volume of the outer diameter of the balloon ( $4/3\pi R^3$ ) and the volume of the inner-diameter of the balloon ( $4/3\pi(R-t)^3$ ). The first yields

$$4\pi R^2 t \text{ while the latter yields}$$

$$4/3\pi t (3R^2 - 3Rt + t^2)$$

Even if the balloon radius were  $25\mu\text{m}$ , the difference would be on the order of  $10^{-13}$  and would approach 0 for larger radii

$$\text{Mass of air displaced} = \rho_d \cdot 4/3\pi R^3$$

$$\text{Mass of air taking its place} = \rho_{\text{He}} \cdot 4/3\pi R^3$$

$$F_{\text{buoyancy}} = g(\rho_d - \rho_{\text{He}})\pi R^3$$

And this must balance the total weight of the balloon,  $(m_{\text{payload}} + m_{\text{fabric}})$  if it is floating:

$$F_{\text{gravity}} = g(m_{\text{payload}} + m_{\text{fabric}}) = g(100 \text{ kg} + 25 \times 10^{-6} \cdot 4\pi R^2 \cdot 1000) \\ = g(100 + 0.1 \cdot \pi R^2)$$

So we need to solve for  $R$ :

$$g(\rho_d - \rho_{\text{He}})\pi R^3 = g(100 + 0.1 \cdot \pi R^2)$$

$$(0.015 - 0.0021)\pi R^3 = (100 + 0.1 \cdot \pi R^2)$$

I could also say that since they are both at the same pressure and temperature:

$\rho_{\text{He}} = \rho_d R_d / R_{\text{He}}$ , so my left hand side of the equation becomes:

$$g \cdot \rho_d (1 - R_d / R_{\text{He}}) \cdot 4/3\pi R^3 = g \cdot \rho_d (1 - M_{\text{He}} / M_d) \cdot 4/3\pi R^3 = g \cdot \rho_d (1 - 27/4) \cdot 4/3\pi R^3 \\ = g \cdot \rho_d (0.86) \cdot 4/3\pi R^3$$

Either way, I can now solve the cubic for  $R$ , or if you just come down to something like:

$$\rho_d (0.86) \cdot 4/3\pi R^3 = (100 + 25 \times 10^{-6} \cdot 4\pi R^2 \cdot 1000)$$

or since the density of air at the float altitude is  $0.014 \text{ kg/m}^3$

$$(0.015 \cdot 0.86 \cdot 4/3\pi) \cdot R^3 = 100 + 0.314 \cdot R^2$$

Or

$$100 = 0.056 \cdot R^3 - 0.314 \cdot R^2$$

If you can solve cubic equations, this comes out to be  **$R=14.3 \text{ m}$** , but really, any of the above equations would give full points as long as you had it in terms of one density and  $R$ .

Now, due to the wonders of the new digital exam system, someone in the chain cut and pasted the question and lost the formatting. Thus, a 25  $\mu$  balloon came out to be 25 mm! This makes the mass of the balloon  $100 \cdot \pi R^2$ , and the diameter of the **balloon 5618 m**! Obviously, this is ridiculous, but that is what it comes out to be with the mistake in the digital copy that was sent to you. So, it will be accepted.

- c) Assuming the mean temperature of 255 K between the surface, where the pressure is 1000 hPa, and the balloon altitude at 10 hPa, what is the altitude of the balloon in metres? (4%)

Ah, a problem where we have a constant temperature between two pressure levels and you are asked to calculate the difference in height between these pressure levels. This cries out for the hypsometric equation:

$Z_2 - Z_1 = R \cdot \langle T \rangle / g \cdot \ln(p_1/p_2)$ . So taking the 1 subscript to be 1000 hPa = 100000 Pa, and the 2 subscript to be 10 hPa = 1000 Pa, we have:

$Z_2 - 0 = 287 \cdot 255 / 9.8 \cdot \ln(100) = \mathbf{34390.7 \text{ m or about 34 km}}$ , well into the stratosphere.

- d) Assuming the atmosphere is stable with respect to vertical motion, estimate the temperature at the surface where the pressure is 1000 hPa. (4%)

Here you need to use the potential temperature. If the atmosphere is neutrally stable, then the potential temperature would be constant in altitude. It is most convenient to use the float altitude where the temperature is 255 K and the pressure is 10 hPa to calculate that  $\theta = 225 \text{ K}$ . Then the temperature at any other altitude, where  $\theta$  is still 225 K, is.

$T = \theta \cdot (P/P_0)^\kappa$ , where  $\kappa = R_d/C_p = 0.286$ . If the altitude is the surface, where  $P = 1000 \text{ hPa}$ , then  $P_0 = 10 \text{ hPa}$  gives  $T$  at the surface to be  $T = 839 \text{ K}$ . This is, of course, the maximum temperature that would create stable conditions, so any temperature lower than this would also be stable. That is, a parcel at 839 K on the ground, if transported up to 34 km, would just be at the temperature of the surrounding air. Below 34 km, it would be cooler than the surrounding air. Thus, any parcel at a lower temperature at the ground would be cooler than its surroundings up to and including 34 km, so the atmosphere is very stable. So we could say,  **$T_{\text{ground}} \leq 839 \text{ K}$**

If one tries to use the idea of the first order approximation to a constant  $\theta$ , a constant lapse rate of  $\Gamma_{\text{dalr}} = 9.8 \text{ K/km}$ , one comes up with about 562 K. Since this is only a first order approximation, it is unlikely to hold over 34 km! However, I will give 1 point for this solution.

Either way, it shows how Hollywood gets it wrong in movies like “The day after tomorrow”, where they say that the storm is bringing air down from the stratosphere and will freeze everything. In fact, that air would be over 900 K, and freezing would be the last of your problems!

#### 4) (20%) Atmospheric thermodynamics, water vapour and structure

- a) Calculate the period of oscillation of an air parcel given that  $dT/dz = -6.5 \text{ K} \cdot \text{km}^{-1}$  and  $T = 270 \text{ K}$ . (6%)

The formula for the Brunt-Väisälä frequency is  $N^2 = g/T * (\Gamma_d - \Gamma)$ , where  $\Gamma_d$  is the dry adiabatic lapse rate,  $g/C_p$ , and  $\Gamma$  is the atmospheric lapse rate is:

$$\Gamma = -dT/dZ = 6.5 \text{ K} \cdot \text{km}^{-1} = 0.0065 \text{ K/m}.$$

Substituting in the values

$$N^2 = 9.8/270 * (0.00976 - 0.0065) \text{ (remember, both in K/km or K/m, and } g/C_p \text{ will give K/m)}$$

This gives  $N^2 = 1.183 \times 10^{-4} \text{ (rad/s)}^2$ , or the buoyancy frequency  $N = 0.0109 \text{ rad/s}$ .

For a frequency in Hz, divide by  $2\pi$  to get  $N = 0.0017 \text{ Hz}$ . This makes the period

$$\tau_{\text{Brunt}} = \mathbf{577.53 \text{ sec or } 9.62 \text{ minutes.}}$$

- b) Air at a temperature  $20^\circ\text{C}$  and pressure  $1000 \text{ hPa}$  has a dew point of  $15^\circ\text{C}$ . What is (1) its relative humidity and (2) its water vapour mass mixing ratio? (6%)

Consider this practice for the problem below. The expression for the saturation vapour pressure of water vapour is listed in the equations, giving at  $T_d = 288.15$ :  $e_s(T_d) = 17.37 \text{ hPa}$ . Now, we know that  $\mu(T_o, P_o) = \mu_s(T_d, P_o)$ , which, when cancelling all the  $\epsilon$  and  $P_o$  factors in the expression for mass mixing ratio means that  $e(T_o) = e_s(T_d)$ , so  $e(T_o) = 17.37 \text{ hPa}$ . How much water could the air hold at  $T_o$ ? Well just calculate  $e_s(T_o)$  using the formula to get  $e_s(T_o = 293.15) = 23.94 \text{ hPa}$ . The **Relative humidity is  $e(T_o)/e_s(T_o) = 17.37/23.94 = 72.5\%$** . To find the mass mixing ratio of water in the air, just multiply  $e_s(T_d) = 17.37 \text{ hPa}$  by  $\epsilon = M_v/M_d$  and divide by  $P_o = 1000 \text{ hPa}$ . This gives  **$\mu(T_o, P_o) = 0.0107$ , which is the mixing ratio,  $\mu$ , in the parcel at  $T = 293.15 \text{ K}$ ,  $P = 1000 \text{ hPa}$ .**

- c) Air initially at sea level with a temperature  $20^\circ\text{C}$  and dew point  $15^\circ\text{C}$  is forced to rise over a mountain of height  $1000 \text{ m}$ . What are the temperature, dew point and relative humidity of the air when it has sunk to a level  $200 \text{ m}$  above sea level on the other side of the mountain? (assume no precipitation takes place). (8%)

Since we are allowed to assume that no precipitation takes place, the process will be reversible and the conditions  $0 \rightarrow 1000 \text{ m} \rightarrow 200 \text{ m}$  will be the same as if the parcel went from  $0 \rightarrow 200 \text{ m}$ . We will also assume the surface pressure to be  $1000 \text{ hPa}$ .

The question is whether the air will be saturated at  $200 \text{ m}$ . To do this we need to know what the environmental conditions are.

First we assume that no condensation takes place. This will allow the parcel to be transported to  $200 \text{ m}$  and cool according to the adiabatic lapse rate. Then we can calculate the pressure at this altitude again assuming no condensation takes place by conserving potential temperature. We can then check to see if the saturated mass mixing ratio at this new temperature pressure,  $T_1$  and  $P_1$ , is greater than the mass mixing ratio in the parcel. If it is, then our assumption of no condensation holds. If not, we should adjust  $T$  by the latent heat of vaporization for the amount of water vapour turned into liquid water, and then adjust  $P$  accordingly.

The parcel moving upward will decrease temperature according to the dry adiabatic lapse rate,  $T = T_o - g/C_p * Z$ . For the pressure at this altitude, we can derive from the hydrostatic equation and the ideal gas law that  $dP/P = -$

$g/(R \cdot T) \cdot dZ$  and integrate from 0 to  $Z$ . However, we could also use the fact that an atmosphere with a temperature gradient equal to the dry adiabatic lapse rate will have a constant potential temperature at each altitude. Thus, at  $P_0$ ,  $\theta = T_0 \cdot (P_0/(P=P_0))^{\kappa}$ , or  $\theta = T_0 = 293.15$  K. To get the pressure at any other altitude, we can solve the potential temperature equation for  $P$ :

$P = P_0 \cdot (T/\theta)^{1/\kappa} = P_0 \cdot (T/\theta)^{(C_p/R)}$ , where  $T$  is given by  $T = T_0 - g/C_p \cdot Z$ . This is exactly the result you get if you integrate the hydrostatic equation.

So, at  $Z=200$  m,  **$T=291.2$  K, giving  $P=976.897$  hPa**

**If no condensation takes place in the parcel, this will also be the temperature of the parcel at this pressure.** If condensation has taken place, the parcel will be warmer.

Now we just need to determine if the moisture in the parcel becomes saturated as it moves to 200 m. We have assumed the surface pressure;  $P_0$ , to be 1000 hPa, so, first calculate the saturated mass mixing ratio of water at the surface, which is the mass mixing ratio of water in the parcel.

We are given  $T_0 = 20^\circ\text{C} = 293.15\text{K}$ , and  $T_d = 15^\circ\text{C} = 288.15$ . The expression for  $\mu_s = \epsilon \cdot e_s(T_d)/P_0$ , where  $\epsilon$  is (approximately) equal to the ratio of the mass of water vapour to that of dry air, or  $18/29 = 0.62$ .

The expression for the saturation vapour pressure of water vapour is listed in the equations, giving at  $T_d=288.15$ :  $e_s=17.37$  hPa. This would give a saturation mass mixing ratio at a temperature of  $T_d=288.15\text{K}$ ,  $P=1000$  hPa of  $\mu_s=0.0107$ , which is the mixing ratio,  $\mu$ , in the parcel at  $T_0=293.15$  K,  $P=1000$  hPa.

Now calculate the saturated mixing ratio at 200 m. If the saturated mixing ratio at 200 m is greater than the amount of water we have in the parcel, then we are safe to just calculate the adiabatic temperature of a parcel moving with constant  $\theta$ .

We have calculated that at 200 m the air temperature is 291.2 K and the pressure is 976.9 hPa. Calculating  $\mu_s$  at this pressure and temperature gives  $\mu_s = 0.0134$ . Since this is greater than the amount of water in the parcel,  $\mu=0.0107$ , the parcel has not saturated, **no condensation has taken place and the parcel pressure and temperature are equal to the air pressure and temperature,  $T=291.2$  K,  $P=976.9$  hPa.**

You might try this with an isothermal profile. Now  $P=P_0 \cdot \exp(-Z/H)$ , where  $H$  is the scale height  $H=R \cdot T_0/g=8585.1$  m, or  $P=976.973$  hPa. In fact, this only differs from the potential temperature calculation in the second decimal place by 8 parts in 100. This will yield a slightly higher pressure, giving a lower value of  $\mu_s$  at 200 m that differs in the 7<sup>th</sup> decimal place. This would also be acceptable.

## 5) (20%) Radiation

a) (4%) Define (that is give the units) and the connection between:

1. Spectral radiance

$\text{W} \cdot \text{m}^{-2} \cdot \text{sr}^{-1} \cdot \text{nm}$ . The energy flux in a unit wavelength unit that is emitted in a direction within a unit solid angle. Could be given as  $W=\text{photon/s}$ , and



wavelength units (nm, m,  $\mu$ , etc) or frequency or wavenumber units (Hz or  $\text{cm}^{-1}$ ) for all answers.

2. Radiance

$\text{W}\cdot\text{m}^{-2}\cdot\text{sr}^{-1}$ . The energy flux integrated over wavelength that is emitted in a direction within a unit solid angle. This would be the Spectral radiance integrated over wavelength to represent the total energy within that wavelength band.

3. Irradiance

$\text{W}\cdot\text{m}^{-2}$ . The energy flux integrated over wavelength and solid angle. Represents the total energy flux into a hemisphere.

- b) Mars has a radius of 3389.5 km and the distance to the sun is 1.524 AU (astronomical units). If its surface albedo is 0.250, calculate its radiative equilibrium temperature assuming no atmosphere. (6%)

If one remembers the solar constant,  $F_s = 1367 \text{ W}\cdot\text{m}^{-2}$ , one could scale that. However, the solar temperature is given so the total power emitted at each unit area of the surface of the sun would be  $F_s = \sigma T^4$ , or  $5.67 \times 10^{-8} \text{ W}\cdot\text{m}^{-2}\cdot\text{K}^{-4} \cdot (5786 \text{ K})^4 = 6.35 \times 10^7 \text{ W}\cdot\text{m}^{-2}$ .

So the total energy emitted by the sun in watts would be this times the surface area of the sun  $= 4\pi R_s^2$ . This yields  $3.9 \times 10^{26}$  Watts. This energy is emitted over  $4\pi \text{ sr}$ , or into a sphere. At the orbit of the Earth, this energy would be spread over a sphere of area of  $4\pi (R_{\text{eo}})^2$ , where  $R_{\text{eo}} = 150 \times 10^9 \text{ m}$ . That is, one would divide by this area, so that the solar flux at any orbital distance is given by:

$F_s (R_s/R_{\text{orbit}})^2$ . So at the Earth's orbit we have  $F_e = 6.35 \times 10^7 \text{ W}\cdot\text{m}^{-2} \cdot (6.958 \times 10^8 / 1.5 \times 10^{11})^2 = 1367.36 \text{ W}\cdot\text{m}^{-2}$ . At the orbit of Mars,  $R_{\text{mo}} = R_{\text{eo}} \cdot 1.524 = 2.29 \times 10^{11} \text{ m}$ , so the using  $R_{\text{mo}}$  as  $R_{\text{orbit}}$  yields a flux at Mars  $F_m = 588.7 \text{ W}\cdot\text{m}^{-2}$ .

If one remembered the  $F_e$ , and then scaled the Earth value, one would use  $F_e \cdot 4\pi R_e^2 / (4\pi R_m^2)$ , which comes out to be  $F_e \cdot (1/1.524)^2$ , giving the same value  $F_m = 588.7 \text{ W}\cdot\text{m}^{-2}$ .

Now, we have  $F_m$ , and an albedo of 0.25. With no other information, we need to take the no atmosphere situation and calculate radiative balance.

$F_m \cdot (1-A) \cdot \pi (R_m)^2 = \sigma (T_m)^4 \cdot 4\pi (R_m)^2$ . Solving for  $T_m$ , we obtain:

$T_m = (1/4 \cdot (1-A) \cdot F_m / \sigma)^{1/4}$ . Substituting in values we get  **$T_m = 210 \text{ K}$**

- c) The surface temperature of Mars is measured to be 242 K, what is the long-wavelength optical depth and transmission of the Martian atmosphere assuming the short-wavelength transmission is 1? (4%)

So here we are asking you to include an atmosphere. I will refer you to the notes of lecture 3, where the beam coming to the earth is reduced by  $T_{s\lambda}$ , in this case 1, and the outgoing radiation reduced by  $T_{L\lambda}$  by atmospheric absorption. One also has to take into account that the energy absorbed in the long wavelengths will heat the atmosphere, which itself will radiate in both directions as  $\sigma (T_a)^4$ . This yields radiative balance equations (input equal to output) at the top of the atmosphere

and the surface of the earth. Eliminating  $T_a$  from these two equations and solving for  $T_m$ , we get the equation:

$$\sigma_B \cdot T_m^4 = \frac{1}{4}(1-A) \cdot F_m \cdot \frac{1+T_{s\lambda}}{1+T_{L\lambda}}$$

We can solve for  $T_{L\lambda}$  in this equation as:

$$T_{L\lambda} = (1/4 \cdot (1-A) \cdot F_m \cdot (1+T_{s\lambda}) / (\sigma(T_m)^4)) - 1$$

With  $T_{s\lambda}=1$ , and  $T_m = 242$  K, and albedo  $A=0.25$ , we get

$$T_{L\lambda} = (1/4 \cdot (1-A) \cdot F_m \cdot 2) / (\sigma(T_m)^4) - 1$$

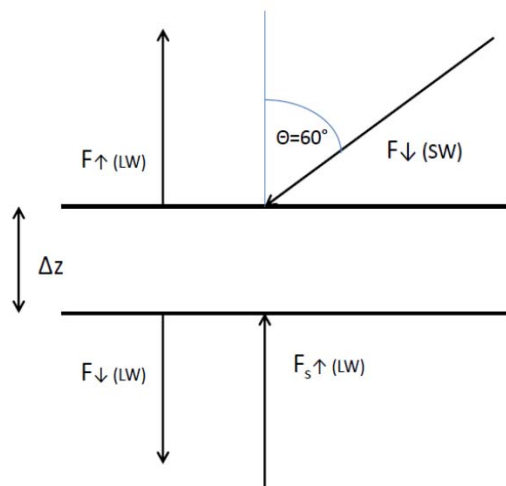
or

**$T_{L\lambda} = 0.13527$ , or about 13% transmission.**

Since  $T_{L\lambda} = \exp(-\tau)$ , where  $\tau$  is the optical depth, we can solve for  $\tau$  to be:

**$\tau = -\ln(T_{L\lambda}) = 2$** , so the atmosphere is optically thick.

- d) In the figure below, a layer in the atmosphere of thickness  $\Delta z$  and density  $\rho$  receives and emits radiation. The layer receives short-wavelength radiation  $F_{\downarrow}(\text{SW})$  that is at a zenith angle of  $\theta=60^\circ$ , and long-wavelength radiation  $F_s\uparrow(\text{LW})$  from directly below. In the layer, the species that absorb short wavelength radiation have a mixing ratio of  $v_{\text{abs}}(\text{SW})$  and a mass-absorption coefficient of  $k_{\text{abs}}(\text{SW})$ . The mixing ratio of species that absorb long wavelength radiation have a mixing ratio of  $v_{\text{abs}}(\text{LW})$  and a mass-absorption coefficient  $k_{\text{abs}}(\text{LW})$ . The flux of short wavelength radiation,  $F_{\downarrow}(\text{SW})$ , is given for a surface normal ( $90^\circ$ ) to the direction of the incoming radiation. Assuming the layer is in radiative equilibrium, calculate the temperature of the layer. (6%)



$$\rho = 0.2 \text{ kg} \cdot \text{m}^{-3}$$

$$F_{\downarrow}(\text{SW}) = 800 \text{ W} \cdot \text{m}^{-2}$$

$$k_{\text{abs}}(\text{SW}) = 100 \text{ m}^2 \cdot \text{kg}^{-1}$$

$$k_{\text{abs}}(\text{LW}) = 100 \text{ m}^2 \cdot \text{kg}^{-1}$$

$$v_{\text{abs}}(\text{SW}) = 1.0 \times 10^{-5}$$

$$v_{\text{abs}}(\text{LW}) = 3.0 \times 10^{-4}$$

$$\Delta z = 500 \text{ m}$$

$$F_s\uparrow(\text{LW}) = 200 \text{ W} \cdot \text{m}^{-2}$$

The situation we have is that the layer is heated by absorption of both short and long wavelength radiation, but is cooled by the emission of long wavelength radiation.

Absorption of short wavelength radiation

The energy per  $m^2$  incident on the layer from short wavelength radiation would be  $F_{\downarrow}(SW) \cdot \cos(\theta)$ , where  $F_{\downarrow}(SW)$  is the vertical downward flux.

The optical depth of the layer is given by:

$$\tau_{sw} = k_{abs}(SW) \cdot \rho \cdot v_{abs}(SW) \cdot \Delta z = 0.10$$

Remember, the optical depth is defined as the vertical distance, and one needs to account for the horizontal motion with the angle  $\theta$ . Thus, the transmission will be:

$$T_{sw} = e^{-\tau_{sw}/\cos(\theta)} = 0.82 \text{ and of course the absorption would be}$$

$$\alpha_{sw} = 1 - T_{sw} = 1 - e^{-\tau/\cos(\theta)} = 0.18$$

So, the short wavelength radiative energy absorbed is the layer,  $SW$  ( $\Delta F_a(SW)$ ):

$$\Delta F_a(SW) = \alpha_{sw} \cdot F_{\downarrow}(SW) \cdot \cos(\theta) = 72.5 \text{ Wm}^{-2}$$

Absorption of long wavelength radiation

The energy per  $m^2$  incident on the layer from long wavelength radiation would be  $F_s \uparrow(LW)$ , the vertical upward flux from the surface.

The optical depth for long wavelength radiation is given by:

$$\tau_{LW} = k_{abs}(LW) \cdot \rho \cdot v_{abs}(LW) \cdot \Delta z = 3.0$$

Again, the transmission and absorption long wavelength radiation for this vertical beam is:

$$T_{LW} = e^{-\tau} = 0.05, \quad \alpha_{LW} = 1 - T_{LW} = 1 - e^{-\tau_{LW}} = 0.95$$

And as a result, the long wavelength radiative energy absorbed in the layer,  $LW$  ( $\Delta F_a(LW)$ ) is:

$$\Delta F_a(LW) = \alpha_{LW} \cdot F_s \uparrow(LW) = 190.0 \text{ Wm}^{-2}$$

The final term we need is the cooling of the layer by the emission of long wavelength radiation

The layer at temperature  $T$  will emit long wavelength radiation per square metre as:

$$\Delta F_E(LW) = \epsilon \sigma T^4$$

Where  $T \epsilon$  is the long wavelength emissivity. Now, Mr. Kirchoff has told us that emissivity is equal to absorptivity,  $\alpha_{LW}$ , and we have already calculated that above to be 0.95.

Now, at the top-side of the layer (taking energy leaving the layer as negative) we have:

$$-\Delta F_E(LW) + \Delta F_a(SW) = 0 = -\epsilon \sigma T^4 + \alpha_{sw} \cdot F_{\downarrow}(SW) \cdot \cos(\theta)$$

And at the bottom side:

$$-\Delta F_E(LW) + \Delta F_a(LW) = 0 = -\epsilon \sigma T^4 + \alpha_{LW} \cdot F_s \uparrow(LW).$$

Combining these, we have

$$\alpha_{sw} \cdot F_{\downarrow}(SW) \cdot \cos(\theta) + \alpha_{LW} \cdot F_s \uparrow(LW) = 2 \cdot \epsilon \sigma T^4$$

$$\text{Or } T = ((\alpha_{sw} \cdot F_{\downarrow}(SW) \cdot \cos(\theta) + \alpha_{LW} \cdot F_s \uparrow(LW)) / (2 \epsilon \sigma))^{1/4}$$

Substituting in values, we get **T=222 K**



There is only **one** correct answer so you must **choose the best answer**.

Answer A, B, C... (Capital letters).

Correct answer gives +3; incorrect or blank answers give 0.

Write the answers for the multiple choice questions **on the answer sheet you turn in** using a table similar to the following:

Question	a	b	c	d	e	f	g	h	i	j
Answer										

- 6) At which wavelength does the Earth's blackbody irradiance peak?
- A) ~150 nm   B) ~550 nm   C) ~1500 nm   **D) ~15000 nm**   E) ~150000 nm
- 7) If the atmospheric absorption of carbon dioxide at 15  $\mu\text{m}$  becomes saturated, what happens if carbon dioxide levels continue to increase?
- A) Total absorption stays the same because it is saturated  
 B) Total absorption begins to decrease near the band centre as it saturates  
 C) The absorption continues to increase but only near the band centre  
**D) Total absorption increases as lines farther from the band centre begin to saturate**  
 E) None of the above
- 8) If an atmospheric lapse rate of 6 K/km is measured, which of the following is true?
- A) The temperature falls with altitude and the atmosphere is absolutely unstable  
**B) The temperature falls with altitude and the atmosphere is conditionally unstable**  
 C) The temperature rises with altitude and the atmosphere is conditionally stable  
 D) The temperature rises with altitude and the atmosphere is conditionally unstable  
 E) The temperature rises with altitude and the atmosphere is absolutely stable
- 9) Describe the change in the Earth's visible albedo if the polar ice caps melt.
- A) It will increase  
 B) It will first increase and then decrease  
 C) It will not change  
 D) It will first decrease and then increase  
**E) It will decrease**  
 F) None of the above.

- 10) In which layer of the atmosphere is ozone the major species?
- A) Stratosphere
  - B) Mesosphere.
  - C) Troposphere.
  - D) Thermosphere.
  - E) Exosphere.
  - F) **None of the above.**
- 11) A small cloud droplet will evaporate \_\_\_\_\_ a large cloud droplet?
- A) At the same rate as
  - B) More slowly than
  - C) **Faster than**
  - D) It will not evaporate
  - E) None of the above
- 12) Which two atmospheric layers would the mean temperature profiles be stable against convection?
- A) Mesosphere and Stratosphere
  - B) Mesosphere and Thermosphere
  - C) Mesosphere and Troposphere
  - D) **Stratosphere and Thermosphere**
  - E) Stratosphere and Troposphere
  - F) The atmosphere is never stable against convection
- 13) From what phenomenon does the Coriolis effect arise?
- A. Curvature of the Earth's surface
  - B. **Rotation of the spherical Earth around its axis**
  - C. Rotation of the spherical Earth around the sun
  - D. Effect of winds high in the atmosphere.
  - E. Motion of the oceans in their basins
  - F. None of the above.
- 14) In late afternoon at the Equator, in which direction would you look to find a rainbow?
- A) North
  - B) South
  - C) **East**
  - D) West
  - E) There are no rainbows at the equator due to the high sun angles
- 15) If no feedbacks are included, what would the climate sensitivity factor be for a  $\pm 1\%$  change in solar irradiance?
- A) **72 °C per fractional change in solar irradiance**
  - B) 30 °C per fractional change in solar irradiance
  - C) 200 °C per fractional change in solar irradiance
  - D) -72 °C per fractional change in solar irradiance
  - E) -30 °C per fractional change in solar irradiance
  - F) -200 °C per fractional change in solar irradiance