Radiation

1) A parallel beam of visible radiation at a single wavelength is passing vertically upward through a layer of 1 km in thickness, containing a homogeneous gas with a density of 0.03 kg m−3 with an absorption coefficient k=0.5 m2 kg−1 and a zero scattering coefficient. The absorption coefficient is zero outside of this region. What is the optical thickness of the layer?

15

-15

0.015

-0.015

1

-1

The optical depth is given by:

$$\tau = \int_{Z}^{\infty} k\, \rho\, dz$$

Here, where k and rho are constant, they can be pulled from the integral, and the integration goes from Zs to Ze:

$$\Delta\tau = k \cdot \rho \int_{z_s}^{z_e} dz = k \cdot \rho(z_e - z_s)$$

The integral over z extends from the bottom layer, say Z=0, to the top of the layer where the optical depth is zero (either k or ρ are 0). In this case, we integrate from 0 to 1 km (=1000 m) to find that $\tau = k \cdot \rho \cdot (1000\text{ m} - 0) = 0.5\text{ m}^2 \cdot \text{kg}^{-1} * 0.03\text{ kg} \cdot \text{m}^{-3} \cdot 1000\text{ m} = 15$.

2) at a different wavelength, the optical depth of a one kilometre thick layer is 5 for a vertical collimated beam, what is the transmission of the beam

0.7%

20%

1.5%

13.5%

The transmission is given by $T = \exp(-\tau) = \exp(-5) = 0.007$, or 0.7%

3) At what latitude and during what season is the daily-averaged insolation (amount of energy received from the Sun) at the top of the atmosphere the greatest?

South pole on Dec. 21

Equator at Equinox

Equator on Jan. 5

North pole on June 21

During NH winter, SH summer, the Earth is closest to the Sun due to its eliptical orbit and thus the solar "constant" is about 7% greater than at NH summer when the Earth is farthest from the Sun. During summer at the South Pole, the atmosphere is illuminated 25 hrs per day, so this region of the atmosphere receives the largest daily-averaged insolation.

4) On a spacecraft, one only has radiation as a cooling mechanism. A spacecraft's thin, rectangular 10 m^2 solar panel is inclined at an angle of 30^o to the direct solar beam (the direction toward the Sun). The panel has an albedo for solar radiation of 0.1 and an emissivity for thermal infrared radiation of 0.9. If the panel is such a good heat conductor that both its faces always have equal temperature, calculate the temperature of the solar panel at radiative equilibrium if the total solar irradiance on the panel is 1367 Wm^{-2}.

278 K

298 K

331 K

0 C

The solar panel presents a cross-sectional area of A=10m^2·sin(30°) = 5 m^2. The energy onto the panel is therefore 1367 ·5 Watts., but the albedo, a=0.1, means that 10% is reflected away, leaving only 0.9 of this being absorbed by the panel. Thus,

energy in = F·A·sin(θ)·(1-a)=1367·10·0.5·(1-0.1) = 6151.5 W

Every square meter of the panel emits as ε·σ·T^4, where ε is the emissivity, 0.9, and σ is the Stefan-Boltzmann constant=5.67x10-8 W·m^{-2}·K^{-4}. Remember that both sides of the panel emit:

energy out = 2·A·ε·σ·T^4 = 2·10·0.9·5.67x10^{-8}·T^4. Solving Input = output for T gives:

$$T = \left(\frac{1}{2}\cdot\frac{F\cdot sin(\theta)\cdot(1-a)}{\epsilon\cdot\sigma}\right)^{\frac{1}{4}}$$

Substituting in the values yields T = 278.6 K, or 5.5 °C

Meteoology Graphs

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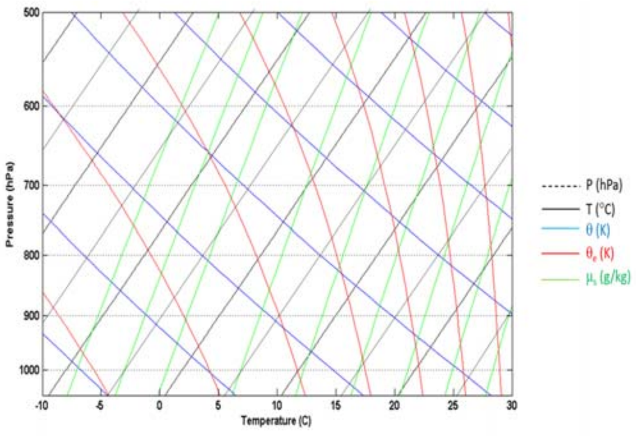
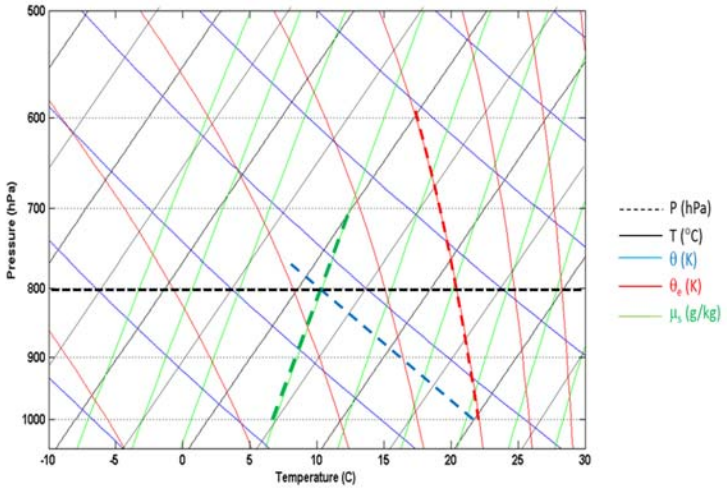
5) On the Skew-T log-P diagram, at a pressure of 1000 hPa an air parcel has a temperature T=20 C and a dewpoint temperature of Td=5 C.

a) At what pressure would the parcel's Lifting Condensation Level (LCL) be?

b) If the parcel at 1000 hPa had a dew point temperature of 20 C and a temperature of 20 C, what is the air parcel's temperature when it is lifted to 600 hPa?



Part a	Part b
800 hPa	0 C
850 hPa	-20 C
750 hPa	-10 C
550 hPa	-5 C
900 hPa	5 C



a) Lifting a parcel at 1000 hPa that was 20 C and had a dew point of 5 C, the parcel would travel adiabatically until the water in the parcel was equal to the saturated mass mixing ratio.
Since $\mu_s(T_d, P) = \mu(T,P)$, the parcel will move parallel to the dry adiabat (blue line) until it reaches the saturated mixing ratio it had at $\mu_s(T_d, 1000 \text{ hPa})$, the green line, which is approximately 800 hPa. At this point, the water may saturate, and this is the LCL.

b) If the dew point is equal to the temperature, the water in the parcel is already saturated. Thus, lifting the parcel will condense more water, and the parcel must move along the saturated adiabat (the red line) until it reaches 600 hPa. The temperature at this point is approximately 0C.

Spectroscopy

6) At a wavelength of 500 nm, the scatter from a 50 nm radius particle is approximately:

Rayleigh

Mie

Geometric

In the forward direction

Able to create a rainbow

The ratio of R to λ is 0.1, which means that the particle is much smaller than the wavelength of light. Thus, the scatter would be Rayleigh scatter, characterized by nearly symmetric scatter in all directions. If the particles were on the order of the wavelength ($R/\lambda \approx 1.0$) the scatter would be Mie-like, with most of the scattered light moving in the direction of the incoming beam.

7) For light at a wavelength of 4300 nm in a dry isothermal atmosphere at T_o=275 K with a surface pressure of 1000 hPa, at what altitude does the natural line width equal the Lorentz, collision broadened line width.

80 km

30 km

50 km

100 km

Light at 4300 nm is a wavenumber of 2325.6 cm^{-1}. Equating the natural and Lorentz line widths, $\alpha_N = \gamma_L$, gives:

$$\alpha_N = 3.3x10^{-8}\,cm^{-1}\cdot\left(\frac{\nu}{1000\,cm^{-1}}\right)^3 = \gamma_L = 0.01\,cm^{-1}\cdot\left(\frac{p}{1000\,hPa}\right)\left(\frac{273\,K}{T}\right)^{1/2}$$

Gives us: P = 1000 hPa· (3.38x10^{-8} cm^{-1}/0.01 cm^{-1})· (sqrt(T)/sqrt(273 K)) ·(ν^3/(1000 $cm^{-1})^3$)

Or P= 2x10^{-11} hPa ·cm^3·$K^{-½}$ · sqrt(T) ·ν^3, with T in K and ν in cm^{-1}, resulting in P in hPa.

At 275 K, and ν=2325.6 cm, this is P=0.042 hPa.

Since it is an isothermal atmosphere at 275 K, P is also equal to:

$P=P_o\cdot\exp(-Z/H)$, where H = R*T_o/g = 8054 m, and P_o=1000 hPa. Solving for Z, with P=0.042 hPa and P_o=1000 hPa, gives

Z = -H*ln(P/P_o) =81166 m, or 82 km

8) Molecular nitrogen does not absorb infrared dipole radiation to make vibrational transitions because:

N_2 has no permanent dipole moment

The dipole moment of N_2 is perpendicular to dipole radiation

N_2 actually does absorb infrared dipole radiation to make vibrational transitions

N_2 only makes rotational transitions at longer wavelengths instead of vibrational transitions.

If a molecule has no permanent dipole moment, the only way to make a dipole transistion is for the E-field of the photon wave to induce a temporary one. While it is possible to distort the electron cloud relative to the nuclei to induce a temporary dipole moment and make an electronic transition, it is not possible to induce a dipole moment between the nuclei as they are the same (in the case of N_2).

9) The average surface temperature of Venus is a balmy 480°C. At what wavelength is the planet's spectral irradiance greatest?

3850 nm

6040 nm

10000 nm

15000 nm

Just Wein displacement law, λ_{max}=2897.8/T will give the peak irradiance of a blackbody at temperature T in microns (10^{-6} m). At T = 480 + 273.15 K = 3.847 μ = 3847 nm.

10) Carbon dioxide is an important absorbing gas. Two fundamental wavelengths at which carbon dioxide absorbs are 4300 nm and 15000 nm. Absorption at these wavelengths is associated with excitation into an upper vibrational (and rotational) level. Is it possible for an average collision to excite these energy levels at atmospheric temperatures of 300 K?

No

Yes

Yes for 15000 nm, No for 4300 nm

No for 15000 nm, Yes for 4300 nm

The energy associated with collisions is given by $E_1=k\cdot T$ (monatomic gas), $E_2=3/2\cdot k\cdot T$ (diatomic gas) and $E_3=5/2\cdot k\cdot T$ (polyatomic gas), where k is Boltzmann's constant in J/K and T is the temperature. For a gas at 300 K, $E_1=4.14\times10^{-21}$ J, $E_2=6.21\times10^{-21}$ J, $E_3=1.04\times10^{-20}$ J.

The energy level seperation associated with a radiative transition is $E_r=h\cdot\nu^*$, where h is Planck constant in J·s and ν^* is the frequency of the transition = c/λ (where c is the speed of light).

For a energy associated with the transition at λ_1=4300 nm, the energy is 4.63×10^{-20} J, greater than E_1, E_2 or E_3. Similarly for λ_2=15000 nm, the transition energy is 1.33×10^{-20} J, greater than the collisional energies. Thus, collisions do not have enough energy to excite these transitions in the carbon dioxide molecule.

DYNAMICS

11) The geostrophic wind results from a balance between:

pressure gradient force and Coriolis force.

Coriolis force and centripetal force.

centripetal force, pressure gradient force, and Coriolis force.

pressure gradient force, Coriolis force, and friction.

The geostrophic wind describes the condition where the flow is parallel to lines of constant pressure (isobars). This occurs when the pressure gradient force, trying to drive flow across the isobars, is balanced by the Coriolis force, which always acts perpendicular to the flow velocity.

STRUCTURE

12) The ____________ and the ___________ are unstable layers of the atmosphere.

mesosphere, troposphere

troposphere, ionosphere

stratosphere, thermosphere

mesosphere, cryosphere

Whenever the evironmental temperature decreases with altitude, instabilities can occur. That is because a parcel displaced upward will expand and cool either adiabatically (about 9.6 K/km) or diabatically (4-6 K/km) if its water vapour is condensing. However, if the environmental temperature had cooled more than this, the parcel will be warmer than its surroundings and continue to rise-that is, become unstable. The troposphere and mesosphere are the regions of the atmosphere where the environmental temperature decreases with altitude.

13) Which of the following is true of a parcel of air but not true of the environment.

changes temperature with altitude at approximately the dry or moist adiabatic rates

temperature values differ erratically from one level to the next

consists of different air molecules at each level

humidity values differ erratically from one level to the next

The environmental temperature is what it is. It will be set by radiative and dynamic effects. Since the atmosphere is well mixed to 105 km, the same type air molecules, including water vapour, will occur at each level. Above 3 km, the humidity will drop continuously (not erratically) with altitude.

14) Which of the following would **NOT** be true concerning temperature change taking place in a parcel of dry air in an atmospheric temperature profile equal to the dry adiabatic lapse rate.

the temperature rises as the parcel rises

the parcel does not gain or receive heat from its surroundings

condensation does not occur during the process

the temperature falls as the parcel rises

The definition of an air parcel is that it does not exchange heat with its surroundings but will equilibrate its pressure. Since the atmosphereic pressure drops with altitude, the parcel will expand and cool as it rises into a less dense atmosphere. It is a dry paracel and so there is no water to condense. The false answer is that its temperature will rise as it ascends.

15) In the movie *The Day After Tomorrow*, the premise was that cold air in the mesosphere would descend to the surface, normally at 1000 hPa and 288 K, and freeze everything. Apparently an atmospheric scientist pointed out their mistake, and in later versions you can see they dubbed in “stratosphere” instead of “mesosphere”. If straosphereic air with a temperature of 270 K at 50 km, where the pressure is 0.8 hPa, were to descend adiabatically to the surface, what would its temperature be?

2070 K

6385 K

1712 K

15 K

35 K

If the air were to descend adiabatically, it would maintain its potential temperature, θ. If we take the initial conditions as those in the mesosphere, $P_o=2\text{x}10^{-3}$ hPa and $T_o=\theta=150$ K, then at its new pressure of 1000 hPa, we could calculate its temperature from the potential temperature relation:

$$\theta := T\left(\frac{Po}{P}\right)^{\kappa} \quad \text{or solving for T:} \quad T = \frac{\theta}{\left(\frac{Po}{P}\right)^{\kappa}}$$

Where κ = R/Cp = 287/1004 = 0.286. Substituting in for P=1000 hPa (note, keep Po and P in the same units), we calculate T = 2073 K. Admittedly still a problem, but I would not put on another jumper!

16) In which layer of the atmosphere is ozone the major species?

Stratosphere.
Mesosphere.
Troposphere.
Thermosphere.
Exosphere.
None of the above

Ozone is a minor species throughout the atmosphere, never rising to more than 5-10 parts per million by volume.

17) Commercial passenger aircraft flies must maintain a cabin pressure of 756 hPa at flight level. Usually, the cabin is at a temperature of 20 Celsius at flight level and maintains this temperature as it lands. However, the cabin pressure equilibrates to the outside pressure once it is above 756 hPa. If the plane lands at sea level (P=1000 hPa), what is the maximum the percentage relative humidity the cabin can be at flight level to prevent fog forming in the cabin as it lands.

76%

100%

50%

25%

5%

Here, we have that Po = 75600 Pa and To=20 +273.15 = 293.15 K. The saturation vapour pressure of water vapour under these conditions is e_s(To), and the partial pressure of water vapour is e(To). These are related by the percentage relative humidity, RH, by e(To)=RH/100·e_s(To), since RH is defined as the percentage ratio between the partial pressure of water and the saturation partial pressure.

Now, we know that the mass mixing ratio of water vapour will be constant until condensation occurs (i.e. fog forms). The mass mixing ratio of water vapour at flight level is μ(To,Po) = ε·e(To)/Po, which is, from the above can be expressed as the fraction of the saturation partial pressure, μ(To, Po) = ε· RH/100·e_s(To)/Po.

As the plane lands, the temperature is constant at To, but the pressure varies from 756 hPa to 1000 hPa at landing. In order to prevent fogging, μ(To, Po) must be less than the saturation mass mixing ratio all the way to 1000 hPa = 100000 Pa. Since To is held constant, the saturated mass mixing ratio is a function of pressure only, μ_s(To, P) = ε·e_s(To)/P. Thus:

$$\mu(T_o, P_o) < \mu_s(T_o, P)$$

Or

$$\epsilon \cdot \frac{RH}{100} \cdot \frac{e_s(T_o)}{P_o} < \epsilon \cdot \frac{e_s(T_o)}{P}$$

Most everything cancels, and solving for RH gives:

$$RH < 100 \cdot \frac{P_o}{P}$$

The maximum RH in the cabin at flight level to avoid reaching saturation at sea level (P=100000 Pa) would be 75.6%. I suspect that a 20 °C cabin at that level of humidity would be more uncomfortable than a typical Ryanair flight.

18) A desert site has a temperature of 38 C and a relative humidity of 14%. At the same time, an arctic site has a temperature of -2C and a relative humidity of 98%. If both sites are at 1000 hPa, which site has the highest density of water vapour in the air.

The desert has a higher density of water vapour

The arctic has a higher density of water vapour

The two sites have an identical amounts of water vapour

The arctic has a higher mass mixing ratio than the desert since the air is more dense

The partial pressure of water vapour in the air is given by e(T) = RH/100·e_s(T), where e_s(T) is the saturation vapour pressure. However, to examine the density, we employ Dalton's Law that says the ideal gas law applies to each constituent gas independently. Thus: e(T) = $\rho_v \cdot R_v \cdot T$, where Rv is the specific gas constant for water vapour, 461 J/K/kg.

Using the Clausius–Clapeyron relation the saturation partial pressure of water vapour can be calculated:

$$e_s = 6.112\ hPa \cdot e^{\frac{L_v}{R_v}\cdot\left(\frac{1}{273}-\frac{1}{T}\right)}$$

Where Lv is the latent heat of vaporization of water, 2.5x10^6 J/K/kg.

For the desert site, T=38+273.15=311.15 K, giving e_s=69.8 hPa. At 14% relative humidity, the partial pressure of water vapour in the desert site is 9.77 hPa. Using the ideal gas law for water vapour, we find that the density of water vapour in the air is 6.8x10^{-5} kg/m^3. Even relative to the mass of air at P=1000 hPa, the mass mixing ratio of water vapour μ=ε·e(T)/P = 6g/kg.

For the Arctic site, T=-2+273.15 = 271.15 K, giving es= 5.33 hPa. At 98% relative humidity, this means that the partial pressure of water vapour in the Arctic site is 0.75 hPa. Here the ideal gas law gives the density of water vapour in the air is 4.18x10^{-5} kg/m^3. Here the mass mixing ratio of water vapour is μ=3.2 g/kg.

Thus, there are more water molecules in the air in the desert than in the arctic. However since the warm air can hold much more water vapour than cold air before saturating, the relative humidity is much less in the desert. The difference between the densities at the two sites is small, but there is a large difference in μ since there are fewer molecules of water vapour in the more dense cold air.

19) (***10 pts***) For an isothermal atmosphere at To=288 K and a surface pressure of 1000 hPa, how much has the total mass of ***carbon*** in the atmosphere increased as the carbon dioxide mixing ratio changed from 350 to 400 parts per million by volume (ppm). You may take the mixing ratio of carbon dioxide to be constant in altitude up to 80 km, above which it is zero. (Answers are in giga-tonnes, where 1 tonne = 1000 kg).

100 Gt

400 Gt

10 Gt

2 Gt

Isothermal should trigger your brain to say that the scale height is constant. Therefore, the combination of the hydrostatic and ideal gas law can be integrated to give:

$$P = P_o \cdot e^{-\frac{Z}{H}} \; with \; H = \frac{R_d \cdot T}{g}$$

R_d the specific gas constant for a dry atmosphere, 287 J/K/g, and g the acceleration of gravity, 9.8 m/s^2. The partial pressure of CO_2, on the other hand, depends on its mixing ratio, ν_{CO2}, which varies from 350 to 400 ppmv (or a fraction of the total number of molecules from 3.5×10^{-6} to 4×10^{-6}). Hence the partial pressure of CO^2 is

$$P_{CO_2} = \nu_{CO_2} \cdot P_o \cdot e^{-\frac{Z}{H}}$$

Dalton's Law tells us that each gas in a mixture behaves as if it is the only gas present. Thus, the density of CO_2 would be the partial pressure of CO_2 divided by $R_{CO2} \cdot T$, where $R_{CO2} = M_d/M_{CO2} \cdot R_d =$ 189 J/K/kg.

$$\rho_{CO_2} = \frac{P_{CO_2}}{R_{CO_2} \cdot T} = \frac{\nu_{CO_2}}{R_{CO_2} \cdot T} \cdot P_o \cdot e^{-\frac{Z}{H}}$$

This can also be arrived at using $\rho_d = P/(R_d \cdot T)$ and the mass mixing ratio of CO_2, $\mu_{CO2} = M_{CO2}/M_d \cdot \nu_{CO2}$. Then the density of CO_2 is just $\rho_{CO2} = \mu_{CO2} \cdot \rho_d$. Since the specific gas constant of gas i is: $R_i = R^*/M_i$, with R^* being the universal gas constant, this is the same as the equation above.

So, we have the mass density as a function of altitude, which we can integrate from 0 to 80000 m. This will give us the mass in a column of atmosphere 1 m^2 area. Now we could do this for 350 ppmv and 400 ppmv and difference, but let us work out the column mass of carbon dioxide per 1 ppmv, and just multiply by 400-350= 50 ppmv. That is take $\nu_{CO2} = 1\times10^{-6}$. So, first integrate ρ_{CO2} in altitude:

$$\rho_{CO_2}^{col} = \int_0^{80\,km} \rho_{CO_2} \cdot dZ = \int_0^{80\,km} \frac{\nu_{CO_2}}{R_{CO_2} \cdot T} \cdot P_o \cdot e^{-\frac{Z}{H}} \cdot dZ$$

This integrates rather easily to:

$$\rho_{CO_2}^{col} = \frac{\nu_{CO_2}}{R_{CO_2} \cdot T} \cdot P_o \cdot H \cdot \left(1 - e^{-\frac{80000}{H}}\right)$$

Assuming H is in metres, and for Po=100000Pa, T= 288K, νCO2=1x10-6, R_{CO_2}=189 J/K/kg, and H=R_d·T/g=287 J/K/kg*288 K/9.8 m/s^2 = 8434 m, gives:

$$\rho_{CO_2}^{col} = 0.0155 \, \frac{kg}{m^2}$$

The surface area of the Earth is Ae = 4·π·Re^2, with Re=6370 x10^3 m ⇒ Ae=5.1x10^{14} m^2. Multiplying this by the atmospheric CO2 column density gives the total mass of 1 ppmv of CO2 in the atmosphere as

Atmospheric Mass of CO_2 per ppmv = 7.9x10^{12} kg,
or in tonnes (1 tonne = 1000 kg) 7.9x10^9 tonnes/$ppmv_{CO2}$

BUT, we were asked for the mass of CARBON in the atmosphere. Each CO2 molecule contains 1 Carbon atom, so the mass of carbon in the atmosphere would be the total mass of carbon dioxide times the molecular weight of carbon/the molecular weight of carbon dioxide, Mc/M_{co_2} = 12/44. That is because you could convert the mass of CO_2 into moles by dividing by its molecular weight, and each mole of CO_2 contains one mole of C. Then you convert moles of C into mass by multiplying by the molecular weight of C.

So, the atmospheric mass of carbon in the atmosphere for each ppmv of CO_2 is:

Mass of carbon = 12/44·7.9x10^9 tonnes = 2.2x10^9 tonnes/$ppmv_{co2}$.

With the addition of 50 ppmv CO_2, the mass of CO_2 in the atmosphere will increase by 7.9*50=395 Gt.

Atmospheric carbon mass increase for 50 ppmv increase in CO_2 = 1.08x10^{11} tonnes, or 108 Gt.

(note, that failing to convert Gt of CO2 to Gt of C will still give 5 points).