NTNU

Department of Physics (IFY)

Examination paper for FY3201/FY8902 Atmospheric physics and climate change

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Examination date: 9 June 2023

Examination time (from-to): 09:00-13:00

Permitted examination support material: Code G:

Textbook (or printed pdf of textbook) with no annotations inside

One SIDE of A5 paper with handwritten or printed notes

ALL calculators are allowed.

Other information:

Language: English

Number of pages (front page excluded): 7 pages

Number of pages enclosed: 7 pages (plus cover sheet)

Informasjon om trykking av eksamensoppgave

Originalen er:

1-sidig X 2-sidig □

sort/hvit □ farger X

skal ha flervalgskjema □

Checked by:

Date Signature

For multiple choice questions, each incorrect or blank answer will score zero points. Answers have been randomized and are not exact. You must choose the best answer. For all calculations use SI units!

You may take:

Molar mass of dry air: ~29 kg/kmole
Molar mass of helium: ~4 kg/kmole
Molar mass of H_2O: ~18 kg/kmole
Molar mass of CO_2: ~44 kg/kmole

273 K = 0 °C 1 hPa = 10^2 Pa = 10^2 N m^{-2} 1 atm = 1013 hPa g=9.8 m s^{-2} constant in z c=3 x10^8 m·s^{-1}

Avagadro's number: $N_A = 6.02 \times 10^{23}$ molecules/mole
Stefan–Boltzmann constant: $\sigma = 5.67\times10^{-8}$ W·m^{-2}·K^{-4}
Solar photospheric temperature, T_s = 5786 K
Radius of the Earth = 6370 km
Radius of Venus = 6051 km
Radius of Mars = 3396 km
Latent heat of vaporization water: $L_v=2.5\times10^6$ J· kg^{-1}
Latent heat of sublimation ice: $L_i=2.8\times10^6$ J· kg^{-1}
Gas constant for water vapour: R_v=461 J·K^{-1}·kg^{-1}

Boltzmann's constant k = 1.38×10^{-23}J/K
Planck Constant: h=6.63×10^{-34} J·s
Radius of the Sun = 695800 km
1 AU (Earth-Sun distance) =150×10^6 km
Venus-Sun distance = 0.72 AU
Mars-Sun distance = 1.52 AU
Density of liquid water = 1000 kg·m^{-3}
Density of water vapour = 5×10^{-3} kg·m^{-3}
Surface tension of water droplet 75×10^{-3} N·m^{-1}

Values for dry air: C_p=1004 J·K^{-1}·kg^{-1} C_v=718 J·K^{-1}·kg^{-1} R_d=287 J·K^{-1}·kg^{-1}

$\gamma = C_p / C_v$ $\kappa = R_d / C_p$ $R_d=C_p - C_v$ Γ_{dalr}=9.8 K/km

Clausius–Clapeyron relation: $e_s = 6.112\ hPa \cdot exp\left[\frac{L_v}{R_v}\cdot\left(\frac{1}{273\ K}-\frac{1}{T}\right)\right]$

Some integrals that may be of use:

$$\int x^m e^{(a x)}\, dx = \frac{x^m e^{(a x)}}{a} - \frac{m \int x^{(m-1)} e^{(a x)}\, dx}{a}$$

$$\int x\, e^{(a x)}\, dx = \frac{e^{(a x)}(a x - 1)}{a^2}$$

$$\text{For a} > 0 \quad \int_o^\infty e^{(-a x)}\, dx = \frac{1}{a}$$

$$\int_X^\infty e^{(-a x)}\, dx = \frac{e^{(-a X)}}{a}$$

$$\int \frac{1}{a + b\, x}\, dx = \frac{\ln(a + b\, x)}{b}$$

PERIODIC TABLE OF ELEMENTS

Atomic #
Symbol
Name
Weight

C Solid
Hg Liquid
H Gas
Rf Unknown

Metals
- Alkali metals
- Alkaline earth metals
- Lanthanoids (Lanthanides)
- Actinoids (Actinides)
- Transition metals
- Post-transition metals

Metalloids

Nonmetals
- Other nonmetals
- Noble gases

Pnictogens (15) · Chalcogens (16) · Halogens (17)

Shells: K L M N O P Q

	1	2	3	4	5	6	7	8	9	10	11	12	13	14	15	16	17	18
1	1 **H** Hydrogen 1.008 1																	2 **He** Helium 4.0026 2
2	3 **Li** Lithium 6.94 2,1	4 **Be** Beryllium 9.0122 2,2											5 **B** Boron 10.81 2,3	6 **C** Carbon 12.011 2,4	7 **N** Nitrogen 14.007 2,5	8 **O** Oxygen 15.999 2,6	9 **F** Fluorine 18.998 2,7	10 **Ne** Neon 20.180 2,8
3	11 **Na** Sodium 22.990 2,8,1	12 **Mg** Magnesium 24.305 2,8,2											13 **Al** Aluminium 26.982 2,8,3	14 **Si** Silicon 28.085 2,8,4	15 **P** Phosphorus 30.974 2,8,5	16 **S** Sulfur 32.06 2,8,6	17 **Cl** Chlorine 35.45 2,8,7	18 **Ar** Argon 39.948 2,8,8
4	19 **K** Potassium 39.098 2,8,8,1	20 **Ca** Calcium 40.078 2,8,8,2	21 **Sc** Scandium 44.956 2,8,9,2	22 **Ti** Titanium 47.867 2,8,10,2	23 **V** Vanadium 50.942 2,8,11,2	24 **Cr** Chromium 51.996 2,8,13,1	25 **Mn** Manganese 54.938 2,8,13,2	26 **Fe** Iron 55.845 2,8,14,2	27 **Co** Cobalt 58.933 2,8,15,2	28 **Ni** Nickel 58.693 2,8,16,2	29 **Cu** Copper 63.546 2,8,18,1	30 **Zn** Zinc 65.38 2,8,18,2	31 **Ga** Gallium 69.723 2,8,18,3	32 **Ge** Germanium 72.630 2,8,18,4	33 **As** Arsenic 74.922 2,8,18,5	34 **Se** Selenium 78.971 2,8,18,6	35 **Br** Bromine 79.904 2,8,18,7	36 **Kr** Krypton 83.798 2,8,18,8
5	37 **Rb** Rubidium 85.468 2,8,18,8,1	38 **Sr** Strontium 87.62 2,8,18,8,2	39 **Y** Yttrium 88.906 2,8,18,9,2	40 **Zr** Zirconium 91.224 2,8,18,10,2	41 **Nb** Niobium 92.906 2,8,18,12,1	42 **Mo** Molybdenum 95.95 2,8,18,13,1	43 **Tc** Technetium (98) 2,8,18,13,2	44 **Ru** Ruthenium 101.07 2,8,18,15,1	45 **Rh** Rhodium 102.91 2,8,18,16,1	46 **Pd** Palladium 106.42 2,8,18,18	47 **Ag** Silver 107.87 2,8,18,18,1	48 **Cd** Cadmium 112.41 2,8,18,18,2	49 **In** Indium 114.82 2,8,18,18,3	50 **Sn** Tin 118.71 2,8,18,18,4	51 **Sb** Antimony 121.76 2,8,18,18,5	52 **Te** Tellurium 127.60 2,8,18,18,6	53 **I** Iodine 126.90 2,8,18,18,7	54 **Xe** Xenon 131.29 2,8,18,18,8
6	55 **Cs** Caesium 132.91 2,8,18,18,8,1	56 **Ba** Barium 137.33 2,8,18,18,8,2	57–71	72 **Hf** Hafnium 178.49 2,8,18,32,10,2	73 **Ta** Tantalum 180.95 2,8,18,32,11,2	74 **W** Tungsten 183.84 2,8,18,32,12,2	75 **Re** Rhenium 186.21 2,8,18,32,13,2	76 **Os** Osmium 190.23 2,8,18,32,14,2	77 **Ir** Iridium 192.22 2,8,18,32,15,2	78 **Pt** Platinum 195.08 2,8,18,32,17,1	79 **Au** Gold 196.97 2,8,18,32,18,1	80 **Hg** Mercury 200.59 2,8,18,32,18,2	81 **Tl** Thallium 204.38 2,8,18,32,18,3	82 **Pb** Lead 207.2 2,8,18,32,18,4	83 **Bi** Bismuth 208.98 2,8,18,32,18,5	84 **Po** Polonium (209) 2,8,18,32,18,6	85 **At** Astatine (210) 2,8,18,32,18,7	86 **Rn** Radon (222) 2,8,18,32,18,8
7	87 **Fr** Francium (223) 2,8,18,32,18,8,1	88 **Ra** Radium (226) 2,8,18,32,18,8,2	89–103	104 **Rf** Rutherfordium (267) 2,8,18,32,32,10,2	105 **Db** Dubnium (268) 2,8,18,32,32,11,2	106 **Sg** Seaborgium (269) 2,8,18,32,32,12,2	107 **Bh** Bohrium (270) 2,8,18,32,32,13,2	108 **Hs** Hassium (277) 2,8,18,32,32,14,2	109 **Mt** Meitnerium (278) 2,8,18,32,32,15,2	110 **Ds** Darmstadtium (281) 2,8,18,32,32,16,2	111 **Rg** Roentgenium (282) 2,8,18,32,32,17,2	112 **Cn** Copernicium (285) 2,8,18,32,32,18,2	113 **Nh** Nihonium (286) 2,8,18,32,32,18,3	114 **Fl** Flerovium (289) 2,8,18,32,32,18,4	115 **Mc** Moscovium (290) 2,8,18,32,32,18,5	116 **Lv** Livermorium (293) 2,8,18,32,32,18,6	117 **Ts** Tennessine (294) 2,8,18,32,32,18,7	118 **Og** Oganesson (294) 2,8,18,32,32,18,8

For elements with no stable isotopes, the mass number of the isotope with the longest half-life is in parentheses.

6	57 **La** Lanthanum 138.91 2,8,18,18,9,2	58 **Ce** Cerium 140.12 2,8,18,19,9,2	59 **Pr** Praseodymium 140.91 2,8,18,21,8,2	60 **Nd** Neodymium 144.24 2,8,18,22,8,2	61 **Pm** Promethium (145) 2,8,18,23,8,2	62 **Sm** Samarium 150.36 2,8,18,24,8,2	63 **Eu** Europium 151.96 2,8,18,25,8,2	64 **Gd** Gadolinium 157.25 2,8,18,25,9,2	65 **Tb** Terbium 158.93 2,8,18,27,8,2	66 **Dy** Dysprosium 162.50 2,8,18,28,8,2	67 **Ho** Holmium 164.93 2,8,18,29,8,2	68 **Er** Erbium 167.26 2,8,18,30,8,2	69 **Tm** Thulium 168.93 2,8,18,31,8,2	70 **Yb** Ytterbium 173.05 2,8,18,32,8,2	71 **Lu** Lutetium 174.97 2,8,18,32,9,2
7	89 **Ac** Actinium (227) 2,8,18,32,18,9,2	90 **Th** Thorium 232.04 2,8,18,32,18,10,2	91 **Pa** Protactinium 231.04 2,8,18,32,20,9,2	92 **U** Uranium 238.03 2,8,18,32,21,9,2	93 **Np** Neptunium (237) 2,8,18,32,22,9,2	94 **Pu** Plutonium (244) 2,8,18,32,24,8,2	95 **Am** Americium (243) 2,8,18,32,25,8,2	96 **Cm** Curium (247) 2,8,18,32,25,9,2	97 **Bk** Berkelium (247) 2,8,18,32,27,8,2	98 **Cf** Californium (251) 2,8,18,32,28,8,2	99 **Es** Einsteinium (252) 2,8,18,32,29,8,2	100 **Fm** Fermium (257) 2,8,18,32,30,8,2	101 **Md** Mendelevium (258) 2,8,18,32,31,8,2	102 **No** Nobelium (259) 2,8,18,32,32,8,2	103 **Lr** Lawrencium (266) 2,8,18,32,32,8,3

Ptable.com

1) Using κ instead of $1/\kappa$ in solution for pressure gives -2

2) Obtaining 125% or 31% RH due to having $e(T_1)/e_s(T_2)$ or $e_s(T_1)/e_s(T_2)$ gives -3 (-2 if they recognize it is not right).

3) a) Saying neutrally or conditionally stable instead of stable gives -1.
e) Saying neutrally stable instead of conditionally stable gives -2. Saying unstable gives full marks.

6) Mismatched units in radiance and wavelength scales gives -1. Not having spectral radiance units gives -3.

7) Due to unclear wording of the questions, all answers give full marks.

9) Mistaking absorption for emission, and failure to mention increasing height of $\tau=1$ into the stratosphere will give -3 (more deductions can occur if other incorrect statements are made).

10) Only describe greenhouse gas heating (correctly) and mentioning other heat sources responsible for the excess radiative cooling gives -2

11) Incorrect definition of Dobson Unit gives -3 if they then carry out the calculation. With only the definition with no calculation, it is -5.

12) a) Due to a poorly copied periodic table, I will not deduct points for incorrect molar masses being used.
b) Not converting to number density correctly, or using R_d in stead of R for NO_2, gives -3.
c) Not deducting the new 2% contribution from the original estimate of mass gives -3.

13) a) Using Γ_e in km and others in m gives -1. Using T at 0 km rather than at 59 km gives -1. Adding Γ_e instead of subtracting gives -1.
b) Dropping a sign gives -1. Using potential temperature or Hypsometric equation gives -4. Mixing km and m, or hPa and Pa (other than in division cancelling units) gives -2.
c) Using wrong integration limits gives -2. Assuming isothermal gives -4, and mixing km and m units, except where division cancels units, gives -2
d) Using $\tau \cdot \cos(\chi_s)$ instead of $\tau/\cos(\chi_s)$ gives -2. Division by ρ_a instead of total ρ gives -1, while forgetting to divide by C_p gives -3.

1) (5 Pts) A parcel of air at 900 hPa has a temperature of 20 °C. It is lifted adiabatically until the parcel temperature drops to 13 °C. What is the pressure at this altitude?

Using the reference level of $\theta = T_1 = (273 + 20)$ K at P_1=900 hPa, use the potential temperature equation to solve for P_2 where $T_2 = (273 + 13)$ K:

$$\theta = T_2 \cdot \left(\frac{P_1}{P_2}\right)^{\kappa} \; where \; \kappa = \frac{R}{C_p}$$

Solving for P2 yields

$$P_2 = P_1 \cdot \left(\frac{\theta}{T_2}\right)^{-\frac{1}{\kappa}} = 90000\ Pa \cdot \left(\frac{293\ K}{286\ K}\right)^{-\frac{1004\ J\cdot K^{-1}\cdot kg^{-1}}{287 J\cdot K^{-1}\cdot kg^{-1}}} = 827\ hPa$$

2) (5 Pts) A parcel of air at 1000 hPa and 25 °C is lifted adiabatically to its lifting condensation level at 800 hPa where the parcel temperature is T=7 °C, what was the relative humidity of the parcel before lifting?

The fractional RH is defined as: $RH = \frac{e(T_1)}{e_s(T_1)} = \frac{\varepsilon\cdot e(T_1)/P_1}{\varepsilon\cdot e_s(T_1)/P_1} = \frac{\mu(T_1,P_1)}{\mu_s(T_1,P_2)}$

Now, the saturated partial pressure of water vapour at a temperature T is:

$$e_s(T) = 6.112\ hPa \cdot exp\left[\frac{L_v}{R_v}\cdot\left(\frac{1}{273\ K} - \frac{1}{T}\right)\right]$$

where L_v and R_v are the latent heat of vaporization and specific gas constant for water vapour, respectively. The key here is that we can only calculate saturated quantities. Thus, the vapour pressure at T_1 we have to get from the fact that at the LCL, where the parcel is at T_2 and P_2, the water vapour in the parcel is saturated. Thus:

$$\mu(T_1, P_1) = \mu_s(T_2, P_2)$$

Or, using the definition of μ, we have that:

$$\varepsilon \cdot \frac{e(T_1)}{P_1} = \varepsilon \cdot \frac{e_s(T_2)}{P_2} \rightarrow e(T_1) = e_s(T_2) \cdot \left(\frac{P_1}{P_2}\right)$$

So now, the RH at T1 and P1 can be expressed in saturated quantities (which we can calculate) as:

$$RH = \frac{e_s(T_2)}{e_s(T_1)} \cdot \left(\frac{P_1}{P_2}\right)$$

For T_1=25+273 K = 298 K, T_2=7+273 K=280 K, P1=1000 hPa and P2=800 hPa, using the expression for $e_s(T)$, this becomes:

$$RH = \frac{e_s(280)}{e_s(298)} \cdot \left(\frac{100000\ Pa}{80000\ Pa}\right) = \frac{1004\ Pa}{3235\ Pa} \cdot \left(\frac{100000\ Pa}{80000\ Pa}\right) = 0.3880 = 38.8\%$$

3) (20 Pts) In the balloon sounding on the SkewT-LnP diagram below, the atmospheric temperature is plotted in red as a function of pressure. At 1000 hPa is 20 °C and the dew point is 7 °C.

a) At what pressure is the lifting condensation level (LCL)?

Since $\mu_s(T_d,P) = \mu_p(T,P)$, to find the point where the water vapour in the parcel, $\mu_p(T,P)$, becomes saturated, one should follow a line of constant μ_s upward from T_d, then follow a line of constant θ up from T (since before saturation the motion is adiabatic). Where they cross is where the water vapour in the parcel becomes saturated. This is approximately 825 $\pm$5 hPa

State the stability conditions (stable, unstable, neutrally or conditionally stable) of the atmosphere to small vertical displacements in the following pressure ranges::

b) Between 1000 and 900 hPa

Stable: Here parcels follow θ and $\Gamma_{atm} < \Gamma_{dalr}$

c) Between 900 and 850 hPa

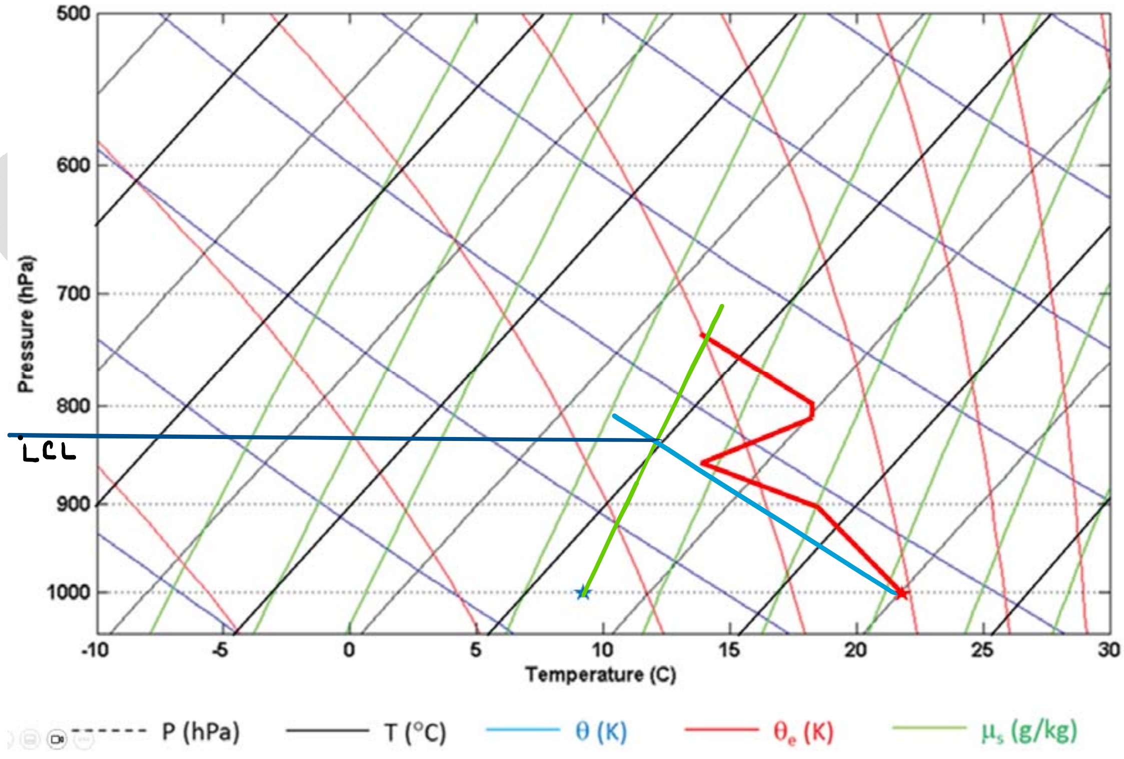
Unstable: Here parcels follow θ and $\Gamma_{atm} > \Gamma_{dalr}$

d) Between 850 and 800 hPa

Stable: Here parcels follow θ below LCL, θ_e above, and $\Gamma_{atm} < \Gamma_{dalr}$ and $\Gamma_{atm} < \Gamma_{walr}$

e) Between 800 and 750 hPa

Unstable or conditionally stable: Here saturated parcels follow θ_e (not θ) and $\Gamma_{atm} > \Gamma_{walr}$. If the parcel has not yet begun to condense, it is neutrally stable since $\Gamma_{atm} = \Gamma_{dalr}$



4) (4 pts) The ____________ and the ___________ are layers of the atmosphere that can become unstable.

   a) troposphere, ionosphere
   b) mesosphere, troposphere
   c) stratosphere, thermosphere
   d) mesosphere, cryosphere

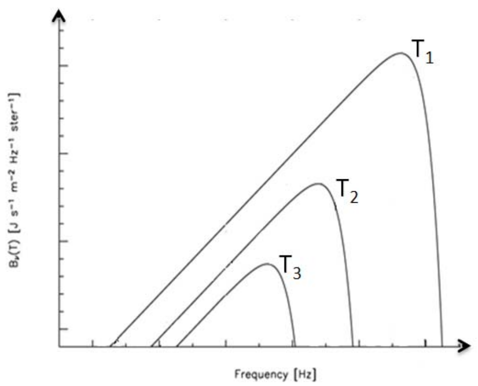
   The temperature falls with altitude in both the troposphere and mesosphere, which can lead to an unstable atmosphere if the lapse rate exceeds the dry or wet adiabatic lapse rate

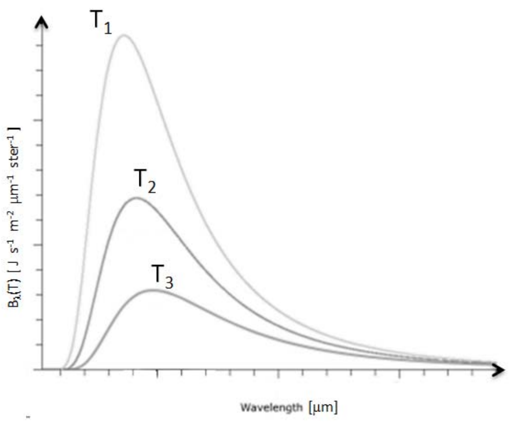
5) (4 pts) What is the major species in the stratosphere?

   a) Molecular Oxygen
   b) Atomic Oxygen
   c) Molecular Nitrogen
   d) Ozone
   e) None of the above.

   Molecular nitrogen is the major species in the atmosphere from the ground up to approximately 120 km. Ozone is a minor species, making up only 5 ppmv.

6) (4 pts) Sketch the relative spectral radiance as a function of wavelength for three blackbodies at temperatures $T_1 > T_2 > T_3$. Label the curves with their temperatures and put units on the axes, but you do not need to put numerical values on the axes.





Radiance or Irradiance units would be correct, asI would either wavelength or wavenumber units (either the left or right plot). I do not need the actual temperatures on the curves or the numbers on the axes. Curves should not cross, and the hottest temperature, T_1, should peak at shorter wavelengths (higher frequencies).

7) (4 pts) Relative to the Earth's surface, what effect does the Coriolis force have on masses of air or water that are changing latitude?

- a) The results are unpredictable; currents can veer right or left in either hemisphere.
- b) They turn to the left in the northern hemisphere and to the right in the southern hemisphere.
- *c) They turn to the right in the northern hemisphere and to the left in the southern hemisphere.*
- d) They turn to the right in both hemispheres.
- e) They turn to the left in both hemispheres.

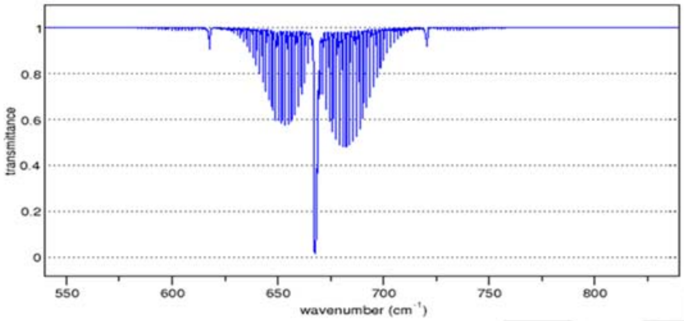
The Coriolis force is given by $-2\vec{\Omega} \times \vec{u}$, *where* Ω *points "upward" toward Polaris (the North Star), or northward. Thus, in the northern hemisphere, for a velocity, u, pointing "downward" or southward toward the equator, the Coriolis force is < 0 or westward. If I am travelling southward, the west is to my right. For a velocity, u, northward toward the pole, Coriolis is > 0, and eastward. If I am travelling northward, east is to my right. In the southern hemisphere, the Coriolis force still points northward. Thus, a velocity pointing northward toward the equator still results in a Coriolis force to the west. However, when travelling northward, the west is toward my left.*

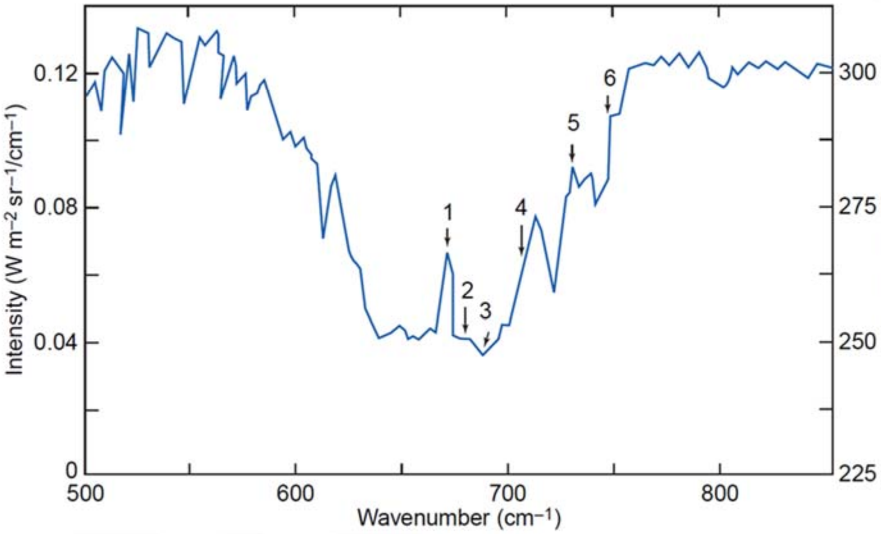
8) (4 pts) What wavelengths of sunlight are absorbed by molecular nitrogen in the troposphere?

- a) Infrared.
- b) Ultraviolet.
- c) Radio waves.
- d) Microwaves.
- e) Visible.
- *f) None of the above.*

The nitrogen molecule does not have a dipole moment and therefore cannot absorb the sunlight that reaches the troposphere.

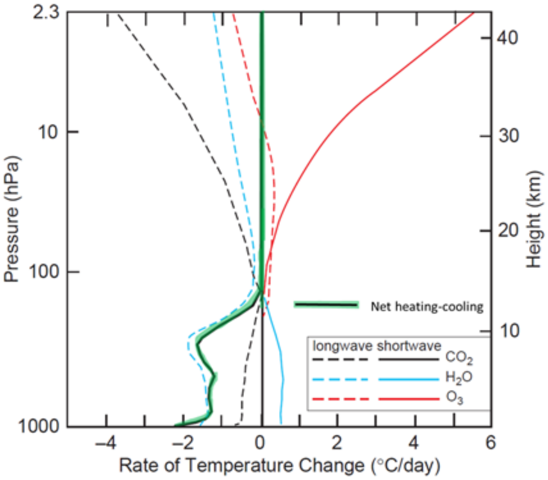
9) (5 Pts) Shown below is the carbon dioxide 15-micron band. The top figure is the absorption spectrum taken in the laboratory, and below is the intensity of the outgoing radiation from Earth measured by satellite showing the carbon dioxide. Can you explain why the intensity (and temperature) of the outgoing radiation at point 1 is higher than that at points 2 and 3?





From the upper spectrum it is clear that the region around the band centre, point 1, has higher absorption coefficient, and therefore a higher optical depth, than the regions around points 2 and 3. That is, $k\nu_1 > k\nu_2$ *or* $k\nu_3$ *Thus, the radiation at point 1 will reach optical depth one, where it can escape to space, at a higher altitude. The narrowness of the spectrum in region 1 also indicates that the* CO_2 *is radiating to space from a higher altitude. In this case, that altitude must be in the stratosphere, where temperatures are higher than in the upper troposphere. On the other hand, the lower absorption at points 2 and 3 mean that the radiation at these wavelengths will reach optical depth one at lower altitudes where the temperatures are cooler.*

10) (5 Pts) Shown below are the vertical profiles of the time rate of change of temperature due to the absorption of solar radiation (solid curves) and the absorption and emission of infrared radiation (dashed curves) by water vapor (blue), carbon dioxide (black), and ozone (red). The heavy black/green solid curve represents the net heating or cooling effect of the three gases. The figure shows that the net effect of the absorption and emission of radiation by greenhouse gases is to cool the air in the troposphere about $1^{o}C$ per day. Can you explain why an increase of greenhouse gases would, nevertheless, result in warming the air in the troposphere?



The figure shows the radiative heating and cooling, not the total heating and cooling of the atmosphere. Additional, non-radiative sources of heat, for example tropospheric convection and latent heat release, will warm temperatures, resulting in stronger radiative emission and cooling to space. The net effect will be for the increased radiative cooling to balance the total heat input at the higher equilibrium, or stagnation temperature.

The long-wavelength greenhouse gas cooling emission escapes to space at and above the point where the optical depth is one. Adding additional greenhouse gases will increase their optical depth, raising the altitude at which their radiation can escape to space to higher in the troposphere where temperatures are cooler. The colder emission temperatures at this escape point reduce the intensity of the radiation and cooling to space, resulting in warming of the atmosphere. These warmer temperature at this escape point will increase the radiative emission and cooling to space, once again balancing radiative cooling and total heating at a new, higher, equilibrium or stagnation temperature.

11) (5 Pts) What ozone column (in molecules·m^{-2}) corresponds to 1 Dobson Unit (DU)?

Given n ozone molecules in volume V at temperature T, we have $pV = nk_BT$ *by the ideal gas law. By definition, 1 DU corresponds to a column of height* 10^{-5} *m and horizontal cross-sectional area 1* m^2*, so put* $V = 10^{-5}$ m^3*. Then for STP, p = 1e5 Pa (1 atm) and T = 273 K. This gives* $n=2.7\times10^{20}$ *molecules·*m^{-2}.

12) Nitrogen dioxide, (NO_2) is a pollutant released by diesel and petrol internal combustion engines as well as industrial processes that causes smog. Mainly produced by traffic in cities, the narrow valleys and fjords where Norwegian cities are located tend to concentrate levels. However, the yearly average mass density across Norway 32.33 μg·m^{-3}, which is low by European standards.

a) (5 Pts) What is the average number density of NO_2 in molecules·m^{-3}?

Here we first need the weight of the NO_2 *molecule. From the molar masses (the atomic number in g·mole*$^{-1}$*) from periodic table, we can see this is:*
$M_N + 2\cdot M_O = 14.007+2(15.999) = 46.005\times10^{-3}$ *kg·mole*$^{-1}$.
To convert this to a density, we need to convert moles to molecules using Avogadro's number, 6.02×10^{23} *molecules·mole*$^{-1}$*, and multiply by the concentration given in kg·m*$^{-3}$*:*

$$N_{NO_2} = \frac{6.02 \times 10^{23}\ molecules \cdot mole^{-1}}{46.003 \times 10^{-3}\ kg \cdot mole^{-1}} \cdot 32.33\ \mu g \cdot m^{-3} \cdot 10^{-9}\ kg \cdot \mu g^{-1}$$

Which yields: $N_{NO_2} = 4.23 \times 10^{17}\ molecules \cdot m^{-3}$

b) (5 Pts) At 1000 hPa and 22 C, the density of NO_2 measured one day in Bergen was 29.95 μg·m^{-3} what is the volume mixing ratio of NO_2?

Here we need to remember that the volume mixing ratio is defined as: $\nu_i = \frac{n_i}{n} = \frac{P_i}{P}$ *and we have the ideal gas laws for dry air and specific species:* $P = \rho RT$ *and* $P_i = \rho_i R_i T$*, where the specific gas constant is the ideal gas constant R* divided by the molecular mass, M. Thus:*

$\nu_i = \frac{P_i}{P} = \frac{\rho_i R_i T}{P} = \frac{\rho_i R_d \left(\frac{M_d}{M_i}\right) T}{P}$*, where we have assumed the total gas mixture has the molar mass of dry air. Again:*

$M_{NO_2} = M_N + 2 \cdot M_O = 14.007 + 2 \cdot 15.999 = 46.005 \times 10^{-3}\ kg \cdot mole^{-1}$*, and* $M_d = 29\times10^{-3}$ *kg·mole*$^{-1}$*. With Rd given on the help sheet as 287 J·K*$^{-1}$*·kg*$^{-1}$*, we have:*

$\nu_{NO_2} = \frac{29.95\times10^{-9}\ kg\cdot m^3 \cdot 287\ J\cdot K^{-1} kg^{-1} \left(\frac{46}{29}\right)\cdot(22+273)\ K}{100000\ Pa}$ *or* $\nu_{NO_2} = 1.6 \times 10^{-8} = 16\ ppbv$

c) (5 pts) Originally, the Martian atmosphere was thought to consist only of 95% CO_2, 3% N_2 and 2% Ar. However, recently, it was discovered that it also contains 0.2% O_2. By how much did the initial estimate of the mean molecular weight of the Martin atmosphere change when this additional species, O_2 was taken into account.

The molecular weight of a mixture of gases is the sum of the products of the mixing ratios of each component times its molar mass $M_m = \Sigma \nu_i \cdot M_{mi}$. *For Mars, we can get the molar masses from the periodic table where we have that* M_m *of* CO_2*=44,* N_2 *= 28, Ar, = 40, and for later,* O_2 *= 32 all in g/mole.*
The molecular weight of the Martian atmosphere without O2 is therefore:

$$M_{m_{no\,O_2}} = 0.95 \cdot 44 + 0.03 \cdot 28 + 0.02 \cdot 40 = 43.44\ g \cdot mole^{-1}$$

Now, if we replace 0.2% of this mixture with O_2*, we calculate the new molecular weight by:*

$$M_{m_{O_2}} = \left(1 - \nu_{O_2}\right) \cdot M_{m_{no_{O_2}}} + \ \nu_{O_2} \cdot M_{M_{O_2}} = (1 - 0.002) \cdot 43.44 + 0.002 \cdot 32 = 43.41\ g \cdot mole^{-1}$$

Thus, the molecular mass has gone down, decreasing by 0.03 kg/kmole

13) On Venus g_v=8.87 m·s^{-2}, the specific gas constant for dry air is R_{dv} = 195.5 J·K^{-1}·kg^{-1}, and the specific heat at constant pressure, C_{pv} = 846 J·K^{-1}·kg^{-1}. The pressure at the surface is 92000 hPa, and the temperature 753 K. The temperature falls off linearly with a lapse rate of 8 K/km (note non-SI units of hPa and km).

a) (2 Pts) What is the buoyancy period at 59 km, and is the atmosphere stable with respect to vertical motion there?

The Brunt frequency, or buoyancy frequency, in radians/sec is given by:

$$N = \sqrt{\frac{g\left(\frac{g}{Cp} + \frac{dT}{dz}\right)}{T}}$$

Of course, we have a different g, Cp, and we need to calculate T(z=59 km) and dT/dz there. We are asked for a period, and the brunt period is given by:

$$\mathrm{T}_{Brunt} = \frac{2\pi}{N}\ in\ seconds$$

Here we are given the lapse rate of 8 K/km, which is the negative of the temperature gradient. Thus, we know:

$$T = T_o - \Gamma \cdot z\ \ and\ \frac{dT}{dz} = -\Gamma$$

Putting in the values for Venus, and calculating the temperature and temperature gradient:

At 59 km, we get values of T=281 K, Γ_d*=g/Cp= 0.01K/m,* $\Gamma_{env}=-\frac{dT}{dz} = 0.008$ *K/m, giving N=0.0089 rad/sec and* T_{Brunt}*=709.5 s = 11.8 minutes. Since N is real, the atmosphere is stable with respect to vertical motions.*

b) (6 Pts) What is the temperature and pressure at 59 km?

Here we have the temperature dropping linearly with altitude. One can use the hysrostatic equation along with the ideal gas law:

$$\frac{dP}{dz} = -g \cdot \rho \ \text{ and } P = \rho \cdot R_{dv} \cdot T$$

To get:

$$\frac{dP}{P} = -\frac{g}{R_{dv} \cdot T} dz$$

Since T is T(z) this has to be integrated to give the pressure as a function of altitude:

$$P = P_o \cdot e^{-\left[\left(\frac{g_v}{R_{dv}}\right) \cdot \int_0^z \frac{1}{(T_o - \Gamma \cdot z')} dz'\right]}$$

With the density given by:

$$\rho = \frac{P}{R_{dv} \cdot (T_o - \Gamma \cdot z)}$$

This gives:

$$P = P_o \cdot \left[\frac{T_o - \Gamma \cdot z}{T_o}\right]^{\frac{g_v}{R_{dv} \cdot \Gamma}}$$

And ρ given as above. At z=59000 m, g_v=8.87 m·s^{-2}, R_{dv}=195.5 J·K^{-1}·kg^{-1}, *Γ=0.008 K·m^{-1}, T_o=753 K and Po=9200000 Pa we obtain:*

P= 343.5 hPa and T=281 K. For completeness, the density ρ= 0.625 kg·m^{-3}. It is interesting to note that at around 50 km, the temperature and pressure are similar to that at the surface of the Earth.

The Venusian atmosphere is composed of 96.5% CO_2 and approximately 3.5% N_2, neither of which absorb solar radiation. However, there is a 150-ppm mixing ratio of sulfur dioxide (SO_2) at the surface that absorbs at in the UV/visible with an absorption coefficient k = 0.11 m^2·kg^{-1}. The Sun is at a solar **ZENITH** angle (angle of the Sun from overhead) of χ= 45 degrees. You may assume no scattering, and take the optical depth of incoming solar radiation (averaged over all wavelengths), the density of air, and the mass mixing ratio of the absorber to be defined as:

$$\tau(Z) = \int_Z^\infty \rho(z)\, \mu(z)\, k\, dz$$

$$\rho(z) = \rho 0\, \mathbf{e}^{\left(-\frac{z}{Hm}\right)}$$

$$\mu(z) = \mu 0\, \mathbf{e}^{\left(-\frac{z}{Hv}\right)}$$

Where $\rho(z)$ is the mass density of air, $\mu(z)$ is the mass mixing ratio of SO_2. The scale height of the atmosphere is H_m = 13 km, and the scale height of the SO_2, is H_v = 8 km. At the surface, the pressure is 92000 hPa, the total density, ρ0=85 kg·m-3 and the temperature is 753 K

c) (6 Pts) Calculate the optical depth τ and its derivative with height $d\tau/dz$ at a height of 5 km in the atmosphere.

So, the first thing is to substitute in for ρ and μ into the integral

$$\tau = \int_{Z}^{\infty} k \cdot \rho 0 \cdot e^{-\frac{z'}{H_m}} \cdot \mu 0 \cdot e^{-\frac{z'}{H_v}} \cdot dz'$$

This is not too hard to integrate as the exponents of the exponentials add and can be factored out. With k a constant, we get:

$$\tau = \frac{\rho 0\ \mu 0\ k\ Hm\ Hv\ \mathrm{e}^{\left(-\frac{Z\,(Hv+Hm)}{Hm\ Hv}\right)}}{Hv + Hm}$$

and of course, its derivative

$$\frac{d\tau}{dZ} = -\rho 0\ \mu 0\ k\ \mathrm{e}^{\left(-\frac{Z\,(Hv+Hm)}{Hm\ Hv}\right)}$$

But, now we need to put in values for $\rho 0$ (given) and $\mu 0$, for which we only have the volume mixing ratio. No worries, the mass mixing ratio is given by:

$$\mu 0 \approx \frac{M_{SO_2}}{M_{dv}} \cdot \nu$$

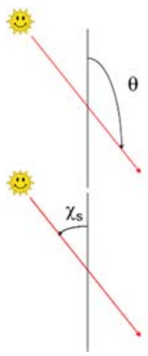
Where the mass of the atmosphere is approximately the mass of dry air without SO_2, and ν is the volume mixing ratio. Now, the mass of SO_2 is, form the periodic table given: $M_S + 2 \cdot (M_O) = 32.065 + 2 \cdot 15.999 = 64.064$ g/mole. The mass of the atmosphere is: $0.965 \cdot M_{CO_2} + 0.035 \cdot M_{N_2} = 0.965 \cdot (12.011 + 2 \cdot 15.999) + 0.035 \cdot (2 \cdot 14.007) = 43.449$ g/mole.

Therefore the mixing ratio of SO_2 at the surface is $\mu 0 = 1.474 \cdot 150 \times 10^{-6} = 0.00022$. Now, plug in this value along with k, $\rho 0$, Hm=13000 m and Hv=8000 m, and we get at 5 km altitude:

$\tau = 3.73$, and $\frac{d\tau}{dZ} = -0.00075\ m^{-1} = -0.75\ km^{-1}$

d) (6 Pts) Calculate the heating rate in Kelvin·day^{-1} due to short wavelength radiation at 11 km if the downward solar flux at the top of the atmosphere is 2600 W/m^2 and the solar ZENITH angle is χ=45 degrees. At 11 km, the total density is ρ=36.5 kg·m^{-3}, the absorber mass mixing ratio is 0.0559 g·kg^{-1}, the optical depth is τ=1 and its derivative, $d\tau/dz$, is -0.22 km^{-1}.

First things first. The ZENITH angle vs $\mu = \cos(\theta)$ is shown here:



As you can see $\cos(\chi) = -\mu$. Therefore, we can use the equation for downward flux from the short wavelength heating lecture, but the one for χ instead of μ. We could, of course, use the equation for $\mu = \chi + 90^o = 135^o$. either way, this is given by:

$$F_{\nu}^{\downarrow}(z) = -F_{\nu\infty}^{\downarrow} \cdot \mu \cdot e^{\frac{\tau(z)}{\mu_s}} \quad = \quad F_{\nu\infty}^{\downarrow} \cdot \cos(\chi_s) \cdot e^{-\frac{\tau(z)}{\cos(\chi_s)}}$$

Where $F_{\nu\infty}^{\downarrow} = 2600\ W \cdot m^{-2}$

All we have to do now is pay attention to the fact that we want the net UPWARD flux, which is:

$$F_\nu(z) = \left[F_\nu^\uparrow(z) - F_\nu^\downarrow(z)\right] = F_{\nu\infty}^\downarrow \cdot \mu \cdot e^{\frac{\tau(z)}{\mu_s}} = -F_{\nu\infty}^\downarrow \cdot \cos(\chi_s) \cdot e^{-\frac{\tau(z)}{\cos(\chi_s)}}$$

We also need the optical depth given, $\tau=1$. *Putting in the numbers, we get the net upward flux* $F_\nu(Z=11km) = -446.9\ W \cdot m^{-2}$.

We need to get the heating rate in W/kg, then divide by the specific heat C_p *in J/kg/K, will give the heating rate in K/sec. That heating rate at altitude z is given in terms of the net upward flux as:*

$$\frac{dQ(z)}{dt} = -\frac{\frac{dF(z)}{dz}}{\rho(z) \cdot C_p}$$

Where $\rho(z)$ *it the total atmospheric density, and above we have an expression for F (z):*

$$F(z) = -F_\infty^\downarrow \cdot \cos(\chi_s) \cdot e^{-\frac{\tau(z)}{\cos(\chi_s)}}$$

When we differentiate this with respect to z, we get:

$$\frac{dQ(z)}{dt} = -\frac{-F_\infty^\downarrow \cdot \frac{d\tau(z)}{dz} \cdot e^{-\frac{\tau(z)}{\cos(\chi_s)}}}{\rho(z) \cdot C_p}$$

Now, kindly old professor Espy actually gave you everything you need to calculate this without differentiating τ. *That's because you needed to do that in part a. and if you didn't, then you shouldn't lose points twice! Plugging in the numbers, and remembering SI units so that* $d\tau/dz = -0.22 \times 10^{-3}\ m^{-1}$ *you get:*

$$\frac{dQ}{dt} = 4.3 \times 10^{-6} \frac{K}{s} = 0.00026 \frac{K}{min} = 0.016 \frac{K}{hr} = 0.37 \frac{K}{day}$$