Each incorrect or blank answer will score zero points. Answers have been randomized and are not exact. You must choose the best answer.

For all calculations use SI units!

You may take:

Molar mass of dry air: ~29 kg/kmole

Molar mass of helium: ~4 kg/kmole

Molar mass of H_2O: ~18 kg/kmole

Molar mass of CO_2: ~44 kg/kmole

273 K = 0 °C 1 hPa = 10^2 Pa = 10^2 N m^{-2} 1 atm = 1013 hPa g=9.8 m s^{-2} constant in z c=3 x10^8 m·s^{-1}

Avagadro's number: N_A = 6.02x10^{23} molecules/mole Boltzmann's constant k = 1.38x10^{-23}J/K

Stefan–Boltzmann constant: σ = 5.67×10^{-8} W·m^{-2}·K^{-4} Planck Constant: h=6.63x10^{-34} J·s

Solar photospheric temperature, T_s = 5786 K Radius of the Sun = 695800 km

Radius of the Earth = 6370 km 1 AU (Earth-Sun distance) =150x10^6 km

Radius of Venus = 6051 km Venus-Sun distance = 0.72 AU

Radius of Mars = 3396 km Mars-Sun distance = 1.52 AU

Latent heat of vaporization water: L_v=2.5x10^6 J· kg^{-1} Density of liquid water = 1000 kg·m^{-3}

Latent heat of sublimation ice: L_i=2.8x10^6 J· kg^{-1} Density of water vapour = 5x10^{-3} kg·m^{-3}

Gas constant for water vapour: R_v=461 J·K^{-1}·kg^{-1} Surface tension of water droplet 75x10^{-3} N·m^{-1}

Values for dry air: C_p=1004 J·K^{-1}·kg^{-1} C_v=718 J·K^{-1}·kg^{-1} R_d=287 J·K^{-1}·kg^{-1}

$\gamma = C_p / C_v$ $\kappa = R_d / C_p$ $R_d = C_p - C_v$ Γ_{dalr}=9.8 K/km

Clausius–Clapeyron relation: $e_s = 6.112\ hPa \cdot exp\left[\frac{L_v}{R_v} \cdot \left(\frac{1}{273\ K} - \frac{1}{T}\right)\right]$

Some integrals that may be of use:

$$\int x^m e^{(a x)}\, dx = \frac{x^m e^{(a x)}}{a} - \frac{m \int x^{(m-1)} e^{(a x)}\, dx}{a}$$

$$\int x\, e^{(a x)}\, dx = \frac{e^{(a x)}(a x - 1)}{a^2}$$

$$\text{For a} > 0 \quad \int_o^\infty e^{(-a x)}\, dx = \frac{1}{a}$$

$$\int_X^\infty e^{(-a x)}\, dx = \frac{e^{(-a X)}}{a}$$

$$\int \frac{1}{a + b x}\, dx = \frac{\ln(a + b x)}{b}$$

General Knowledge Questions (4 points each)

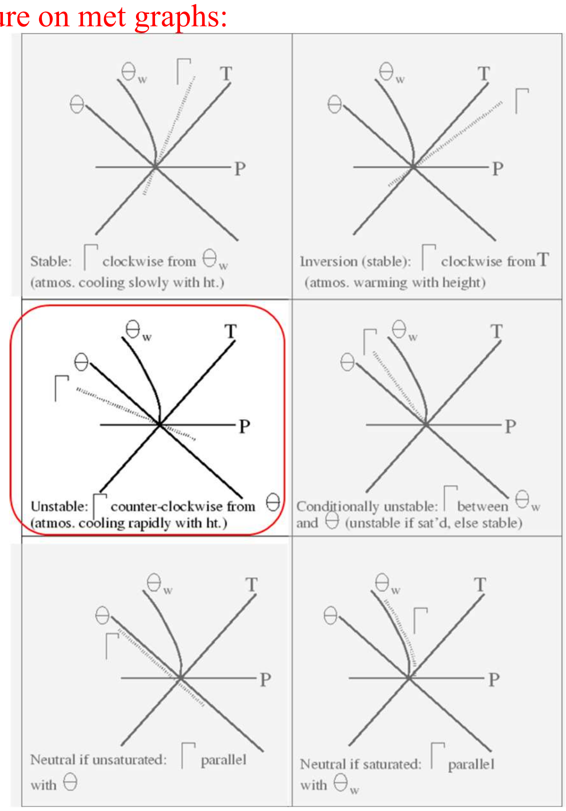
1) Matching radiometric units:

Radiance	$\frac{J}{s\ m^2\ sr}$
Spectral Irradiance	$\frac{Watts}{m^2\ nm}$
Spectral Radiance	$\frac{Watts}{m^2\ sr\ nm}$
Irradiance	$\frac{Watts}{m^2}$

Radiance is the energy flux in a direction, Irradiance is integrated over the emission hemisphere. Spectral includes wavelength units.

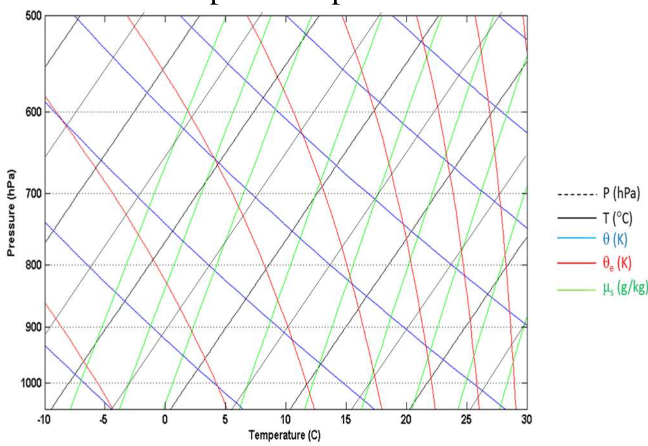
2) Which of the following graphs depicts unstable conditions on a skew T-P diagram, where Γ is the atmospheric lapse rate?

From Lecture on met graphs:



As the air parcel at T and P is displaced, it follows either θ, if it is not saturated, or θ_w if it is. In either case, if it is displaced upward/downward and is *warmer/cooler than the surrounding atmosphere it will continue to rise/fall. This is an unstable condition. If it follows* θ_w*, it is stable when dry and neutrally stable (will not rise or fall after displacement) when condensing/evaporating water. If it follows* θ*, it is conditionally stable (neutrally stable if dry, unstable if condensing water).*

3) On the Skew-T log-P diagram, at a pressure of 1000 hPa an air parcel has a temperature T=20 C and a dewpoint temperature of Td=5 C



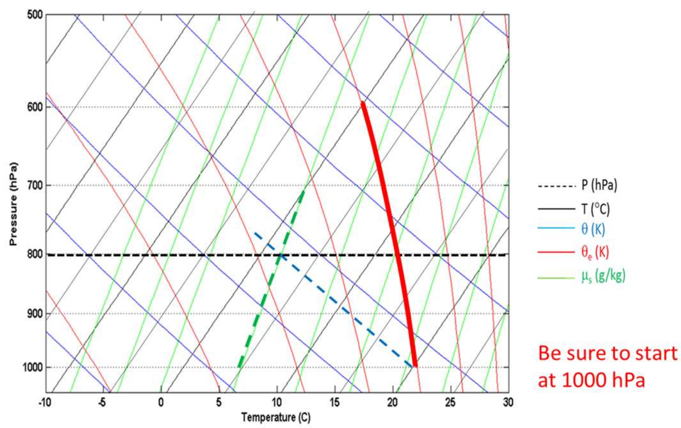
.

a) At what pressure would the parcel's Lifting Condensation Level (LCL) be?

800 hPa 550 hPa 750 hPa 850 hPa 900 hPa

b) If the parcel at 1000 hPa had a dew point temperature of 20 C and a temperature of 20 C, what is the air parcel's temperature when it is lifted to 600 hPa?

0 C -20 C -10C -5C 5 C



Part a

Lifting a parcel at 1000 hPa that was 20 C and had a dew point of 5 C, the parcel would travel adiabatically until the water in the parcel was equal to the saturated mass mixing ratio.

Since $\mu_s(T_d, P) = \mu(T,P)$*, the parcel will move parallel to the dry adiabat (blue line) until it reaches the saturated mixing ratio it had at* $\mu_s(T_d, 1000\ hPa)$*, the* *green line, which is approximately 800 hPa. At this point, the water may saturate, and this is the LCL.*

Part b:
If the dew point is equal to the temperature, the water in the parcel is already saturated. Thus, lifting the parcel will condense more water, and the parcel must move along the saturated adiabat (the red line) until it reaches 600 hPa. The temperature at this point is approximately 0 C.

4) Which is not the case in baroclinic stratification?
   a. is the lowest energy state of the atmosphere
   b. isentropes are not parallel to isobars
   c. potential energy can be converted to kinetic energy
   d. can be caused by horizontal temperature gradients

Barotropic stratification is when the height of a pressure surface does not vary horizontally. As a result, there are no horizontal pressure gradients and, thus, no pressure force to initiate motions. From the hypsometric equation, we know that to keep a uniform vertical spacing between pressure surfaces, the temperature cannot vary horizontally either. Under these conditions, the lines of constant potential temperature (isentropes) are parallel to the lines of constant pressure (isobars), and there is no further conversion of potential energy to kinetic energy. Baroclinic stratification occurs when there are horizontal gradients in the height of a pressure surface. Again, the hypsometric equation tells us that there must also be horizontal temperature gradients. Now, there is a pressure gradient force at each altitude that can initiate the conversion of potential energy into kinetic energy, and the temperature gradients cause a displacement of the isentropes from the isobars. Since potential energy can be converted to kinetic energy, this is not the lowest energy state of the atmosphere.

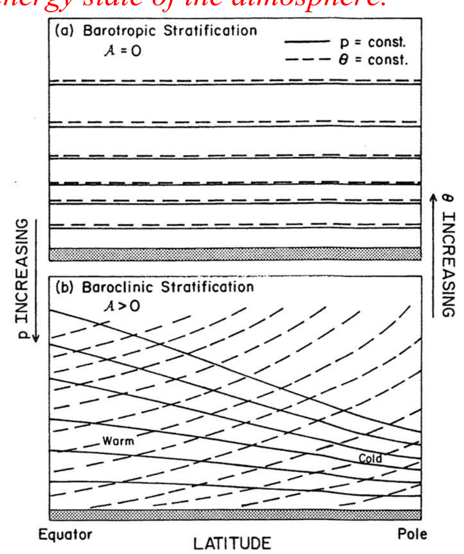


Figure 12.5 Thermal structure corresponding to (a) barotropic stratification, wherein isentropic surfaces coincide with isobaric surfaces and available potential energy $\mathcal{A}$ is zero (Sec. 15.1.3), and (b) baroclinic stratification, wherein isentropic surfaces do not coincide with isobaric surfaces and $\mathcal{A}$ is positive. The rotation of isobaric and isentropic surfaces from their positions under barotropic stratification is symbolic of atmospheric heating at low latitude and cooling at middle and high latitudes (refer to Fig. 1.29c).

5) Of the gases listed below, which is NOT believed to be responsible for enhancing the earth's greenhouse effect?

a. molecular oxygen (O_2)
b. chlorofluorocarbons (CFCs)
c. nitrous oxide (N_2O)
d. carbon dioxide (CO_2)
e. methane (CH_4)

A greenhouse gas has high transmission in the visible, but absorbs appreciably in the infrared. Since infrared wavelengths interact with molecular vibrations and rotations, the molecule must possess a permanent dipole moment, or one that is induced by vibration or rotation. O2 is a symmetric (homonuclear) molecule that does not possess a permanent dipole moment. Since the only vibrations are along the internuclear axis, one cannot be induced. Hence it cannot interact with infrared light.

6) The most abundant gas in the stratosphere is:

a. nitrogen (N_2).
b. oxygen (O_2).
c. carbon dioxide (CO_2).
d. ozone (O_3).
e. chlorofluorocarbons (CFCs).

Ozone is never the major species, reaching a maximum of about 5 to 10 parts per million by volume. That means there is a maximum of only 1-10 ozone molecules in any volume. At altitudes below 120 km, the most abundant gas is N_2.

7) Which two atmospheric layers have temperature profiles that allow convection?

a. Mesosphere and Troposphere.
b. Mesosphere and Stratosphere.
c. Mesosphere and Thermosphere.
d. Stratosphere and Thermosphere.
e. Stratosphere and Troposphere.
f. The correct answer is not shown.

The Troposphere and Mesosphere both become cooler with higher elevation. This would allow a parcel moved upward to become convectively unstableIf it does not cool to at least the same temperature of the atmosphere now surrounding it.

8) Relative to the Earth's surface, what does the Coriolis effect have on masses of air or water that are displaced northward or southward?

   a) **Relative to the direction of travel, they turn to the right in the northern hemisphere and to the left in the southern hemisphere.**
   b) Relative to the direction of travel, they turn to the left in the northern hemisphere and to the right in the southern hemisphere.
   c) The results are unpredictable; currents can veer right or left relative to the direction of travel in either hemisphere.
   d) Relative to the direction of travel, they turn to the right in both hemispheres.
   e) Relative to the direction of travel, they turn to the left in both hemispheres

   The Coriolis force is given by $-2\vec{\Omega} \times \vec{u}$, where Ω points "upward" toward Polaris (the North Star), or northward. Thus, in the northern hemisphere, for a velocity, u, pointing "downward" or southward toward the equator, the Coriolis force is < 0 or westward. If I am travelling southward, the west is to my right. For a velocity, u, northward toward the pole, Coriolis is > 0, and eastward. If I am travelling northward, east is to my right. In the southern hemisphere, the Coriolis force still points northward. Thus, a velocity pointing northward toward the equator still results in a Coriolis force to the west. However, when travelling northward, the west is toward my left.

9) Molecular nitrogen (N_2) does not absorb infrared dipole radiation to make vibrational transitions because:

   a. N_2 has no permanent dipole moment.
   b. N_2 only makes rotational transitions at longer wavelengths.
   c. False, N_2 does absorb infrared dipole radiation to make vibrational transitions.
   d. The dipole moment of N_2 is perpendicular to dipole radiation.
   e. The correct answer is not shown.

   The only way to make a dipole transition is for the E-Field of the photon wave to interact with the dipole moment of the molecule. Due to its symmetry, N_2 does not have a permanent dipole moment and cannot make dipole transitions involving its nuclei (i.e. vibrational or rotational transitions). However, the E-field of the photon wave can distort the electron cloud relative to the nuclei to induce a temporary dipole moment and make an electronic transition.

10) A small cloud droplet will evaporate ____________ a large cloud droplet?

   a. faster than.
   b. at the same rate as.
   c. more slowly than.
   d. it will not evaporate.
   e. the correct answer is not shown.

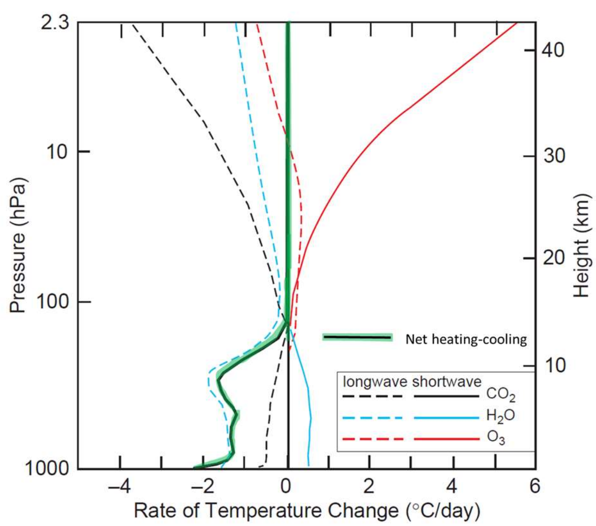
   When the surface of the water is "bent" into a droplet, the water molecules near the surface are not as densely packed and allow interior molecules to reach the *surface and evaporate. The smaller the droplet radius, the more pronounced this effect.*

11) If the atmospheric absorption of carbon dioxide at 15 μm becomes saturated, what happens if carbon dioxide levels continue to increase?

a. Total absorption increases as lines farther from the band centre begin to saturate.
b. Total absorption stays the same because it is saturated.
c. Total absorption begins to decrease near the band centre after it saturates.
d. The absorption continues to increase but only near the band centre.

Since carbon dioxide absorption is a vibrational-rotational transition, it consists of a central band centre at the wavelength of the vibrational energy difference, with individual rotational transitions that take place at the same time spread out on either side of the band centre. The population of the individual rotational levels is determined by collisions and follows a Boltzmann distribution. Thus, lines farther from the band centre, which come from higher rotational levels, have low population and therefore low absorption. As the levels of CO_2 are increased, so too are the populations of all the rotational levels, and thus the absorption of lines farther from the band centre increases as their population increases.

12) If the greenhouse effect produces a temperature warming in the troposphere, why do we find a net 2 K/day radiative cooling there as shown in the figure?



a. **The cooling offsets non-radiative processes that heat the atmosphere in this region**
b. The greenhouse heating is offset by this cooling, resulting in a steady temperature.
c. That cooling is only affecting the radiation and not the temperature.
d. There is, in fact, a net cooling of the troposphere
e. The correct answer is not shown

The key is that the radiative cooling/heating rate does not drive temperature. Rather, the temperature drives the radiative cooling/heating rate (σT^4). If there is a net radiative cooling, it is because another, non-radiative process is heating the atmosphere and driving away from radiative equilibrium temperature. Since it is hotter, it radiates away energy at a faster rate in order to balance the total (radiate plus non-radiative) energy input.

13) In the two-stream approximation, the integral over wavelength and angle can best be approximated as two streams at which angles to the vertical?

a. **53 and 127 degrees**
b. 24 and 156 degrees
c. 35 and 145 degrees
d. 42 and 138 degrees
e. 60 and 120 degrees
f. The correct answer is not shown

*The two-stream approximation is the result of taking the point where $\Delta\tau = \pm 1$, the maximum contribution to the integral over wavelength and angle, as the total contribution. Thus, integrating the 2 * (third exponential integral) over angle with $\Delta\tau = \pm 1$, and setting the result equal to $e^{-1/\mu}$, we get $\mu = \cos(\theta) = \pm 3/5$, or $\theta = 53°$ and $\theta = 127°$ where θ is the angle between the ray and the vertical.*

14) At a wavelength of 500 nm, the scatter from a 50 nm radius particle is approximately:

a) **Rayleigh**
b) Mie
c) Geometric
d) In the forward direction
e) Able to create a rainbow
f) The correct answer is not shown

The ratio of R to λ is 0.1, which means that the particle is much smaller than the wavelength of light; 10 times smaller, in fact. Thus, the scatter would be Rayleigh scatter, characterized by nearly symmetric scatter in all directions. If the particles were on the order of the wavelength ($R/\lambda \approx 1.0$) the scatter would be Mie-like, with most of the scattered light moving in the direction of the incoming beam.

Short Calculations (4 points each)

15) In the movie The Day After Tomorrow, the premise was that cold air in the mesosphere would descend to the surface, normally at 1000 hPa and 288 K, and freeze everything. Apparently, an atmospheric scientist pointed out their mistake, and in later versions you can see they dubbed in "stratosphere" instead of "mesosphere". If stratospheric air with a temperature of 270 K at 50 km, where the pressure is 0.8 hPa, were to descend adiabatically to the surface, what would its temperature be?
    - **a. 2070 K**
    - b. 1712 K
    - c. 13 K
    - d. 3.5 K
    - e. 6385 K

    If the air were to descend adiabatically, it would maintain its potential temperature, θ. If we take the initial conditions as those in the stratosphere, $P_o=2x10^{-3}$ hPa and $T_o=\theta=150$ K, then at its new pressure of 1000 hPa, we could calculate its temperature from the potential temperature relation:

    $$\theta := T\left(\frac{Po}{P}\right)^{\kappa} \text{ or solving for T: } T = \frac{\theta}{\left(\frac{Po}{P}\right)^{\kappa}}$$

    *Where $\kappa = R/Cp = 287/1004 = 0.286$. Substituting in for P=1000 hPa (note, keep Po and P in the same units), we calculate **T = 2073 K**. Admittedly still a problem, but I would not put on another jumper!*

16) A person perspires. How much liquid water (as a percentage of the person's mass) must evaporate to lower the temperature of the person by 5.0 C. Take the specific heat of the human body to be that of water, Cpw = 4200 J/kg/K
    - **a. 1%**
    - b. 3%
    - c. 5%
    - d. 9%

    Evaporating a mass of water, M_w, will release an amount of heat, $dQ = L_v \cdot M_w$ into the environment
    Losing an amount of heat, dQ, from a body of mass M_p with a heat capacity C_{pw}= 4200 J/kg/K will drop its temperature by $dQ= C_{pw}\cdot M_p \cdot dT$
    Equating the dQ's and solving for $M_w/M_p= dT\cdot C_{pw}/ L_v$
    *For dT=5 K (remember a change in temperature is the same in K as in C) and the values for C_{pw} and L_v we get that **M_w/M_p= 0.0075=0.75%***

17) If the atmospheric pressure at the surface of the Earth is 1000 hPa, what is the mass of the atmosphere in kg?

   a. **$5x10^{18}$**
   b. $5x10^{15}$
   c. $5x10^{16}$
   d. $5x10^{17}$
   e. $5x10^{19}$

   *Pressure in Pa is force/unit area, or Nt/m^2. So the force is $P_{surface} \cdot (4\pi \cdot R_e^2)$. In SI units this is $100000\ Nt/m^2 \cdot 4\pi \cdot (6370x10^3)^2\ m^2 \approx 5x10^{19}$ Nt. And since $F=M \cdot g$, the mass in kg is **$M=F/g \approx 5x10^{18}$ kg.***

18) An air mass of temperature +10° C and pressure 1013 hPa contains 7 g/kg water vapour. Calculate the relative humidity.

   a. **90%**
   b. 60 %
   c. 0.1 %
   d. 135 %

   A unit of g/kg indicates a mass mixing ratio, and we have $\mu = M_v/M_d \cdot e/p$. We solve for e at a pressure of 1013 hPa given that $\mu = 0.007$. This gives:

$$e = \mu \cdot M_d/M_v \cdot p = 0.007 \cdot 29/18 \cdot 1013\ hPa$$

$$e = 11.42\ hPa$$

   and from the Clausius–Clapeyron relation listed on the first page at $T = 273+10\ K = 283$, we get the saturation vapour pressure e_s to be:

$$e_s = 12.33\ hPa$$

   and the relative humidity is e/e_s, usually expressed in %

$$RH=100 \cdot (11.42/12.33) = 93\%$$

Long Calculations 6 Points each

19) For an isothermal atmosphere at 20 °C and a surface pressure of 1000 hPa, how many molecules are there in the Earth's atmosphere?

   a. **10^{44}**
   b. 10^{42}
   c. 10^{38}
   d. 10^{29}
   e. 10^{27}

   We can calculate the number of molecules in a column of area 1 m^2, and then multiply by the surface area of the earth = $4\pi \cdot Rearth^2$. We have:

$$N(z) = no\ e^{\left(-\frac{z}{H}\right)}$$

And need to integrate from z=0 to z= ∞ to get the column density in molecules/m^2:

$$Ncol = no \int_0^{\infty} e^{\left(-\frac{z}{H}\right)} dz$$

Now this integral is of the form seen on the help sheet:

$$\int_o^{\infty} e^{(-a\,x)}\,dx = \frac{1}{a}$$

*With a=1/H, this becomes Ncol=no*H = 2.3x10^{25}molecules/m^3·8880 m, and we get Ncol=2x10^{29} molecules/m^2. 22)   Ae=4π·R_{earth}^2 = 5.1x10^{14} m^2, we get* ***<u>Ntot=Ncol*Ae=1.04x10^{44} molecules</u>***

With more math and more complicated integrals, one can formulate the problem as a set of spherical shells of volume 4π·r^{2}dr going from the R_{earth} up to infinity and integrate the volume this way.

However, if you are lazy, like me, you can realize that problem 17 has already answered this question by saying the surface pressure, or force per unit area, is 100000 Pa, or 100000 Nt/m^2. Multiplying by the surface area of the earth gets you the total mass, and there are approximately 29 kg/kmole throughout the well-mixed atmosphere (assuming the total atmosphere above about 105 km is not going to make much difference). Then use Avagadro's number to get the number of molecules per mole, and bingo, you still get 1.04x10^{44} molecules!

20) An exoplanet orbits its star at a distance R_{orbit}= 0.41 AU, and has a radius R_p=1.34*R_{earth}. The planet has an albedo of 0.4 and emissivity of 1. The star it orbits has a radius R_{st}= 0.6*R_{sun} and a photosphere blackbody temperature of T_{st}=4400 K. What is the planet's equilibrium temperature assuming it has no atmosphere?

a. -50 C
b. -15 C
c. 273 K
d. 295 K

We need to know the "stellar" constant for this planet. that is the W/m^2 it puts on the planet. The irradiance W/m^2 at the photosphere of the star is given (with σ_{SB} = Stefan Boltzmann constant):

$$F_{st} = \sigma_{SB} \cdot T_{st}^4$$

The star's total power in watts is this times the area of the photosphere. This is the luminosity of the point source radiating in all directions (4π sr).

$$L_{st} = 4\pi \cdot R_{st}^2 \cdot \sigma_{SB} \cdot T_{st}^4$$

*At the planet, this luminosity is distributed over a sphere 4*π*(R_{orbit})^2 giving the planet's stellar constant as:*

$$F_p = \frac{R_{st}^2}{R_{orbit}^2} \cdot \sigma_{SB} \cdot T_{st}^4$$

We can calculate this intermediate step and find that the stellar constant for this planet is 979.3 W/m²

Now the planet absorbs (1-albedo) of this stellar flux over its cross-sectional area of $\pi*(R_p)^2$, and radiates over its surface area = $4*\pi*R_p^2$ as $\varepsilon \cdot \sigma_{SB}*(T_p)^4$

At equilibrium, energy in = energy out and the radius of the planet cancels:

$$\pi \cdot R_p^2 \cdot F_p(1-a) = 4\pi \cdot R_p^2 \cdot \epsilon \cdot \sigma_{SB} \cdot T_{st}^4$$

We can solve for T_p and find:

$$T_p = \frac{1}{2} \frac{\sqrt{2}\,\left(-F_p\,(-1+a)\,\varepsilon^3\,\sigma_{SB}^3\right)^{\left(\frac{1}{4}\right)}}{\varepsilon\,\sigma_{SB}}$$

*Using the values given, **Tp=225.6 K = -47.4 C***

21) A spherical helium ballon, weighing 50 kg when empty, carries an instrument of 100 kg. In a dry atmosphere it floats at an altitude of 40 hPa where the temperature is 230 K. Assuming the temperature of the helium inside the balloon has equilibrated with the temperature of the air outside of the balloon, what is the radius of the balloon?

  a. **9 m**
  b. 40 m
  c. 6 m
  d. 30 m

The balloon will float because the gravitational force on the He and the balloon plus payload (= m_b) is equal to the gravitational force on the air the balloon displaces. At 40 hPa we can safely assume that the air is dry. Thus:

$$m_{air} \cdot g = (m_{He} + m_b) \cdot g, \text{ or}$$

$$m_{air} - m_{He} = m_b.$$

Now, assuming an ideal gas, the $m_{air} / V_b = \rho_{air}$ and $m_{He} / V_b = \rho_{He}$ where V_b is the volume of the balloon. That lets us convert the mass above to volume times density:

$$V_b \cdot (\rho_{air} - \rho_{He}) = m_b.$$

and remember that at its float altitude, the pressure inside and outside of the balloon is that same. Thus, again for an ideal gas:

$$P_{outside} = \rho_{air} \cdot R_{air} \cdot T_{air} \text{ which is equal to } P_{inside} = \rho_{He} \cdot R_{He} \cdot T_{He}, \text{ but we have:}$$

$$P_{inside} = P_{outside} = P, \text{ giving } \rho_{air} = P/(R_{air} \cdot T_{air}) \text{ and } \rho_{He} = P/(R_{He} \cdot T_{He})$$

*Knowing that the pressure of the air is 4000 Pa gives $\rho_{air}=0.06\ kg/m^3$ and $\rho_{He}=0.008\ kg/m^3$ (remember that the gas constants are specific gas constants, so $R_{He} = R_{air} \cdot M_{air}/M_{He}$). With the total mass of the balloon and payload m_b = 150 km, we get that $V_b=2871.4\ m^3$, and since $V_b=4/3\pi \cdot R_b^3$, the radius of the balloon is **R_b=8.8 m**.*

If we wanted a closed-form solution, we could write:

$$m_{air} = \rho_{air} \cdot V_b = P \cdot V_b / (R_{air} \cdot T_{air}), \text{ and } m_{He} = \rho_{He} \cdot V_b = P \cdot V_b / (R_{He} \cdot T_{He})$$

and to get everything into molar masses (i.e. atomic weights), we can replace the specific gas constants R_{air} and R_{He} with

$$R_{air} = N_A \cdot k / M_{air} \text{ and } R_{He} = N_A \cdot k / M_{He},$$

with k = Boltzmann's constant and N_A = Avagadro's number, given on page 1. After all of this, our equation $m_{air} - m_{He} = m_b$ can be written as:

$$m_b = \frac{P \cdot V_b}{N_A \cdot k} \cdot \left(\frac{M_{air}}{T_{air}} - \frac{M_{He}}{T_{He}} \right)$$

Here, we are told that $T_{air} = T_{He} = T$, so we can solve the above equation for V_b as:

$$V_b = \frac{N_A \cdot k \cdot T \cdot m_b}{P \cdot (M_{air} - M_{He})},$$

Giving ***$V_b = 2871\ m^3$***, *or a radius of* ***r = 8.8 m*** *for a spherical balloon ($V = 4/3 \cdot \pi \cdot r^3$)*

22) An incoming downward flux of short-wavelength radiation of 400 W/m^2 is incident at the top of the atmosphere at a 45-degree **zenith** angle (there is no upward flux). The dry atmosphere is isothermal with a temperature of 17 ºC and a surface pressure of 1000 hPa. There is a well-mixed absorber of mass mixing ratio 10 g/kg and a constant attenuation coefficient k = 0.02 m^2 per kg of absorber.
How much power, in Watts/m^2, is absorbed between 2 and 7 km.

a. 50
b. 3
c. 1800
d. 0.7
e. The correct answer is not shown

The amount of absorber throughout the atmosphere is $\rho \cdot \mu$, and since it is an isothermal atmosphere,

$$\rho = \rho_0 \cdot e^{-\frac{Z}{H_a}}$$

And since we are told it is a dry atmosphere, we have $H_a = R_d \cdot T/g$. With T=273+17=290 K and R_d=287 J/K/kg, our scale height H_a=8493 m. Taking the atmosphere to be an ideal gas:

$$\rho_0 = \frac{P_0}{R_d \cdot T}$$

Given P_o = 100000 Pa, we have ρ_0=1.2kg/m^3, and the density of absorber is μ=0.01 times ρ.

Now the optical depth depends on the attenuation (or absorption) coefficient, k, times the density of absorber from some point Z to infinity:

$$\tau = \int_{Z}^{\infty} \rho 0\, e^{\left(-\frac{z}{Ha}\right)} \mu\, k\, dz$$

Using the help sheet, we see that this integrates to:

$$\tau := \rho 0\, \mu\, k\, Ha\, e^{\left(-\frac{Z}{Ha}\right)}$$

At 7000 m and 2000 m, the absorption of 0.02 $m^2/kg_{absorber}$ yields values of optical depth of:

$$\tau_{7km}=0.895 \text{ and } \tau_{2km}=1.61$$

We now have to look at how the flux is attenuated from the top of the atmosphere (at infinity) to these altitudes as it transits the path at ray angle (not zenith angle) of $\theta=180^o \pm 45^o$ (this would be a ray from upper right to lower left, or upper left to lower right). Since there is horizonal symmetry, we can use either 135^o or 225^o. The downward flux, $F^{\downarrow}$, of 400 W/m2 incident at the top of the atmosphere is further diminished by the angle at which it strikes the atmosphere, θ, as

$$F_{incident}= F^{\downarrow}\cdot\cos(\theta), \text{ or } F_{incident}= F^{\downarrow}\cdot\mu,$$

Where $\mu=\cos(135^o)=-0.707$. This incident flux is attenuated along its path by the transmission coefficient:

$$T = e^{\frac{\tau}{\mu}}$$

So the downward fluxes at 7000 and 2000 m altitude are:

$$F_{7km}= F^{\downarrow}\cdot\mu\cdot\exp(\tau_{7km}/\mu) =400\ W/m^2 \cdot (-0.707)\ \cdot\exp(0.895/(-0.707))=-79.8\ W/m^2,$$

and similarly

$$F_{2km}=400\ W/m^2 \cdot (-0.707)\ \cdot\exp(1.61/(-0.707))=-28.9\ W/m^2.$$

The difference in these fluxes is the power that remains in the layer between 2 and 7 km (since there is no upward flux, the net upward flux is the negative of these values):

*Power deposited between 2 and 7 km = $-F_{7km}+ F_{2km}$ = 79.8-28.9 =**50.8 W/m²**.*

Once again, one could calculate the absorption coefficient between 2 and 7 km as

$$A_{2-7\ km} = 1 - T_{2-7km} = 1 - e^{\frac{\Delta\tau}{\mu}}$$

*This yields an absorption coefficient of $A_{2\text{-}7km}$ = 0.638, which applied to the flux at 7 km gives the power absorbed between 2 and 7 km, again **50.8 W/m²***