

Exam FY3490 Observational Astrophysics

Lecturer: Prof. Manuel Linares

Examination date: June 2nd 2022.

Examination time (from-to): 9am – 12pm

Permitted examination support material: Approved calculator + formula sheet

Other information: This exam accounts for 50% of the final grade. Total score: 10 points.

Official formula sheet provided. Read carefully. Good luck!

Problem 1 [2 points]. An “interloper” in optical astronomy is a star whose angular position in the sky is very close to our science target (say $< 0.5''$). The interloper (star 1) and science target (star 2) appear very close in an optical image, even if they can be far from each other.

- In bad observing conditions (e.g. with a seeing $> 1.5''$), we cannot resolve the two stars and we measure an apparent magnitude $m_3 = 18.64$ for the resulting blend of the two sources. Calculate the true apparent magnitude of our science target, m_2 , knowing that the interloper has an apparent magnitude $m_1 = 20$. [1 p]
- Give the distance to the interloper (in kpc), if it has an absolute magnitude $M = 5$ and the extinction in this band is $A = 0.5$ mag. [0.5 p]
- If we observe from space with a 1m telescope (avoiding atmospheric turbulence), what sets the minimum/best possible angular resolution? In which wavelength range can we observe to resolve an interloper which is $0.3''$ from our target? [0.5 p]

Problem 2 [2 points]. Consider the declination (δ), maximum altitude (h) and zenith angle (z) of a star, as seen from a geographical latitude L .

- Derive the relation between h , δ and L . What is the minimum declination observable from Barcelona ($L = 41^\circ$ N) above the horizon? And at an altitude of at least 30° ? [0.5 p]
- In the common plane-parallel atmosphere approximation, calculate the dimensionless airmass (column density relative to its minimum value towards the zenith) for a zenith angle $z = 35^\circ$. [0.5 p]
- Calculate the fraction of the sky which is visible throughout a full year from Barcelona ($L = 41^\circ$ N) above the horizon. [1 p]

Problem 3 [2 points]. On a particular night, the planet Mars has an angular diameter of 15 arcsec and an energy flux of $1.0 \times 10^{-7} \text{ W m}^{-2}$. Two astronomers observe the planet, using identical CCD cameras whose pixels are 25 μm apart. Albert uses a telescope of 0.3m aperture whose focal ratio is f/8. Bertha uses a telescope of 30 m aperture whose focal ratio is f/4.

- Calculate the surface brightness (energy flux per unit area in the sky) of Mars in units of $\text{W m}^{-2} \text{ arcsec}^{-2}$. [0.5 p]
- How much energy accumulates in a single pixel of Albert's CCD image of Mars in a 100 s exposure? Give your answer in Joules. [0.75 p]
- How much energy (in J) accumulates in the same time in a single pixel of Bertha's image of Mars? [0.75 p]

Question 4 [1 point]. The bolometric peak luminosity of thermonuclear bursts is $[3.79 \pm 0.15] \times 10^{38} \text{ erg/s}$. Observing a newly discovered neutron star, we measure during a thermonuclear burst a bolometric peak flux of $[6.7 \pm 0.7] \times 10^{-8} \text{ erg/s/cm}^2$.

- Assuming that this is a standard candle, calculate the distance and its uncertainty, both in kiloparsecs (kpc). [0.5 p]
- Assuming that the measurements are normally distributed and that the quoted uncertainties represent 1-sigma confidence regions: estimate the probability that the newly discovered source is more than 7.64 kpc away. [0.5 p]

Question 5 [1 point]. In a long-slit optical spectrum:

- If the dimensionless spectral resolution is $R=1000$, what is the resolution in $\text{\AA}$ and in km/s at the Hydrogen alpha line (6563 $\text{\AA}$)? [0.25 p]
- If the spectrum covers the range 3650-7110 $\text{\AA}$ with 2035 pixels, what is the average dispersion? [0.25 p]
- From your answers above, how many pixels sample one full resolution element? [0.25 p]
- State the Nyquist criterion and, based on it, explain whether this spectrum is over, under or critically sampled. [0.25 p]

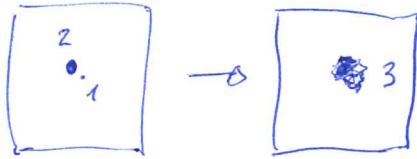
Question 6 [1 point]. A CCD pixel has 95000 e⁻. If the full-well depth of the pixel is 100000 e⁻, the gain is 1.4 e⁻/ADU and a 16-bit ADC is used, will the pixel be saturated? Explain why.

Question 7 [1 point]. Optical and X-ray telescopes.

- Draw schematically a Cassegrain reflector telescope, showing the primary and secondary mirrors and the Cassegrain focus. [0.5 p]
- Explain briefly the difficulties in focusing X-rays and the physical principle used to overcome them. [0.5 p]

OBS - ASTRO - EXAM

P1



a)

(I)	$m_1 - m_2 = 2,5 \log \frac{F_2}{F_1}$	{	$m_3 = 18,64 \text{ mag}$
(II)	$m_1 - m_3 = 2,5 \log \frac{F_3}{F_1}$		$m_1 = 20 \text{ mag}$
(III)	$F_3 = F_1 + F_2 \quad (0,26 \mu)$		$m_2 = ?$

II & III:

$$m_1 - m_3 = 2,5 \log \left(\frac{F_1 + F_2}{F_1} \right) = 2,5 \log \left(1 + \frac{F_2}{F_1} \right)$$

$$\Rightarrow \frac{F_2}{F_1} = 10^{\frac{m_1 - m_3}{2,5}} - 1 = 2,5$$

I:

$$\Rightarrow \boxed{m_2 = m_1 - 2,5 \log \left(\frac{F_2}{F_1} \right) = 19,01 \approx 19 \text{ mag}}$$

b)

$$m - M = 5 \log d_{\text{pc}} - 5 + A$$

$$\boxed{d_{\text{pc}} = 10^{\left(\frac{m - M + 5 - A}{5} \right)} = 7943 \text{ pc} \approx 7,9 \text{ kpc}}$$

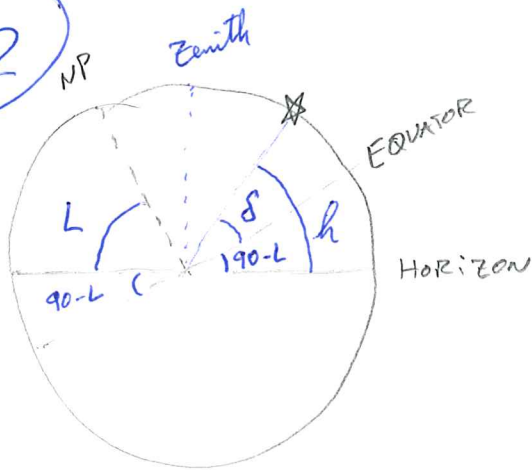
c) DIFFRACTION LIMIT. AIRY DISC RADIUS:

$$\theta_{\min} = 1,22 \frac{\lambda}{D} \Rightarrow \lambda = \frac{D \cdot \theta_{\min}}{1,22} = 1,2 \times 10^{-6} \text{ m}$$

$$\theta_{\min} = 0,3'' = 1,45 \times 10^{-6} \text{ rad}$$

$$\lambda \lesssim 1,2 \mu\text{m}$$

P2



DECLINATION: δ

ALTITUDE: h

ZENITH
ANGLE: z

LATITUDE: L

a)

$$\Rightarrow \boxed{h = 90 - L + \delta}$$

AT HIGHEST POINT:

$$\bullet h = 90 - L + \delta > 0^\circ$$

$$\Rightarrow \underline{\underline{\delta > L - 90^\circ = -49^\circ}}$$

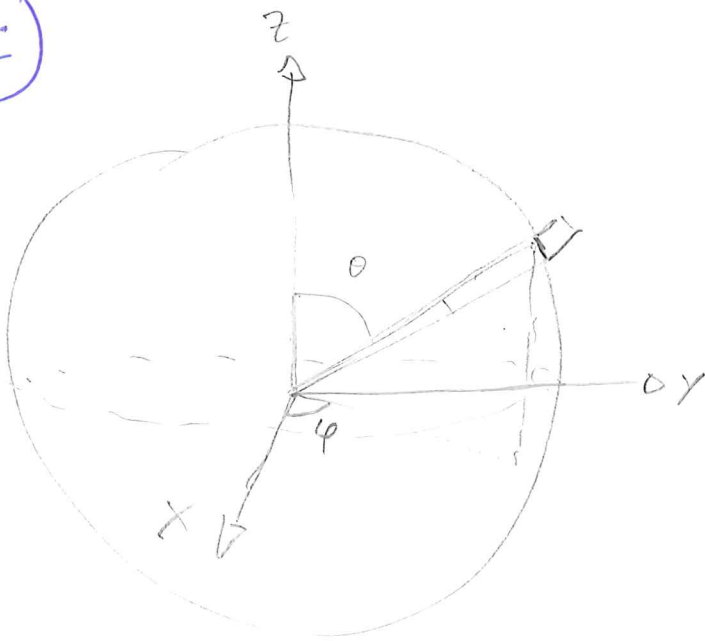
$$\bullet h = 90 - L + \delta > 30^\circ$$

$$\Rightarrow \underline{\underline{\delta > L - 60^\circ = -19^\circ}}$$

b)

$$\underline{\underline{x \approx \frac{1}{\cos z} = 1.22}}$$

c)

BCL ($L=41^\circ$)

above horizon:

$$\delta > -49^\circ$$

$$\Rightarrow \theta = 90 + 49 = 139$$

$$\theta < 139^\circ$$

if only $\theta < 90 + 41 = 131^\circ$
 $\Rightarrow 95\%$

$$d\Omega = \sin\theta \, d\theta \, d\varphi$$

VISIBLE SKY:

$$\Omega_{\text{vis}} = \int_{\text{vis}} d\Omega = \int_0^{2\pi} d\varphi \int_{\theta=0^\circ}^{\theta=\theta_{\text{max}}} \sin\theta \, d\theta = 2\pi (1 - \cos\theta_{\text{max}})$$

$$-\cos\theta \Big|_0^{\theta_{\text{max}}} = \cos 0 - \cos\theta_{\text{max}} = 1 - \cos\theta_{\text{max}}$$

FULL SKY: $\Omega_{\text{sky}} = 4\pi$

$$\Rightarrow \text{FRACTION: } \frac{\Omega_{\text{vis}}}{\Omega_{\text{sky}}} = \frac{1 - \cos\theta_{\text{max}}}{2} = 0,877$$

$$87,7\%$$

P3

$$d \propto \pi \left(\frac{1}{2}\right)^2$$

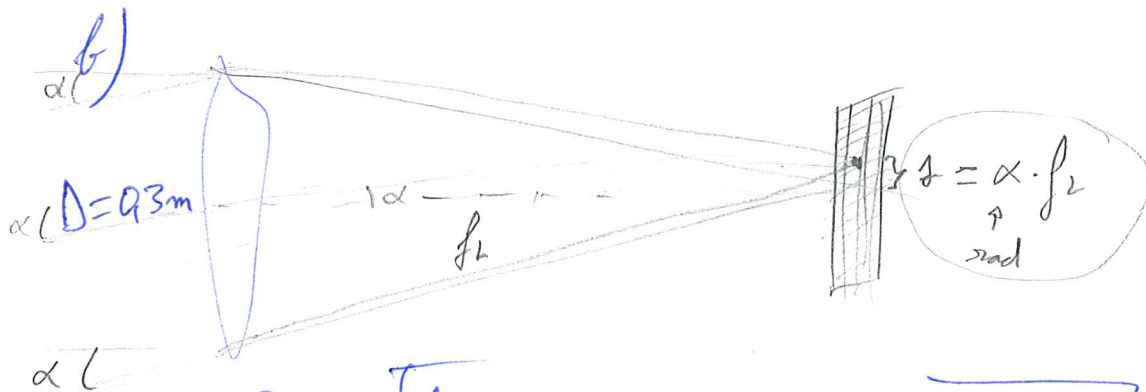
25 μm

25 μm

$$\left\{ \begin{array}{l} d = 15'' \\ F = 10^{-7} \text{ W m}^{-2} \end{array} \right\}$$

$$R = \frac{f_L}{D}$$

$$a) B = \frac{F}{A} = \frac{4F}{\pi d^2} = 5,66 \times 10^{-10} \text{ W m}^{-2} \text{ arcsec}^{-2}$$



$$R = 8 \text{ (f18)}$$

$$f_L = D \cdot R = 0,3 \text{ m} \cdot 8 = 2,4 \text{ m} \quad 0,25$$

AREA PIXEL : 25 $\mu\text{m} \times 25 \mu\text{m}$.

ANGULAR " " : $\Delta\theta \times \Delta\theta$ where $\Delta\theta = \frac{\Delta x}{f_L} = 1,0416 \times 10^{-5} \text{ rad}$

$$\times \frac{180^\circ}{\pi \text{ rad}} \times \frac{3600''}{1^\circ} \rightarrow \Delta\theta = 2,15''$$

$$E = B \times A_{\text{TEL}} \times A_{\text{PIX}} \times T = B \times \pi \left(\frac{D}{2}\right)^2 \times \Delta\theta^2 \times T = 1,85 \times 10^{-8} \text{ J}$$

$$c) \text{ Now } \left\{ \begin{array}{l} D = 30 \text{ m} \\ R = 4 \end{array} \right\} \rightarrow \Delta f_L = D \cdot R = 120 \text{ m} \quad 0,25$$

$$\Delta \theta = \frac{\Delta x}{f_L} = 2,08 \times 10^{-7} \text{ rad} = 0,0429''$$

$$E = B \times A_{\text{TEL}} \times A_{\text{PIX}} \times T = 7,36 \times 10^{-8} \text{ J}$$

Q4

$$L_{\text{peak}} = (3.79 \pm 0.15) \times 10^{38} \text{ erg/s}$$

$$F_{\text{peak}} = (6.7 \pm 0.7) \times 10^{-8} \text{ erg/cm}^2$$

$$F = \frac{L}{4\pi d^2} \Rightarrow d = \sqrt{\frac{L}{4\pi F}} = 2.12 \times 10^{22} \text{ cm} = 6875 \text{ pc}$$

6.88 kpc

$$\delta d = \sqrt{\delta d_F^2 + \delta d_L^2}$$

$$\delta d_F = \frac{\partial d}{\partial F} \delta F = \frac{1}{2} \cdot \sqrt{\frac{L}{4\pi F}} \cdot \frac{\delta F}{F}$$

$$\delta d_L = \frac{\partial d}{\partial L} \delta L = \frac{1}{2} \sqrt{\frac{L}{4\pi F}} \frac{\delta L}{L}$$

$$\Rightarrow \frac{\delta d}{d} = \sqrt{\frac{1}{4} \left(\frac{\delta F}{F}\right)^2 + \frac{1}{4} \left(\frac{\delta L}{L}\right)^2} = \frac{1}{2} \sqrt{\left(\frac{\delta F}{F}\right)^2 + \left(\frac{\delta L}{L}\right)^2} = 0.0556$$

$\frac{1}{2} \sqrt{(0.104)^2 + (0.0396)^2} \quad (5.6\%)$

$$\Rightarrow \delta d = 383 \text{ pc} = 0.38 \text{ kpc}$$

$$\Rightarrow d = 6.88 \pm 0.38 \text{ kpc} = 6.9 \pm 0.4 \text{ kpc}$$

0.25 pc 0.25 pc

b)

$$\frac{x - \bar{x}}{\sigma} = \frac{7,64 - 6,88}{0,38} = 2$$

"2 sigma":

Probab (> +2σ deviation):



From Table 6.2: $P = \frac{0,0455}{2} = 0,0228$

2,3%

2,3%

factor 2 off: 0,25%

correct reasoning but wrong σ: 0,5%

Q5

a) $R = 1000 = \frac{\lambda}{\Delta\lambda} \rightarrow \Delta\lambda = \frac{\lambda}{R} = \underline{6.6 \text{ \AA}}$

$$\frac{\Delta v}{c} = \frac{\Delta\lambda}{\lambda} \rightarrow \Delta v = \frac{c}{R} = 299.8 \text{ km/s}$$

$\approx 300 \text{ km/s}$

b) $\langle \text{disp} \rangle = \frac{7110 - 3650}{2035} = 1.7 \frac{\text{\AA}}{\text{pix}}$

c) 

$$\frac{\Delta\lambda = 6.6 \text{ \AA}}{\text{disp} = 1.7 \text{ \AA/pix}} = 3.86 \approx \underline{3.9 \text{ pixels}}$$

d) Nyquist: $v_{\text{sample}} \geq v_{\text{Nyq}}$ (> 2 pixels per resolution element)

(slightly) OVER SAMPLED

Q6

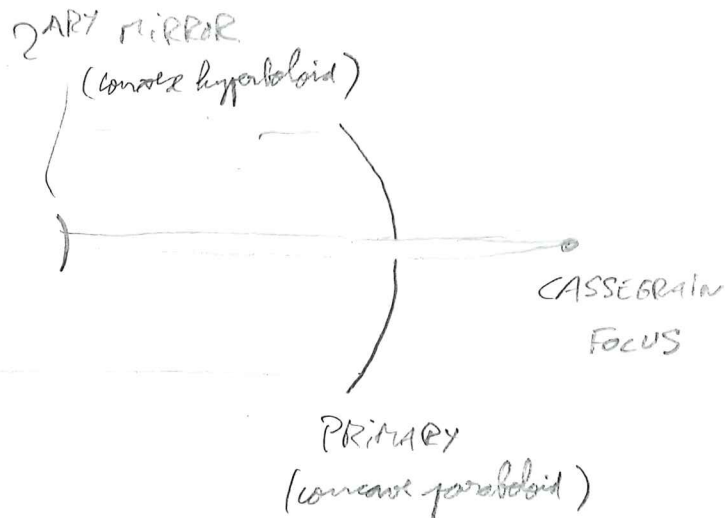
$95.000 e^- < 100.000 e^-$ FULL-WELL DEPTH ✓

~~Pixel~~ PIXEL "COUNTS" = $\frac{95000 e^-}{1.4 e^-/ADU} = \underline{\underline{67857 ADU}}$

16-bit ADC: $2^{16} - 1 = \underline{\underline{65535 MAX}}$

COUNTS > MAX $\Rightarrow$ SATURATED

Q7



b) X-rays penetrate matter/metals, reflected fraction drops drastically with increasing photon energy.

GRAZING INCIDENCE MIRRORS are used to focus X-rays, since reflection is still efficient at very small incident angles ($\ll 1^\circ$).

This results in long focal lengths...