

EXAM - OBSERVATIONAL ASTROPHYSICS - MAY 15, 2023

(I)

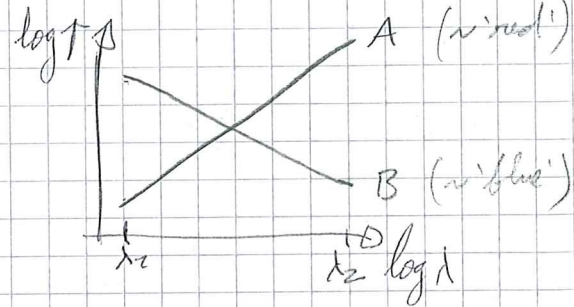
(P1)

2 sources, same band: 400-600 nm

$$\left. \begin{aligned} \lambda_1 &= 400 \text{ nm} = 0,4 \mu\text{m} \\ \lambda_2 &= 600 \text{ nm} = 0,6 \mu\text{m} \end{aligned} \right\}$$

SPECTRAL PHOTON
FLUX DENSITY:

$$\left\{ \begin{aligned} \uparrow_{\lambda A} &= \uparrow_A(\lambda) = A \lambda^3 \\ \uparrow_{\lambda B} &= \uparrow_B(\lambda) = B \lambda^{-2} \end{aligned} \right\}$$



Assume flat response: $\varepsilon(\lambda) = \text{constant} (=1)$

Area of telescope: A_{TEL} (but not needed)

Note:

λ in microns, μm

The two sources will "generate photoelectrons" at a rate ("count rate") R :

$$\frac{R_A}{A_{\text{TEL}}} = \int_{\lambda_1}^{\lambda_2} \uparrow_A(\lambda) d\lambda = A \int_{\lambda_1}^{\lambda_2} \lambda^3 d\lambda = A \left[\frac{\lambda^4}{4} \right]_{0,4}^{0,6} = \frac{0,6^4 - 0,4^4}{4} \cdot A \quad \left[\frac{\text{counts}}{\text{s cm}^2} \right]$$

(0,15)

0,026 · A

$$\frac{R_B}{A_{\text{TEL}}} = \int_{\lambda_1}^{\lambda_2} \uparrow_B(\lambda) d\lambda = B \int_{\lambda_1}^{\lambda_2} \lambda^{-2} d\lambda = B \left[-\lambda^{-1} \right]_{0,4}^{0,6} = \left(\frac{1}{0,4} - \frac{1}{0,6} \right) B \quad \left[\frac{\text{counts}}{\text{s cm}^2} \right]$$

0,833 · B

But they "generate photoelectrons at exactly the same rate":

$$(0,15) \quad R_A = R_B \Rightarrow \left[\frac{A}{B} = \frac{0,833}{0,026} = 32,05 \right] \quad \left[\frac{1}{\mu\text{m}^5} \right]$$

"normalized"
-0,17

Brightness ratio F_B/F_A ? Now it's energy flux: $\times h\nu = \frac{hc}{\lambda}$

$$F_A = \int_{\lambda_1}^{\lambda_2} \uparrow_A(\lambda) \cdot h\nu \cdot d\lambda = Ahc \int_{\lambda_1}^{\lambda_2} \lambda^2 d\lambda = Ahc \left[\frac{\lambda^3}{3} \right]_{0,4}^{0,6} = 0,05067 \cdot Ahc$$

(0,15)

$$F_B = \int_{\lambda_1}^{\lambda_2} \uparrow_B(\lambda) \cdot h\nu \cdot d\lambda = Bhc \int_{\lambda_1}^{\lambda_2} \lambda^{-3} d\lambda = Bhc \left[-\frac{\lambda^{-2}}{2} \right]_{0,4}^{0,6} = 1,736 \cdot Bhc$$

$$\Rightarrow \left[\frac{F_B}{F_A} = \frac{1,736 B}{0,05067 A} = 1,069 \right]$$

(7% difference in F leads to the same R)
(wrong units but ok otherwise: 0,57)

NOTE:

(SPECTRAL) PHOTON FLUX
DENSITY

(SPECTRAL) ENERGY FLUX
DENSITY

$$f_\nu = \frac{\lambda^2}{c} f_\lambda$$

(0.15)

$$f(\lambda)$$

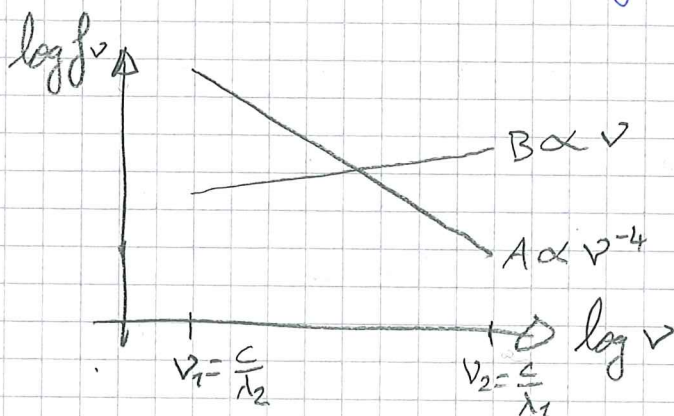
$$\times \frac{hc}{\lambda} \rightarrow f_\lambda = f(\lambda) \text{ "f-lambda"}$$

$$f(\nu)$$

$$\times h\nu \rightarrow f_\nu = f(\nu) \text{ "f-nu"}$$

IF you want to complicate things, this can be also done in ν space:

$$f_\lambda(\lambda) \cdot h\nu \cdot \frac{\lambda^2}{c} = f_A(\nu) = \frac{A h c^4}{\nu^4} ; f_B(\nu) = \frac{B h}{c} \nu = f_\lambda(\lambda) \cdot \lambda \nu \cdot \frac{\lambda^2}{c}$$



$$F_A = \int_{\nu_1}^{\nu_2} f_A(\nu) d\nu = \dots = \frac{A h c}{3} \cdot 9.152$$

$$F_B = \int_{\nu_1}^{\nu_2} f_B(\nu) d\nu = \dots = \frac{B h c}{2} \cdot 3.472$$

$\rightarrow$ And reach the same $\frac{F_B}{F_A} = 1.069$

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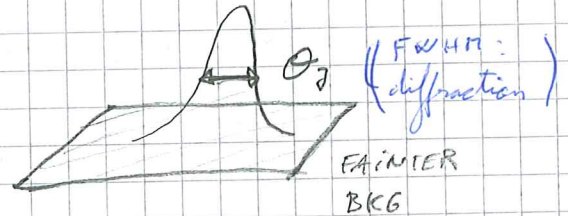
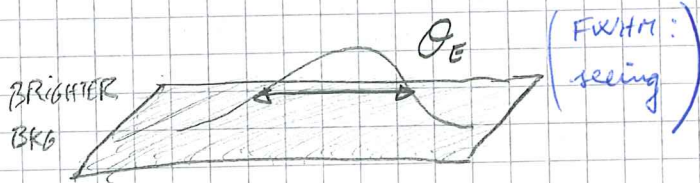
ATMOSPHERE TURBULENCE

ELT: $\Delta_E = 24 \text{ m}$

SPACE

JWST: $\Delta_J = 6.5 \text{ m}$

P2



TO DETECT FAINT SOURCES: PROS & CONS!

• APERTURE:

+PRO (larger $A_E \propto \Delta_E^2$) -CON (smaller $A_J \propto \Delta_J^2$)

• BACKGROUND:

-CON (brighter background lowers S/N)

+PRO (fainter background boosts S/N)

brighter due to atmosphere scattering (e.g. moon light)

"The background for the ELT is 2 magnitudes per square arcsec brighter"

Background flux per square arcsec: B

$$\Rightarrow b_J - b_E = 2 \frac{\text{mag}}{\text{arcsec}^2} = 2.5 \log \frac{B_E}{B_J} \Rightarrow \frac{B_E}{B_J} = 10^{\frac{2}{2.5}} = 10^{0.8} \approx 6.31$$

• POINT-SPREAD FUNCTION (PSF):

(0.29")

with a better (smaller) PSF, the source counts are distributed over a smaller area, so the S/N increases (less background counts in that area).

-CON (seeing spreads counts over larger area) +PRO (diffraction-limited PSF is narrower)

"Magnitude threshold" implies FLUX LIMIT: what is the faintest source flux we need to do something? ("certain application" e.g. detect a source).

• SAME APPLICATION: SAME SIGNAL TO NOISE RATIO (SNR).

Recall: we measure TOTAL (N_T) and BACKGROUND (N_B) counts and then:

$$N_S = N_T - N_B \Rightarrow \sigma_S^2 = \sigma_T^2 + \sigma_B^2 = N_S + 2N_B$$

Poisson: $N_S + N_B$ N_B

AT THE FAINT SOURCE LIMIT, BACKGROUND DOMINATES: $N_B \gg N_S$:

• to the NOISE: UNCERTAINTY ON N_S : $\sigma_S \approx \sqrt{2N_B} \propto \sqrt{B \cdot A \cdot \theta^2}$

"is proportional to $\sqrt{\text{collected background counts in source region}}$ "

↑
bkg
flux
per
source²

↑
collecting
area

↑
angular
area
source
(PSF)

• While our SIGNAL are the net source counts:

$$N_S = K F \cdot A \propto F \cdot D^2 \quad (9.25_1)$$

constant:
bandpass, detector

↑
flux
density

↑
collecting
area

$$\Rightarrow \boxed{\text{SNR} \approx \frac{N_S}{\sigma_S} \propto \frac{F \cdot D^2}{\sqrt{B \cdot A \cdot \theta^2}} = \frac{F}{\sqrt{B}} \cdot \frac{D}{\theta}}$$

$$\text{SNR}_j = \text{SNR}_E \Rightarrow \frac{F_E}{\sqrt{B_E}} \cdot \frac{D_E}{\theta_E} = \frac{F_j}{\sqrt{B_j}} \cdot \frac{D_j}{\theta_j}$$

Ratio of flux limits: $\frac{F_E}{F_j} = \left(\frac{B_E}{B_j} \right)^{1/2} \cdot \frac{D_j}{D_E} \cdot \frac{\theta_E}{\theta_j} = 10^{1/2.5} \cdot \frac{6.5}{24} \cdot 4 = 2.72$

Magnitude difference: $m_j - m_E = 2.5 \log \frac{F_E}{F_j} = 1.087 \quad (9.26_1)$

$$\Rightarrow \boxed{m_E = 26.9 \text{ mag}}$$

P2

NOTE: INCOMPLETE ALTERNATIVES:

A1: considers only aperture : $N_s \propto D^2 F$
 $E \propto D^2 F$ (P1.6 class)

"same N_s " : $F_E \pi \left(\frac{D_E}{2}\right)^2 \Delta t = F_J \pi \left(\frac{D_J}{2}\right)^2 \Delta t$

$$\frac{F_E}{F_J} = \left(\frac{D_J}{D_E}\right)^2 \Rightarrow \boxed{m_E = m_J - 2,5 \log \left(\frac{D_J}{D_E}\right)^2 = 30,8 \text{ mag}}$$

X (+0,57)

A2: considers aperture & PSF : $\frac{N_s}{\text{arcsec}^2} \propto \frac{D^2 F}{\theta^2}$

"same $\frac{N_s}{\text{arcsec}^2}$ " ... $\Rightarrow \frac{F_E}{F_J} = \left(\frac{D_J}{D_E}\right)^2 \left(\frac{\theta_E}{\theta_J}\right)^2 \Rightarrow \boxed{m_E = m_J - 5 \log \left(\frac{D_J}{D_E}\right) - 5 \log \left(\frac{\theta_E}{\theta_J}\right) = 27,8}$

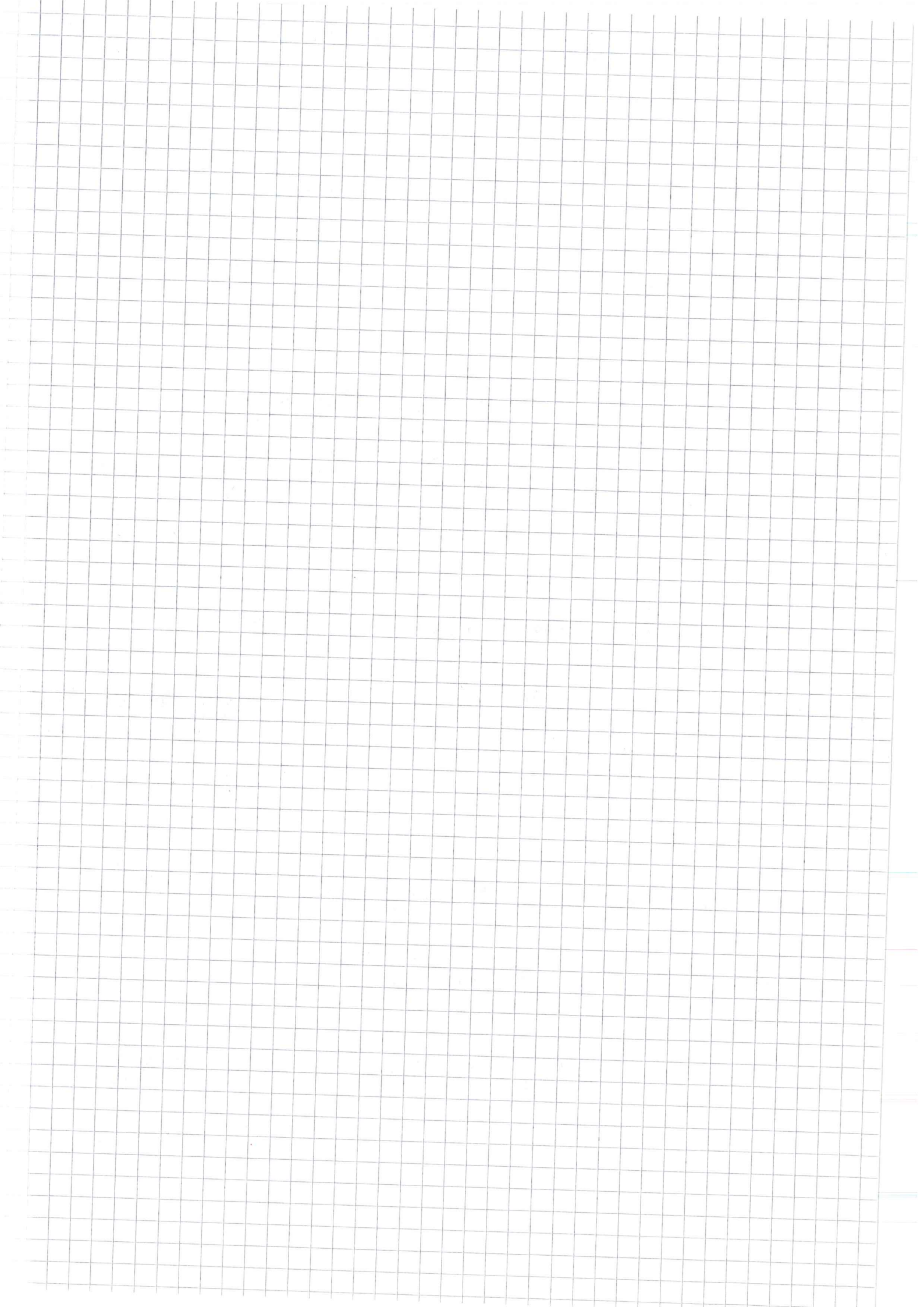
X (+17)

A3: considers aperture, PSF & background : $\frac{N_s}{N_B} \propto \frac{D^2 F}{B \theta^2}$

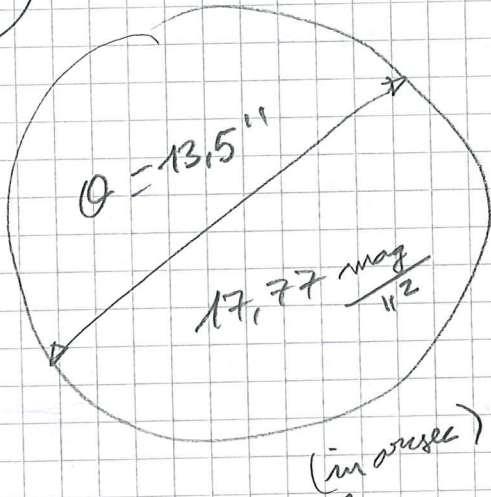
"same $\frac{N_s}{N_B}$ " ... $\Rightarrow \frac{F_E}{F_J} = \frac{B_E}{B_J} \left(\frac{D_J}{D_E}\right)^2 \left(\frac{\theta_E}{\theta_J}\right)^2$

$$\Rightarrow \boxed{m_E = m_J - 5 \log \frac{D_J}{D_E} - 5 \log \frac{\theta_E}{\theta_J} - 2,5 \log \frac{B_E}{B_J} = 25,8}$$

X (+1,57)



P3



Uniform nebula with a surface brightness

$$m_s = 17.77 \frac{\text{mag}}{\text{arcsec}^2}$$

Recall: flux is additive (energy/photon), magnitudes are not!

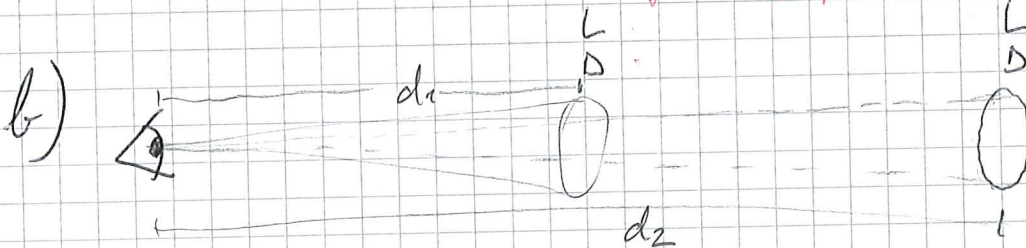
$$F_{\text{nebula}} = \pi \left(\frac{\theta}{2} \right)^2 \cdot F_{\text{ras}}$$

where F_{ras} is the flux in 1 arcsec^2 .

$$m_s - m = 2.5 \log \frac{F_{\text{nebula}}}{F_{\text{ras}}} = 2.5 \log \left(\pi \left(\frac{\theta}{2} \right)^2 \right) = 5.39$$

$$\Rightarrow m = m_s - 5.39 = 12.38 \text{ mag}$$

wrong order: 0.25.



$$\Delta = d_1 \theta_1 = d_2 \theta_2 \Rightarrow \frac{\theta_2}{\theta_1} = \frac{d_1}{d_2} = \frac{1}{2}$$

$$d_2 = 2d_1$$

$$\begin{aligned} A_1 &= 143.1''^2 \\ A_2 &= 35.8''^2 \end{aligned}$$

$$A = \pi \left(\frac{\theta}{2} \right)^2 \propto \theta^2 \Rightarrow \frac{A_2}{A_1} = \left(\frac{\theta_2}{\theta_1} \right)^2 = \left(\frac{d_1}{d_2} \right)^2 = \frac{1}{4}$$

angle but no area: 0.25

The angular area decreases by a factor $\frac{1}{4}$.

c) Flux $F \propto \frac{L}{d^2}$; $m_2 - m_1 = 2,5 \log \frac{F_1}{F_2}$

$$\frac{F_1}{F_2} = \left(\frac{d_2}{d_1}\right)^2 = 4 \rightarrow m_2 - m_1 = 1,505 \text{ mag}$$

$$\Rightarrow \boxed{m_2 = 12,38 + 1,505 = 13,89 \text{ mag}}$$

The apparent magnitude increases by $\approx 1,5 \text{ mag}$ (since it becomes fainter).

d) TOTAL FLUX DECREASES $\times \frac{1}{4}$
 ANGULAR AREA " $\times \frac{1}{4}$ } $\frac{\text{FLUX}}{\text{AREA}} = \text{constant}$

Surface brightness stays constant!

- correct "increase" or "decrease" without quantifying: 0,25!

Q4

The number of pixels to read would be simply 8192×8192 .
At a pixel readout speed of 25 kHz this would take

$$t = \frac{(8192)^2}{25 \times 10^3 \text{ Hz}} = 2684 \text{ s} \approx 45 \text{ min}$$

factor 3 (charge transfer): ≈ 0.25

Q5

a) (see L3.2 class notes)

MADE:

- a metal box with inert gas (Ar, Xe), crossed by a:
- wire/anode at high voltage (attracting e^-), open from one side with a:
- thin transparent ($E \geq 2 \text{ keV}$) window.

OPERATE:

- Photoelectric effect ionizes/ejects inner-shell e^- from gas
- ionization track is formed (outer shell e^- from other atoms): AMPLIFY.
- CHARGE COLLECTED @ ANODE is PROPORTIONAL to E (incoming X-ray photon)

b) $\nu_{\text{Nyquist}} \equiv \frac{1}{2 \cdot \tau} = 500 \text{ Hz}$
 $\uparrow$
 $(\tau = 1 \text{ ms})$

Interpretation: signals with $\nu > \nu_{\text{Nyquist}}$ can't be reliably/fully detected.

c) For $T = 1024 \text{ s}$:
 Frequency resolution, first term in Fourier power spectrum @: $\nu = \frac{1}{T} = 9.8 \times 10^{-4} \text{ Hz} \approx 1 \text{ mHz}$

Q6

$$\left\{ \begin{array}{l} \Delta = 15 \text{ cm} = 150 \text{ mm} \\ \lambda = 650 \text{ nm} \end{array} \right. ; \quad R = \frac{f_L}{\Delta} = 10$$

"f/10"

→ DIFFRACTION-LIMITED RESOLUTION:

Rayleigh Airy disk: $\theta_{\min} = 1,22 \frac{\lambda}{\Delta}$

$$\theta_{\min} = 1,22 \frac{650 \times 10^{-9} \text{ m}}{15 \times 10^{-2} \text{ m}} = 5,29 \times 10^{-6} \text{ rad} \approx 1,1''$$

0,5 p.

→ AT THE FOCAL PLANE:

Focal length: $f_L = R \Delta = 1500 \text{ mm}$

$$\rightarrow s_{\min} = \theta_{\min}(\text{rad}) \cdot f_L = 7,94 \text{ } \mu\text{m} = 7,9 \times 10^{-3} \text{ mm}$$

0,5 p.

Q7

In space: cosmic rays can hit the CCS at almost any angle.

On Earth: from its surface the atmosphere prevents ("shields from") nearly all glancing angle cosmic-ray impacts.
(CCS are typically "facing up" to the sky)

