(I)

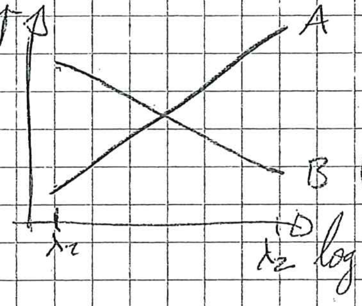
EXAM - OBSERVATIONAL ASTROPHYSICS - ~~MAY 15~~ JUNE 5, 202~~3~~4

(P1)

2 sources, same band: 400–600 nm $\left\{ \begin{array}{l} \lambda_1 = 400\,\mathrm{nm} = 0{,}4\,\mu m \\ \lambda_2 = 600\,\mathrm{nm} = 0{,}6\,\mu m \end{array} \right\}$

SPECTRAL PHOTON FLUX DENSITY: $\left[\frac{\mathrm{phot}}{m^2 \cdot s \cdot \mu m}\right]$

$$\left\{ \begin{array}{l} f_{\lambda A} = f_A(\lambda) = A\,\lambda^3 \\ f_{\lambda B} = f_B(\lambda) = B\,\lambda^{-2} \end{array} \right\}$$



A (~'red') B (~'blue')

- Assume flat response: $\mathcal{E}(\lambda) = \text{constant} \ (\equiv 1)$
- Area of telescope: A_{TEL} (but not needed)

Note: λ in microns, μm (*)

a) (1) The two sources will "generate photoelectrons" at a rate ("count rate") R:

$$\frac{R_A}{A_{TEL}} = \int_{\lambda_1}^{\lambda_2} f_A(\lambda)\,d\lambda = A \int_{\lambda_1}^{\lambda_2} \lambda^3\,d\lambda = A\left[\frac{\lambda^4}{4}\right]_{0{,}4}^{0{,}6} = \underbrace{\frac{0{,}6^4 - 0{,}4^4}{4} \cdot A}_{0{,}026 \cdot A} \quad \left[\frac{\text{counts}}{s\ cm^2}\right]$$

(0,15)

$$\frac{R_B}{A_{TEL}} = \int_{\lambda_1}^{\lambda_2} f_B(\lambda)\,d\lambda = B \int_{\lambda_1}^{\lambda_2} \lambda^{-2}\,d\lambda = B\left[-\lambda^{-1}\right]_{0{,}4}^{0{,}6} = \underbrace{\left(\frac{1}{0{,}4} - \frac{1}{0{,}6}\right) B}_{0{,}833 \cdot B} \quad \left[\frac{\text{counts}}{s \cdot cm^2}\right]$$

But they "generate photoelectrons at exactly the same rate":

(0,15) $$R_A = R_B \Rightarrow \boxed{\frac{A}{B} = \frac{0{,}833}{0{,}026} = 32{,}05} \quad \left[\frac{1}{\mu m^5}\right] (*)$$

"normalized" −0,1

b) Brightness ratio F_B/F_A? Now it's energy flux: $\times\, h\nu = \frac{hc}{\lambda}$

(1)

$$F_A = \int_{\lambda_1}^{\lambda_2} f_A(\lambda) \cdot h\nu \cdot d\lambda = Ahc \int_{\lambda_1}^{\lambda_2} \lambda^2\,d\lambda = Ahc\left[\frac{\lambda^3}{3}\right]_{0{,}4}^{0{,}6} = \underline{0{,}05067 \cdot Ahc}$$

(0,15)

$$F_B = \int_{\lambda_1}^{\lambda_2} f_B(\lambda) \cdot h\nu \cdot d\lambda = Bhc \int_{\lambda_1}^{\lambda_2} \lambda^{-3}\,d\lambda = Bhc\left[\frac{-\lambda^{-2}}{2}\right]_{0{,}4}^{0{,}6} = \underline{1{,}736 \cdot Bhc}$$

$$\Rightarrow \boxed{\frac{F_B}{F_A} = \frac{1{,}736\,B}{0{,}05067\,A} = 1{,}069}$$

(7% difference in F leads to the same R)

(wrong units but ok otherwise: 0,5)

$\boxed{\text{NOTE:}}$ (SPECTRAL) PHOTON FLUX DENSITY $\longrightarrow$ (SPECTRAL) ENERGY FLUX DENSITY

$p(\lambda) \quad \times \frac{hc}{\lambda} \rightarrow f_\lambda = f(\lambda)$ "f-lambda"

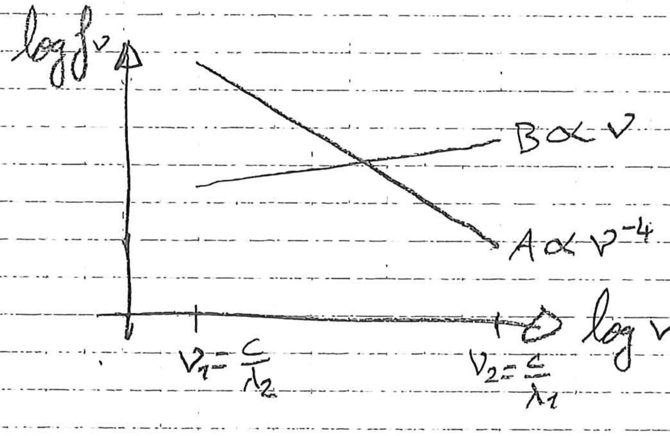
$f_\nu = \frac{\lambda^2}{c} f_\lambda$

$(0,15)$

$p(\nu) \quad \times h\nu \rightarrow f_\nu = f(\nu)$ "f-nu"

IF you want to complicate things, this can be also done in ν space:

$p_A(\lambda)\cdot h\nu \cdot \frac{\lambda^2}{c} = f_A(\nu) = \frac{A h c^4}{\nu^4}$; $f_B(\nu) = \frac{Bh}{c}\nu = p_B(\lambda)\cdot h\nu\cdot\frac{\lambda^2}{c}$

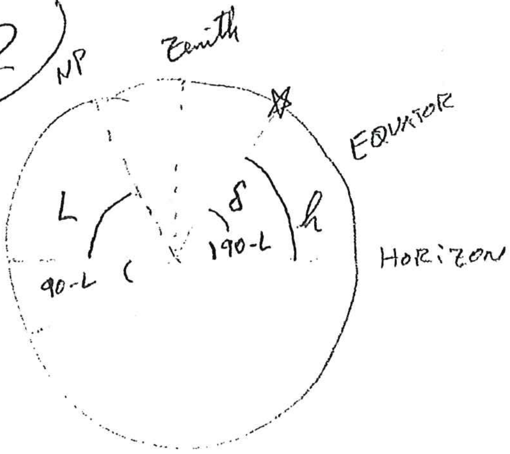


$F_A = \int_{\nu_1}^{\nu_2} f_A(\nu)d\nu = \dots = \frac{Ahc}{3}\cdot 0{,}15$

$F_B = \int_{\nu_1}^{\nu_2} f_B(\nu)d\nu = \dots = \frac{Bhc}{2}\cdot 3{,}47$

$\rightarrow$ And reach the same $\frac{F_B}{F_A} = 1{,}069$

P2



DECLINATION : δ

ALTITUDE : h

ZENITH ANGLE : z

LATITUDE : L

a) $\Rightarrow$ $\boxed{h = 90 - L + \delta}$

0,5p

AT HIGHEST POINT:

- $h = 90 - L + \delta > 0°$

$\Rightarrow \delta > L - 90° = -40°$

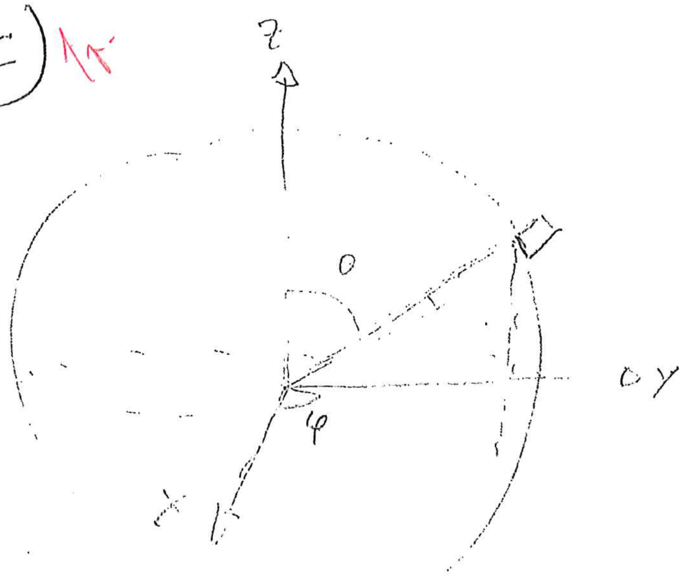
- $h = 90 - L + \delta > 30°$

$\Rightarrow \delta > L - 60° = -10°$

b) $X \simeq \frac{1}{\cos z} =$ $= 1,15$

0,5p

c) 4π



KIEV ($L = 50°$)

above horizon: 30°:

$\delta > -40°$ $\delta > -10°$

$\Rightarrow \theta = 90 + 10 = 100$

$(\theta < 100°)$

[if only $\theta < 90 + 50 = 140°$
$\Rightarrow 95\%$]

$d\Omega = \sin\theta \, d\theta \, d\varphi$

VISIBLE SKY: $\Omega_{vis} = \int_{vis} d\Omega = \underbrace{\int_0^{2\pi} d\varphi}_{2\pi} \underbrace{\int_{\theta=0°}^{\theta=\theta_{max}} \sin\theta \, d\theta}_{} = 2\pi(1 - \cos\theta_{max})$

$-\cos\theta \Big|_0^{\theta_{max}} = \underset{1}{\cos 0} - \cos\theta_{max}$

FULL SKY: $\Omega_{sky} = 4\pi$

$\Rightarrow$ FRACTION: $\dfrac{\Omega_{vis}}{\Omega_{sky}} = \dfrac{1 - \cos\theta_{max}}{2} = 0{,}587$

58,7 %

P3

$d \; \varnothing \; \pi\left(\frac{d}{2}\right)^2$

$25\mu m$

$\square \; 25\mu m$

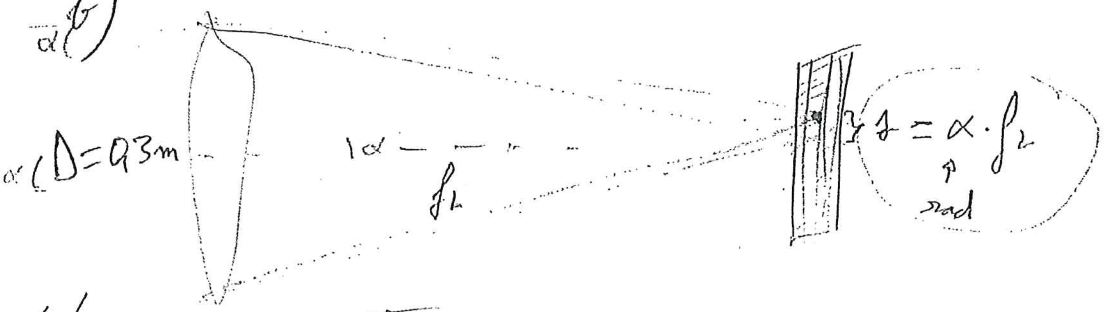
$$\begin{cases} d = 15'' \\ F = 10^{-7} \, W \, m^{-2} \end{cases}$$

$$R = \frac{f_L}{D}$$

a) $B = \frac{F}{A} = \frac{4F}{\pi d^2} = \boxed{5{,}66 \times 10^{-10} \; W \, m^{-2} \, arcsec^{-2}}$

[0,5+]

b)



c

$R = 8$

$(f/8)$

$\boxed{f_L = D \cdot R = 0{,}3m \cdot 8 = 2{,}4 \, m}$

0,25

AREA PIXEL : $25\mu m \times 25\mu m$.

ANGULAR " " : $\Delta\theta \times \Delta\theta$ where $\Delta\theta = \frac{\Delta x}{f_L} = 1{,}0416 \times 10^{-5}$ rad

$\times \frac{180^\circ}{\pi \, rad} \times \frac{3600''}{1^\circ} \rightarrow \boxed{\Delta\theta = 2{,}15''}$

$$E = B \times A_{TEL} \times A_{Pix} \times T = B \times \pi\left(\frac{D}{2}\right)^2 \times \Delta\theta^2 \times T = \boxed{1{,}85 \times 10^{-8} \; J}$$

circular pixel -0,25

c) New $\left\{\begin{array}{l} D = 30\ m \\ R = 4 \end{array}\right\} \rightarrow f_L = D \cdot R = 120\ m$ 0,25

$$\Delta\theta = \frac{\Delta x}{f_L} = 2{,}08 \times 10^{-7}\ rad = 0{,}0429''$$

$$E = B \times A_{TEL} \times A_{PIX} \times T = 7{,}36 \times 10^{-8}\ J$$

Linear pixel -0,25

Q4

95.000 e^- < 100.000 e^- FULL-WELL DEPTH ✓

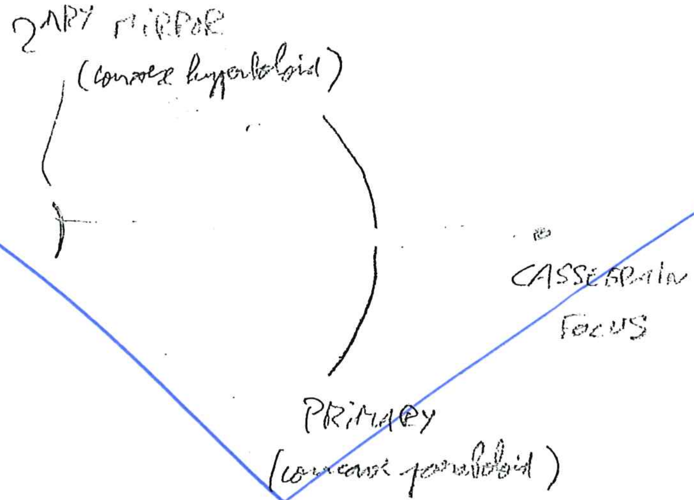
PIXEL "COUNTS" = $\frac{95000\,e^-}{1{,}4\,e^-/ADU}$ = 67857 ADU

16-bit ADC: $2^{16}-1$ = 65535 MAX

COUNTS > MAX ⇒ SATURATED

Q7

a)



b) X-rays penetrate matter (metals), reflected fraction drops drastically with increasing photon energy.

GRAZING INCIDENCE MIRRORS are used to focus X-rays, since reflection is still efficient at very small incident angles (<<1°).

This results in long focal lengths...

Q5

$$L_{peak} = (3{,}79 \pm 0{,}15) \times 10^{38}\ erg/s$$

a)
[0,5]

$$F_{peak} = (1{,}27 \pm 0{,}13) \times 10^{-7}\ erg/s\,cm^2$$

$$F = \frac{L}{4\pi d^2} \Rightarrow d = \sqrt{\frac{L}{4\pi F}} = 1{,}54 \times 10^{22}\ cm$$

$$= 5{,}0\ kpc$$

$$\delta d = \sqrt{\delta d_F^{\,2} + \delta d_L^{\,2}}$$

$$\delta d_F = \frac{\partial d}{\partial F}\delta F = \frac{-1}{2} \cdot \underbrace{\sqrt{\frac{L}{4\pi F}}}_{d} \cdot \frac{\delta F}{F}$$

$$\delta d_L = \frac{\partial d}{\partial L}\delta L = \frac{1}{2} \underbrace{\sqrt{\frac{L}{4\pi F}}}_{d} \frac{\delta L}{L}$$

$$\Rightarrow \frac{\delta d}{d} = \sqrt{\frac{1}{4}\left(\frac{\delta F}{F}\right)^2 + \frac{1}{4}\left(\frac{\delta L}{L}\right)^2} = \frac{1}{2}\sqrt{\left(\frac{\delta F}{F}\right)^2 + \left(\frac{\delta L}{L}\right)^2} = 0{,}0556$$

$$\frac{1}{2}\sqrt{(0{,}104)^2 + (0{,}0396)^2} \qquad (5{,}6\,\%)$$

$$\Rightarrow \delta d = 278\ pc = 0{,}278\ kpc$$

$$\Rightarrow d = 5{,}0 \pm 0{,}3\ kpc = 4{,}99 \pm 0{,}28\ kpc$$

0,25 p. 0,25 p.

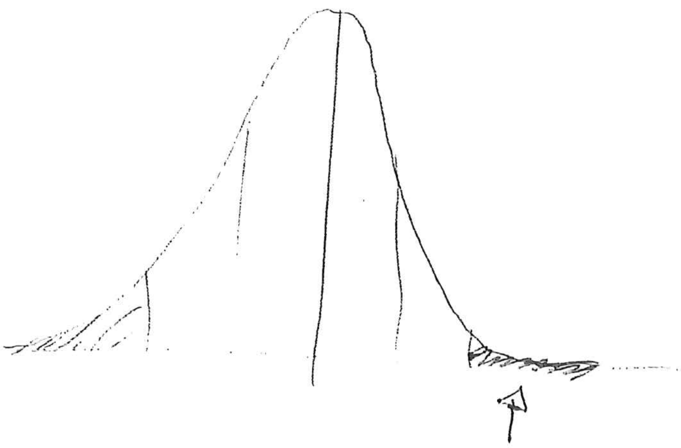
b)

[0,5]

$$\frac{x-\bar{x}}{\sigma} : \frac{5,56-4,99}{0,28} = 2$$

"2 sigma":

Probab (> +2σ deviation) :



From Table 6.2 : $P = \frac{0,0455}{2} = 0,0228$

2,28%

2,3%

factor 2 off : 0,25 p.

Correct reasoning but wrong σ : 0,5 p.

Q6:

$D = 30\,m$,

$f/15 : R = \frac{f_L}{D} = 15$

a)
[0,5p]

$f_L = 15 \times D = 30 \times 15 = 450\,m = 450.000\,mm$

Plate scale: $\frac{1}{f_L} \rightarrow \frac{206265''}{f_L\,(mm)} = 0,458\,''/mm$

b)
[0,5p]
$20 \times 60' = 1200''$ $\Rightarrow$ FoV: $1200'' \times 1200''$

$\times \frac{1}{0,458\,''/mm} \Rightarrow$ FoV: $2620\,mm \times 2620\,mm$

$\times \frac{1\,pix}{30 \times 10^{-3}\,mm} \Rightarrow 87336 \times 87336 = \boxed{7,63 \times 10^{9}\,pix}$

Correct reasoning, wrong plate scale: 0,5p.

Q7

a) 0,25

$R = 2850 = \frac{\lambda}{\Delta\lambda} \to \Delta\lambda = \frac{\lambda}{R} = 1{,}7$ Å

$\frac{\Delta v}{c} = \frac{\Delta\lambda}{\lambda} \to \Delta v = \frac{c}{R} = 105{,}2$ km/s $\simeq 105$ km/s

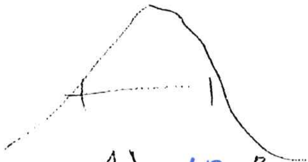
(vel. missing: −0,1)

b) 0,25

$\boxed{\langle disp \rangle = \frac{5730-4000}{1018} = 1{,}7\ Å/pix}$

0,588 pix/Å OK ✓

c) 0,25



$\frac{\Delta\lambda = 1{,}7\ Å}{disp = 1{,}7\ Å/pix} = 1 \Rightarrow 1$ pixels

d) 0,25 Nyquist: $v_{sampl} \gtrsim v_{Nyq}$ ($\gtrsim$ 2 pixels per resolution element)

$\Rightarrow$ (slightly) UNDER-SAMPLED