EXAM FY3215 - JUNE 2025

PROBLEM 1

0,5 a) WIEN's DISPLACEMENT LAW:

$$E_{PEAK} = h\nu_{PEAK} = 2,82\ kT$$

$$\nu_{PEAK}(Hz) = 5,88 \times 10^{10} \times T(K) = 3,40 \times 10^{14}\ Hz$$ 0,25

$$\lambda_{PEAK} = \frac{c}{\nu_{PEAK}} = 8,82 \times 10^{-7}\,m = 8820\ \text{Å}$$ 0,25

(wrong units −0,1)

0,5 b)

L_1, T_1, R_1 $\qquad$ L_2, T_2, R_2

$$\begin{cases} L_1 = 11,3\ L_2 \\ T_1 = 1,4\ T_2 \end{cases}$$

Stefan-Boltzmann: $F = \sigma T^4$

$$\boxed{L = 4\pi R^2 \cdot F = 4\pi R^2 \cdot \sigma T^4} \propto R^2 T^4$$

$$\frac{L_1}{L_2} = 11,3 = \left(\frac{R_1}{R_2}\right)^2 \cdot \left(\frac{T_1}{T_2}\right)^4 = 11,3$$

$$\Rightarrow \boxed{\frac{R_1}{R_2}} = \sqrt{\frac{11,3}{\left(\frac{T_1}{T_2}\right)^4}} = \sqrt{\frac{11,3}{(1,4)^4}} = \boxed{1,7}$$

(OK here 0,25)

1.C) $$\boxed{B_\nu(T) = \frac{2h\nu^3}{c^2} \cdot \frac{1}{e^{h\nu/kT}-1}}$$

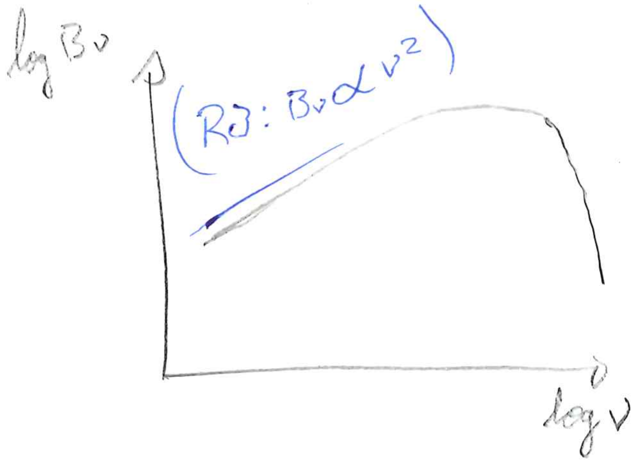
LOW ν : $h\nu \ll kT$

TAYLOR EXPANSION FOR $\nu \to 0$ (MACLAURIN SERIES) :

$$e^{\frac{h\nu}{kT}} \simeq 1 + \frac{h\nu}{kT} + \ldots$$

$$f(x) \simeq f(0) + \frac{f'(0)}{1!}x + \frac{f''(0)}{2!}x^2 + \ldots$$

$$\Rightarrow \boxed{B_\nu(T) \simeq \frac{2\nu^2 kT}{c^2}}$$



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PROBLEM 2

a)

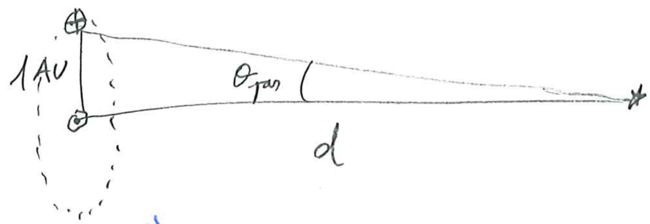
ABSOLUTE MAGNITUDE, M: magnitude that a star would have if it were seen from a distance of $\underline{\underline{10\,pc}}$.

0,25 p

PARSEC, pc: distance at which i) the trigonometric parallax of a star equals one arcsecond ($1''$)

or ii) the mean radius of Earth's orbit around the Sun subtends an angle of $1''$.

0,25 p



b) difference in (apparent) magnitudes:

$$m_1 - m_2 = 2{,}5 \log_{10}\left(\frac{F_2}{F_1}\right)$$

inverse square law:

$$F = \frac{L}{4\pi d^2}$$

$$\underbrace{m - M}_{\Delta M:\ \text{distance modulus}} = 2{,}5 \log_{10}\left(\frac{\frac{L}{4\pi (10\,pc)^2}}{\frac{L}{4\pi d_{pc}^2}}\right) = 2{,}5 \log_{10}\left(\frac{d_{pc}^2}{(10\,pc)^2}\right) =$$

$$= 5 \log \frac{d_{pc}}{10_{pc}} = 5\left(\log d_{pc} - \log_{10}(10\,pc)\right) = \boxed{5 \log_{10}(d_{pc}) - 5}$$

0,5 p

c) Type Ia SNa : $M_V = -19{,}60$ mag (at max)

We measure $m_V = 13{,}25$ (at max)

$$m - M = 13{,}25 + 19{,}60 = 5 \log_{10}(d_{pc}) - 5$$

$$\Longrightarrow d_{pc} = 10^{\frac{13{,}25 + 19{,}60 + 5}{5}} \boxed{= 37{,}15 \text{ Mpc}}$$

d) Correcting for $A_V = 1{,}5$ mag of extinction :

$$m - M = 5 \log_{10} d_{pc} - 5 \underline{+ A}$$

$$d_{pc} = 10^{\frac{13{,}25 + 19{,}60 + 5 - 1{,}5}{5}} = 18{,}62 \times 10^6$$

$$\rightarrow \boxed{d = 18{,}6 \text{ Mpc}}$$

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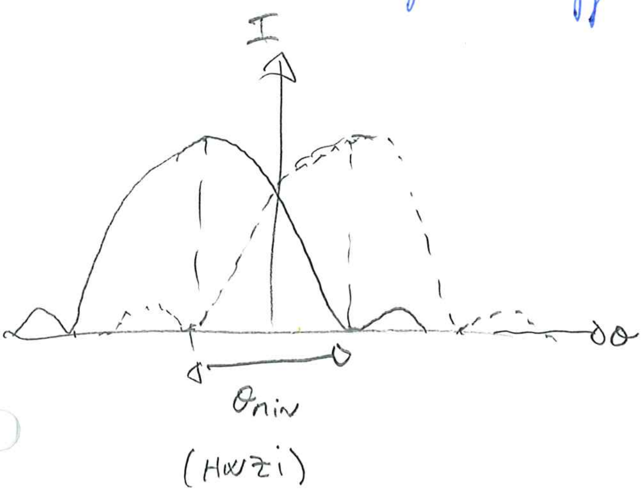
PROBLEM 3

0,5 a) Diffraction-limited angular resolution:

$$\theta_{min} = 1{,}22 \frac{\lambda}{D} \quad (\text{radians}) \qquad \begin{pmatrix} \text{RADIUS} \\ \text{AIRY} \\ \text{DISC} \end{pmatrix}$$

Rayleigh's criterion: 2 point sources are just resolved when the center of one Airy disc falls on the 1st minimum of the other Airy disc diffraction pattern.



$$\theta_{min} = 1{,}22 \frac{5500 \times 10^{-10}\, m}{150 \times 10^{-3}\, m} = 4{,}47 \times 10^{-6}\ \text{rad}$$

$$\underline{\theta_{min} \simeq 0{,}9''}$$

0,5 b) f/5 means focal ratio $R = \frac{f_L}{D} = 5$

$$\Rightarrow \Delta s = \Delta\theta \cdot f_L = \theta_{min} \cdot 5 \cdot D = 3{,}36 \times 10^{-6}\, m$$

$$= \underline{3{,}4\ \mu m}$$

c) AT THE F.P. / DETECTOR :

0,5

$2\Delta x \leq 2\Delta f$

We need at least 2 pixels (resolution elements) across the FWHM :

$$\boxed{\Delta x \lesssim \Delta f = 3{,}4\,\mu m}$$

0,5

d) Seeing : atmospheric turbulence, will set the angular resolution from the ground (except good observing site or excellent conditions)

From sea level $\Delta\theta \gtrsim 3''$

Discuss : 0,25

Quantify : 0,25

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Q4

$$\frac{v_R}{c} = \frac{\Delta\lambda}{\lambda} \Rightarrow v_R = \frac{\Delta\lambda}{\lambda} c = \frac{\lambda_{OBS} - \lambda_0}{\lambda_0} c = \frac{-10{,}87}{6562{,}87} c$$

$$\boxed{v_R = -496{,}6 \text{ km/s}}$$ 0,5 p

NEGATIVE : BLUESHIFT ! (APPROACHING)

MISSING : 0,25 p

$$\boxed{R \gtrsim \frac{\lambda}{\Delta\lambda} = 603{,}8}$$

We need spectral resolution better than $\simeq 600$

0,5 p ($R \gtrsim \frac{\lambda}{2\Delta\lambda}$ OK)

Q5

a) cf. lecture notes : charge transfer & read-out.

0,5

3 CT per pixel

b)

0,5 $$(0{,}999)^{2\times 3\times 2048} = 4{,}6\times 10^{-6}$$

NEARLY ALL CHARGE LOST !

Q6

0,5 a) cf. lecture notes/slides: photoelectron has kinetic energy & creates phonons in the lattice.

TOTAL CHARGE IS $\propto E_{photon}$

0,5 b) cf. PS.2: insert grating or other dispersive element, calibrate spectrum...

OR: FILTERS (PS.1), less precise.

Q7 cf. L1.4:

0,25 a) FROM HORIZON.

0,25 b) ALONG HORIZON, FROM N→E. MISSING ONLY REF. POINT: −0,1

0,25 d) FROM EQUATOR.

0,25 c) ALONG EQUATOR, FROM VERNAL EQUINOX (W→E)

MISSING ONLY REF POINT: −0,1