

Contact person:

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Examination, course FY3403 Particle physics

Monday December 14, 2009

Time: 09.00–13.00

Grades made public: Wednesday December 30, 2009

Allowed to use: Particle physics booklet, calculator, mathematical tables.

A table of physical constants can be found in the Particle physics booklet.

All subproblems are given the same weight in the grading.

Problem 1:

The differential cross section for a two-body scattering process $1 + 2 \rightarrow 3 + 4$ in the centre of mass (CM) frame is

$$\frac{d\sigma}{d\Omega} = S \left(\frac{\hbar c}{8\pi(E_1 + E_2)} \right)^2 \frac{|\vec{p}_f|}{|\vec{p}_i|} |\mathcal{M}|^2.$$

The energy and momentum of particle $j = 1, 2, 3, 4$ are E_j and $\vec{p}_j$, and we have $\vec{p}_1 = -\vec{p}_2 = \vec{p}_i$ and $\vec{p}_3 = -\vec{p}_4 = \vec{p}_f$ in the CM frame. The statistical factor S is 1/2 if particles 3 and 4 are identical and is 1 otherwise. $\mathcal{M}$ is the scattering amplitude.

If the incoming particles 1 and 2 are prepared in a state of unpolarized spins, and if we do not measure the spins of the outgoing particles 3 and 4, then $|\mathcal{M}|^2$ should be replaced by $\langle |\mathcal{M}|^2 \rangle$, which is $|\mathcal{M}|^2$ *averaged* over incoming spins and *summed* over outgoing spins.

Time reversal symmetry implies that $|\mathcal{M}|^2$ *summed* over both incoming and outgoing spins is the same for the process $1 + 2 \rightarrow 3 + 4$ and for the time reversed process $3 + 4 \rightarrow 1 + 2$.

The total cross section for the process $1 + 2 \rightarrow 3 + 4$ is

$$\sigma = \int d\Omega \frac{d\sigma}{d\Omega}.$$

The total cross section for the process $e^+ + e^- \rightarrow \gamma + \gamma$ (annihilation of a positron and an electron into two photons) at low energy (non-relativistic energies for the positron and electron) is

$$\sigma(e^+ + e^- \rightarrow \gamma + \gamma) = \frac{\pi r_e^2 c}{|\vec{v}|}, \quad (1)$$

where $r_e = 2.818 \times 10^{-15}$ m is the classical electron radius (see table 1.1, p. 4 in the Particle physics booklet), and where $\vec{v} = \vec{v}_1 - \vec{v}_2$ is the relative velocity of the positron and the electron.

- a) What is the relation between the total cross section in the CM frame, where $\vec{p}_1 + \vec{p}_2 = 0$, and in the laboratory frame, where $\vec{p}_2 = 0$? Give a brief argument for your answer.
- b) Use time reversal symmetry (which is known experimentally to hold for electromagnetic interactions) in order to find a formula for the total cross section, in the CM frame, of the pair creation process $\gamma + \gamma \rightarrow e^+ + e^-$ at an energy just above the threshold energy.
- c) The cosmic microwave background radiation filling the Universe is electromagnetic black body radiation at a temperature of $T_{\text{CMB}} = 2.73 \text{ K}$.

The average photon energy is of the order of kT_{CMB} , where k is Boltzmann's constant (table 1.1, p. 4 in the Particle physics booklet). What is kT_{CMB} in electronvolt?

A high energy photon, with energy above a certain threshold value, may collide with a photon of the cosmic microwave background and produce an electron positron pair.

What is the threshold energy of the high energy photon for this reaction?

- d) The number density of cosmic microwave background photons is about 400 per cm^3 . Assume that the cross section for the reaction $\gamma + \gamma \rightarrow e^+ + e^-$ is πr_e^2 , and estimate the average distance which a high energy photon can travel before it is destroyed in a collision with a photon of the cosmic microwave background.

Comment?

Problem 2:

- a) A bound state of a spin $1/2$ particle (a quark or an electron) and its antiparticle has parity $P = (-1)^{\ell+1}$ and charge conjugation symmetry $C = (-1)^{\ell+s}$, where ℓ is the relative orbital angular momentum and s is the total spin.

What are the possible values of ℓ and s ?

Why do we expect that the ground state will have $\ell = 0$?

Based on this, what are the quark model predictions for the parity and charge conjugation symmetry of the spin zero mesons π^0 and η , and of the spin one mesons ρ^0 and ω ? Explain your reasoning.

- b) The ground state of positronium (a positron and an electron bound together) can have either $s = 0$ (parapositronium) or $s = 1$ (orthopositronium).

Parapositronium and orthopositronium have very different lifetimes (by a factor of 10^3), because they can not decay into the same number of photons. Why?

Which of them has the shortest lifetime, and why?

- c) The decay rate, or inverse lifetime, for the two photon decay of positronium can be computed as

$$\Gamma = \frac{1}{\tau} = \mathcal{L}\sigma ,$$

where σ is the cross section for annihilation into two photons, and $\mathcal{L}$ is a luminosity,

$$\mathcal{L} = |\psi_{\text{rel}}(0)|^2 |\vec{v}| .$$

Here $\vec{v} = \vec{v}_1 - \vec{v}_2$ is again the relative velocity of the positron and the electron, and ψ_{rel} is the relative wave function, found by solving the Schrödinger equation for the relative motion. The resulting probability density at the origin is

$$|\psi_{\text{rel}}(0)|^2 = \frac{1}{\pi} \left(\frac{\alpha m_e c}{2\hbar} \right)^3 ,$$

where m_e is the electron mass.

The cross section given in equation (1) must be multiplied by 4 here, because the incoming spins are now fixed and there is no averaging. Thus,

$$\sigma = \frac{4\pi r_e^2 c}{|\vec{v}|} = \frac{4\pi c}{|\vec{v}|} \left(\frac{\alpha \hbar}{m_e c} \right)^2 .$$

Derive the formula for the positronium lifetime τ , and compute the numerical value.

Problem 3:

- a) We observe three generations of leptons and quarks, for example with the electron, the electron neutrino, the u and d quarks, and their antiparticles, in the first generation.

How can the particles of one generation be transformed into particles of other generations?

- b) What is meant by asymptotic freedom in quantum chromodynamics?
- c) What is meant by a “Grand Unified Theory”?

Can you mention briefly one or two arguments in favour of grand unification, and one or two arguments against?