



Contact during the exam:
Egil Holvik
Telephone: 93089735

Exam in FY3403 PARTICLE PHYSICS

Friday December 2, 2011
09:00–13:00

Allowed help: Alternativ C

Standard calculator

K. Rottman: *Matematisk formelsamling* (all languages).

Schaum's Outline Series: *Mathematical Handbook of Formulas and Tables*.

Det finnes også en norsk utgave av dette oppgavesettet.

This problem set consists of 3 pages.

Problem 1. Quark model for baryons

Give a qualitative description of how one in the quark model assumes that baryons are made of quarks. In particular try to explain

- how many quarks and antiquarks the baryons are made of,
- which spin S the total baryon system may have,
- which isospin I the total baryon system may have,
- why there are no baryons with charge -2 (in units of the positron charge),

Problem 2.

The normalized spin/flavor wave function for Δ^{++} with spin $S_z = \frac{3}{2}$ is given by

$$|\Delta^{++} \frac{3}{2}\rangle = |u \uparrow\rangle |u \uparrow\rangle |u \uparrow\rangle.$$

- Find the normalized spin/flavor wave function for Δ^+ with spin $S_z = \frac{3}{2}$.
- Find the normalized spin/flavor wave function for Δ^+ with spin $S_z = \frac{1}{2}$.
- The magnetic moment of a baryon with spin/flavor wave function $|\Psi\rangle$ is defined as

$$\mu_z = \langle \Psi | \sum_i \frac{e Q_i}{2 m_i} S_{iz} | \Psi \rangle,$$

where the sum is over the three positions in the wave function. (Note that Q_i , m_i and S_{iz} is *operators* which take different values depending on the states they act on.)

Find the magnetic moment of Δ^{++} with spin $S_z = \frac{3}{2}$. Assume that $m_u = m_d$.

- Find the magnetic moment of Δ^+ with spin $S_z = \frac{1}{2}$. Assume that $m_u = m_d$.

Hint to point a–b): Use the ladder operators.

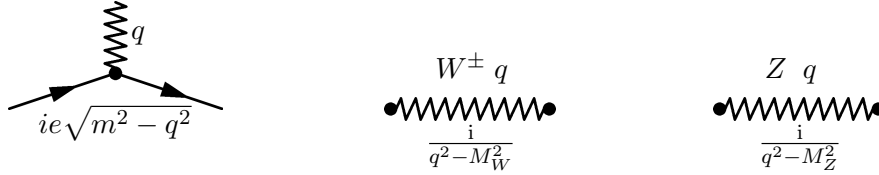
Problem 3. Interaction processes and Feynman diagrams

Draw lowest order Feynman diagrams for the following interaction processes (if the process is possible)

- a) $e^- + \mu^- \longrightarrow e^- + \mu^-$
- b) $e^- + \mu^+ \longrightarrow e^- + \mu^+$
- c) $e^- + \mu^+ \longrightarrow e^+ + \mu^-$
- d) $e^- + e^+ \longrightarrow \mu^+ + \mu^-$
- e) $e^- + e^+ \longrightarrow e^+ + e^-$
- f) $e^- + \nu_\mu \longrightarrow e^- + \nu_\mu$
- g) $e^- + \nu_\mu \longrightarrow \nu_e + \mu^-$
- h) $\tau^- \longrightarrow \mu^- + x$ (replace x by some possible set of particles)
- i) $\nu_e + \nu_\mu \longrightarrow \nu_\mu + \nu_e$
- j) $n \longrightarrow p + x$ (replace x by some possible set of particles)

Problem 4. Elastic $\nu_e + e^- \rightarrow \nu_e + e^-$ scattering

Assume Feynman rules as *indicated* below, where m is the mass of the heaviest fermion involved in the interaction vertex and q is the four-momentum of the virtual messenger particle.



- a) Draw all lowest order Feynman diagrams for the process.
- b) Write down the corresponding algebraic expressions for the scattering amplitude $\mathcal{M}_{fi}$.
- c) Find the total scattering cross-section. You may assume that $|q^2| \ll M_W^2$ and $|q^2| \ll M_Z^2$ to simplify expressions.

Given:

We use *natural units*, i.e. units where $\hbar = c = 1$. The connection between scattering amplitude $\mathcal{M}_{fi}$ and scattering cross-section is

$$\frac{d\sigma}{d\Omega} = \frac{S}{64\pi^2} \frac{|\mathcal{M}_{fi}|^2}{(E_1 + E_2)^2} \frac{|\mathbf{p}_f|}{|\mathbf{p}_i|}. \quad (1)$$

35. CLEBSCH-GORDAN COEFFICIENTS, SPHERICAL HARMONICS, AND d FUNCTIONS

Note: A square-root sign is to be understood over *every* coefficient, e.g., for $-8/15$ read $-\sqrt{8/15}$.

Notation:

J	J	...
M	M	...

m_1	m_2	
m_1	m_2	Coefficients
.	.	
.	.	

$1/2 \times 1/2$

1		
+1/2	+1/2	1
+1/2	-1/2	1/2
-1/2	+1/2	1/2
-1/2	-1/2	1

 $1 \times 1/2$

3/2	3/2	1/2
+1	+1/2	1
+1	-1/2	1/3
0	+1/2	2/3
0	-1/2	1/3
-1	+1/2	2/3
-1	-1/2	1/3

 2×1

3	3	2
+2	+1	1
+2	0	1/3
+1	+1	2/3
+1	0	1/3
0	+1	2/3
0	0	1
-1	+1	2/3
-1	0	1/3
-2	+1	1

 1×1

2	2	1
+1	+1	1
+1	0	1/2
0	+1	1/2
0	0	1
-1	+1	1/2
-1	0	1/2
-2	+1	1

 $3/2 \times 1$

5/2	5/2	3/2
+3/2	+1	1
+3/2	0	2/5
+1/2	+1	3/5
+1/2	0	2/5
-1/2	+1	3/5
-1/2	0	2/5
-3/2	+1	1

 $3/2 \times 3/2$

3	3	2
+3/2	+3/2	1
+3/2	+1/2	1/2
+1/2	+3/2	1/2
+3/2	-1/2	1/5
+1/2	-1/2	3/5
-1/2	+3/2	1/5
-1/2	+1/2	3/5
-3/2	+1/2	1/5
-3/2	-1/2	3/5

 2×2

4	4	3
+2	+2	1
+2	+1	1/2
+1	+2	1/2
+2	0	3/4
+1	+1	4/7
0	+2	3/4
0	0	1
-1	+2	3/4
-1	+1	4/7
-2	+2	1

 $3/2 \times 3/2$

3	3	2
+3/2	+3/2	1
+3/2	+1/2	1/2
+1/2	+3/2	1/2
+3/2	-1/2	1/5
+1/2	-1/2	3/5
-1/2	+3/2	1/5
-1/2	+1/2	3/5
-3/2	+1/2	1/5
-3/2	-1/2	3/5

 2×2

4	4	3
+2	+2	1
+2	+1	1/2
+1	+2	1/2
+2	0	3/4
+1	+1	4/7
0	+2	3/4
0	0	1
-1	+2	3/4
-1	+1	4/7
-2	+2	1

 $3/2 \times 3/2$

3	3	2
+3/2	+3/2	1
+3/2	+1/2	1/2
+1/2	+3/2	1/2
+3/2	-1/2	1/5
+1/2	-1/2	3/5
-1/2	+3/2	1/5
-1/2	+1/2	3/5
-3/2	+1/2	1/5
-3/2	-1/2	3/5

 2×2

4	4	3
+2	+2	1
+2	+1	1/2
+1	+2	1/2
+2	0	3/4
+1	+1	4/7
0	+2	3/4
0	0	1
-1	+2	3/4
-1	+1	4/7
-2	+2	1

 $3/2 \times 3/2$

3	3	2
+3/2	+3/2	1
+3/2	+1/2	1/2
+1/2	+3/2	1/2
+3/2	-1/2	1/5
+1/2	-1/2	3/5
-1/2	+3/2	1/5
-1/2	+1/2	3/5
-3/2	+1/2	1/5
-3/2	-1/2	3/5

 2×2

4	4	3
+2	+2	1
+2	+1	1/2
+1	+2	1/2
+2	0	3/4
+1	+1	4/7
0	+2	3/4
0	0	1
-1	+2	3/4
-1	+1	4/7
-2	+2	1

 $3/2 \times 3/2$

3	3	2
+3/2	+3/2	1
+3/2	+1/2	1/2
+1/2	+3/2	1/2
+3/2	-1/2	1/5
+1/2	-1/2	3/5
-1/2	+3/2	1/5
-1/2	+1/2	3/5
-3/2	+1/2	1/5
-3/2	-1/2	3/5

 2×2

4	4	3
+2	+2	1
+2	+1	1/2
+1	+2	1/2
+2	0	3/4
+1	+1	4/7
0	+2	3/4
0	0	1
-1	+2	3/4
-1	+1	4/7
-2	+2	1

 $3/2 \times 3/2$

3	3	2
+3/2	+3/2	1
+3/2	+1/2	1/2
+1/2	+3/2	1/2
+3/2	-1/2	1/5
+1/2	-1/2	3/5
-1/2	+3/2	1/5
-1/2	+1/2	3/5
-3/2	+1/2	1/5
-3/2	-1/2	3/5

 2×2

4	4	3
+2	+2	1
+2	+1	1/2
+1	+2	1/2
+2	0	3/4
+1	+1	4/7
0	+2	3/4
0	0	1
-1	+2	3/4
-1	+1	4/7
-2	+2	1

 $3/2 \times 3/2$

3	3	2
+3/2	+3/2	1
+3/2	+1/2	1/2
+1/2	+3/2	1/2
+3/2	-1/2	1/5
+1/2	-1/2	3/5
-1/2	+3/2	1/5
-1/2	+1/2	3/5
-3/2	+1/2	1/5
-3/2	-1/2	3/5

 2×2

4	4	3
+2	+2	1
+2	+1	1/2
+1	+2	1/2
+2	0	3/4
+1	+1	4/7
0	+2	3/4
0	0	1
-1	+2	3/4
-1	+1	4/7
-2	+2	1

 $3/2 \times 3/2$

3	3	2
+3/2	+3/2	1
+3/2	+1/2	1/2
+1/2	+3/2	1/2
+3/2	-1/2	1/5
+1/2	-1/2	3/5
-1/2	+3/2	1/5
-1/2	+1/2	3/5
-3/2	+1/2	1/5
-3/2	-1/2	3/5

 2×2

4	4	3
+2	+2	1
+2	+1	1/2
+1	+2	1/2
+2	0	3/4
+1	+1	4/7
0	+2	3/4
0	0	1
-1	+2	3/4
-1	+1	4/7
-2	+2	1

 $3/2 \times 3/2$

3	3	2
+3/2	+3/2	1
+3/2	+1/2	1/2
+1/2	+3/2	1/2
+3/2	-1/2	1/5
+1/2	-1/2	3/5
-1/2	+3/2	1/5
-1/2	+1/2	3/5
-3/2	+1/2	1/5
-3/2	-1/2	3/5

 2×2

4	4	3
+2	+2	1
+2	+1	1/2
+1	+2	1/2
+2	0	3/4
+1	+1	4/7
0	+2	3/4
0	0	1
-1	+2	3/4
-1	+1	4/7
-2	+2	1

 $3/2 \times 3/2$

3	3	2
+3/2	+3/2	1
+3/2	+1/2	1/2
+1/2	+3/2	1/2
+3/2	-1/2	1/5
+1/2	-1/2	3/5
-1/2	+3/2	1/5
-1/2	+1/2	3/5
-3/2	+1/2	1/5
-3/2	-1/2	3/5

 2×2

4	4	3
+2	+2	1
+2	+1	1/2
+1	+2	1/2
+2	0	3/4
+1	+1	4/7
0	+2	3/4
0	0	1
-1	+2	3/4
-1	+1	4/7
-2	+2	1

 $3/2 \times 3/2$

3	3	2
+3/2	+3/2	1
+3/2	+1/2	1/2
+1/2	+3/2	1/2
+3/2	-1/2	1/5
+1/2	-1/2	3/5
-1/2	+3/2	1/5
-1/2	+1/2	3/5
-3/2	+1/2	1/5
-3/2	-1/2	3/5

 2×2

4	4	3
+2	+2	1
+2	+1	1/2
+1	+2	1/2
+2	0	3/4
+1	+1	4/7
0	+2	3/4
0	0	1
-1	+2	3/4
-1	+1	4/7
-2	+2	1

 $3/2 \times 3/2$

3	3	2
+3/2	+3/2	1
+3/2	+1/2	1/2
+1/2	+3/2	1/2
+3/2	-1/2	1/5
+1/2	-1/2	3/5
-1/2	+3/2	1/5
-1/2	+1/2	3/5
-3/2	+1/2	1/5
-3/2	-1/2	3/5

 2×2

4	4	3
+2	+2	1
+2	+1	1/2
+1	+2	1/2
+2	0	3/4
+1	+1	4/7
0	+2	3/4
0	0	1
-1	+2	3/4
-1	+1	4/7
-2	+2	1

 $3/2 \times 3/2$

3	3	2
+3/2	+3/2	1
+3/2	+1/2	1/2
+1/2	+3/2	1/2
+3/2	-1/2	1/5
+1/2	-1/2	3/5
-1/2	+3/2	1/5
-1/2	+1/2	3/5
-3/2	+1/2	1/5
-3/2	-1/2	3/5

 2×2

4	4	3
+2	+2	1
+2	+1	1/2
+1	+2	1/2
+2	0	3/4
+1	+1	4/7
0	+2	3/4
0	0	1
-1	+2	3/4
-1	+1	4/7
-2	+2	1

 $3/2 \times 3/2$

3	3	2
+3/2	+3/2	1
+3/2	+1/2	1/2
+1/2	+3/2	1/2
+3/2	-1/2	1/5
+1/2	-1/2	3/5
-1/2	+3/2	1/5
-1/2	+1/2	3/5
-3/2	+1/2	1/5
-3/2	-1/2	3/5

 2×2

4	4	3
+2	+2	1
+2	+1	1/2
+1	+2	1/2
+2	0	3/4
+1	+1	4/7
0	+2	3/4
0	0	1
-1	+2	3/4
-1	+1	4/7
-2	+2	1

 $3/2 \times 3/2$

3	3	2
+3/2	+3/2	1
+3/2	+1/2	1/2
+1/2	+3/2	1/2
+3/2	-1/2	1/5
+1/2	-1/2	3/5
-1/2	+3/2	1/5
-1/2	+1/2	3/5
-3/2	+1/2	1/5
-3/2	-1/2	3/5

 2×2

4	4	3
+2	+2	1
+2	+1	1/2
+1	+2	1/2
+2	0	3/4
+1	+1	4/7
0	+2	3/4
0	0	1
-1	+2	3/4
-1	+1	4/7
-2	+2	1

 $3/2 \times 3/2$

3	3	2
+3/2	+3/2	1
+3/2	+1/2	1/2
+1/2	+3/2	1/2
+3/2	-1/2	1/5
+1/2	-1/2	3/5
-1/2	+3/2	1/5
-1/2	+1/2	3/5
-3/2	+1/2	1/5
-3/2	-1/2	3/5

 2×2

4	4	3
+2	+2	1
+2	+1	1/2
+1	+2	1/2
+2	0	3/4
+1	+1	4/7
0	+2	3/4
0	0	1
-1	+2	3/4
-1	+1	4/7
-2	+2	1

 $3/2 \times 3/2$

3	3	2
+3/2	+3/2	1
+3/2	+1/2	1/2
+1/2	+3/2	1/2
+3/2	-1/2	1/5
+1/2	-1/2	3/5
-1/2	+3/2	1/5
-1/2	+1/2	3/5
-3/2	+1/2	1/5
-3/2	-1/2	3/5

 2×2

4	4	3
+2	+2	1
+2	+1	1/2
+1	+2	1/2
+2	0	3/4
+1	+1	4/7
0	+2	3/4
0	0	1
-1	+2	3/4
-1	+1	4/7
-2	+2	1

 $3/2 \times 3/2$

3	3	2
+3/2	+3/2	1
+3/2	+1/2	1/2
+1/2	+3/2	1/2
+3/2	-1/2	1/5
+1/2	-1/2	3/5
-1/2	+3/2	1/5
-1/2	+1/2	3/5
-3/2	+1/2	1/5
-3/2	-1/2	3/5

 2×2

4	4	3
+2	+2	1
+2	+1	1/2
+1	+2	1/2
+2	0	3/4
+1	+1	4/7
0	+2	3/4
0	0	1
-1	+2	3/4
-1	+1	4/7
-2	+2	1</