

Suggested solution for Exam FY3403: Particle Physics

NOTE: The solutions below are meant as guidelines for how the problems may be solved and do not necessarily contain all the required details.

PROBLEM 1

(a) Neutrino: nothing lighter to decay into via weak interaction. Photon: nothing lighter to decay into. Electron: lightest charged particle. Proton: lightest baryon (conservation of baryon number saves it). The four-vectors for the electrons q and q' may be written as (neglecting the mass):

$$q = (E/c, E/c, 0, 0), q' = (E'/c, E' \cos \theta/c, E' \sin \theta/c, 0) \quad (1)$$

where we set the scattering to take place in the xy -plane without loss of generality. We may then directly compute:

$$-(q - q')^2 = 4EE' \sin(\theta/2). \quad (2)$$

(b) First: no (unmatched index). Second: yes (transforms like 4-vector). Third: yes (invariant scalar). For the scattering process, the momenta are equal and opposite in the CM frame. Since the initial particles have equal masses, they will have equal energies E . Same argument applies to the final particles which also will have equal energies E . The momenta are found from the standard relation $E^2 = \mathbf{p}^2 + m^2$ for each particle.

(c) Using conservation of 4-momentum we find for the electron $E = (m_W^2 - m_e^2)c^2/(2m_e)$. In numbers, this is 6 PeV, which is an extremely high energy (non-realistic in practice). NB! It was not clear from the problem text whether one should consider the electron-generation or muon- and taon-generations as well. Therefore, full score has been given to those that derived the analytical expression for the threshold energy E (without necessarily inserting numbers).

(d) The high mass of the vector bosons W, Z mediating the weak interaction strongly reduces the amplitude of the Feynman diagrams. However, for high-energy scattering near the pole of the propagator the weak interactions becomes very strong and overtakes the electromagnetic contribution. The gauge group is $SU(2)_L \times U(1)_Y$. Electrons, neutrinos, photons, and Z are colorless. The quarks come in 3 colors whereas the gluon has 8 different color combinations.

(e) Renormalization is the procedure of accounting for higher-order Feynman diagrams by obtaining effective coupling constants and/or masses for the theory. The contribution from higher-order diagrams to these renormalized (effective) quantities may contain both infinite and finite momentum-dependent terms. The renormalized coupling constants and/or masses are what is measured experimentally, which means that there evidently are cancelling infinities. The finite contributions make the coupling constants and/or masses "running" in the sense that they depend on the energy (or equivalently separation distance) of the involved particles. Asymptotic freedom refers to the distance-dependence of the strong coupling constant: for small distances, the strong coupling constant is small whereas it grows larger for larger distances. The interaction strength of quarks thus depends on their separation distance. A gauge theory is a field theory in which the Lagrangian has a continuous symmetry. The different choices exploiting this degree of freedom are called gauges.

(f) The cross-section has a contribution from all the hadrons made up of quarks that have sufficiently small mass to allow for production at a given energy of the initial pair of particles. As this energy is increased and exceeds additional quark masses, the ratio increases. Put differently: as the energy increases above certain thresholds, more and more types of hadrons are allowed for production and this is manifested as a increase (step-up) in the scattering cross section.

PROBLEM 2

(a) The fermion must have charge $+1/3$ due to conservation of charge, which means that it must be an anti-quark of the type $\bar{d}$, $\bar{s}$, or $\bar{b}$.

(b) The amplitude and diagram are given below:

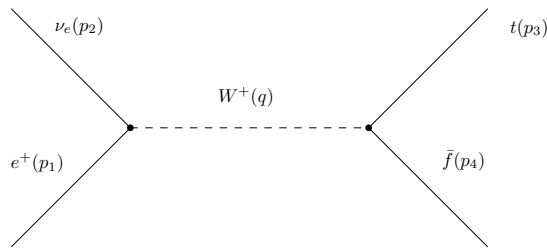


FIG. 1: Feynman diagram.

$$-i\mathcal{M} = \frac{(-ig_W)(ig_c)}{2\sqrt{2}} [\bar{v}_1 \gamma^\mu (1 - \gamma_5) u_2] \frac{-i(g_{\mu\nu} - q_\mu q_\nu / M_W^2)}{q^2 - M_W^2} [\bar{u}_3 \gamma^\nu (1 - \gamma_5) v_4]$$

The second term in the propagator makes no contribution when the positron and neutrino are massless since $\bar{v}_1 q' (1 - \gamma_5) u_2 = 0$ as seen when anticommuting and using the massless Dirac equation. I used the notation $a' \equiv a^\mu \gamma_\mu$. The computation of the spin-averaged Feynman amplitude $\langle |\mathcal{M}|^2 \rangle$ is shown in detail on page 250-252 in the textbook.

(c) See the treatment in the textbook for a detailed explanation. In particular, the gauge field may become massive when spontaneously breaking a local symmetry: the gauge field "eats" the Goldstone bosons.

(d) 1st term: "kinetic term" associated with time- and space-variations of the field. 2nd term: irrelevant constant. 3rd term: mass-term with $m \propto \alpha$. 4th term: interaction term for four ϕ -fields. If the mass-term had opposite sign, we would have to identify the ground-state of the Lagrangian (since $\phi = 0$ would not be the ground-state), reexpress the Lagrangian in terms of deviation from the ground-state, and then identify the mass of the field representing an excitation above the ground-state.