

NTNU Trondheim, Institutt for fysikk

Examination for FY3403 Particle Physics

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Allowed tools: mathematical tables, pocket calculator

Some formulas and data can be found on p. 2ff.

1. Chirality vs. helicity.

a.) Helicity for a free, massive particle is (3 pts)

☐ frame dependent

☐ conserved

b.) Chirality for a free, massive particle is (3 pts)

☐ frame dependent

☐ conserved

2. $SU(2)_L$.

Consider one generation of quarks before spontaneous symmetry breaking, i.e. when they are massless.

a.) How do the quarks transform under $SU(2)_L$ transformations? (5 pts)

b.) Show that the Lagrangian describing these fields (without interactions) is invariant under global, but not under local $SU(2)_L$ transformations. (6 pts)

a.) We have to distinguish left- and right-chiral fields,

$$\psi_L = P_L \psi = \frac{1}{2}(1 - \gamma^5)\psi \quad \text{and} \quad \psi_R = P_R \psi = \frac{1}{2}(1 + \gamma^5)\psi.$$

Left-chiral quark fields live in the fundamental representation of $SU(2)_L$, i.e. $Q = (u, d)_L^T$, while right-chiral fields are singlets, i.e. u_R and d_R . Singlets are invariant under $SU(2)_L$ transformation, while the doublets transform as

$$Q \rightarrow Q' = \exp \left\{ \frac{i\boldsymbol{\alpha} \cdot \boldsymbol{\tau}}{2} \right\} Q$$

where $\boldsymbol{\alpha} = \{\alpha_1, \alpha_2, \alpha_3\}$ parameterise the transformation, and the generators $\boldsymbol{T} = \boldsymbol{\tau}/2$ are given by the Pauli matrices.

b.) Since the right-chiral fields are singlets, we have to consider only the left-chiral fields. Their Lagrangian

$$\mathcal{L} = i\bar{Q}\not{\partial}Q$$

transform under a global transformation as

$$\mathcal{L} \rightarrow \mathcal{L}' = i\bar{Q}U^\dagger \not{\partial}UQ = i\bar{Q}\not{\partial}Q = \mathcal{L},$$

since $U^\dagger U = 1$ and the parameters α are constant. In case of a local transformation, the derivative produce an additional term,

$$\mathcal{L} \rightarrow \mathcal{L} + i\bar{Q}\gamma^\mu U^\dagger(\partial_\mu U)Q$$

which breaks the local invariance.

3. Polarization sum for a massive spin-1 particle.

a.) Write down a possible set of polarisation vectors $\varepsilon_\mu^{(r)}$, $r = 1, \dots, n$ for a massive spin-1 particle. (4 pts)

b.) Derive the completeness relation for a massive spin-1 particle, (6 pts)

$$\sum_{r=1}^n \varepsilon_\mu^{(r)} \varepsilon_\nu^{(r)*} = -\eta_{\mu\nu} + k_\mu k_\nu / m^2.$$

A massive vector field A^μ has four components in $d = 4$ space-time dimensions, while it has only $2s + 1 = 3$ independent spin components. Correspondingly, a four-vector A^μ transforms under a rotation as $(A^0, \mathbf{A})$, i.e. it contains a scalar and a three-vector. Thus the physical components of a massive spin-1 field in its rest-frame are given by $(0, \mathbf{A})$. We can choose the three polarisation vectors in the rest frame e.g. as the Cartesian unit vectors, $\varepsilon_i \propto \mathbf{e}_i$. They satisfy $\varepsilon_\mu^{(r)} \varepsilon^{\mu(r)} = -1$ and, since in the rest-frame $k^\mu = (m, \mathbf{0})$ also $k_\mu \varepsilon_\mu^{(r)} = 0$. Next we evaluate

$$\sum_r \varepsilon^{\mu(r)} \varepsilon^{\nu(r)} = \begin{pmatrix} 0 & 0 & 0 & 0 \\ 0 & 1 & 0 & 0 \\ 0 & 0 & 1 & 0 \\ 0 & 0 & 0 & 1 \end{pmatrix} \quad (1)$$

Next we have to express this using the relevant tensors, here $\eta^{\mu\nu}$ and $k^\mu k^\nu / m^2$, where we divided by m^2 to get the right dimension. This gives for $k^\mu = (m, \mathbf{0})$

$$\sum_r \varepsilon^{\mu(r)} \varepsilon^{\nu(r)} = \begin{pmatrix} 0 & 0 & 0 & 0 \\ 0 & 1 & 0 & 0 \\ 0 & 0 & 1 & 0 \\ 0 & 0 & 0 & 1 \end{pmatrix} = -\eta^{\mu\nu} + k^\mu k^\nu / m^2 \quad (2)$$

If we are not able to guess this, we can derive it formally: We set

$$\sum_r \varepsilon^{\mu(r)} \varepsilon^{\nu(r)} = A\eta^{\mu\nu} + Bk^\mu k^\nu / m^2.$$

Asking then $k_\mu \varepsilon^{\mu(r)} = 0$, gives

$$0 = Ak^\nu + Bk^\nu$$

or $A = -B$. The normalisation condition requires $A = -1$.

4. W decays.

Calculate the squared matrix element $|\bar{\mathcal{M}}|^2$ for the decay rate of a W boson into a massless

fermion pair, summed over final state spins and averaged over initial ones. Express $|\bar{\mathcal{M}}|^2$ as function of invariant scalar products, $f(p_1 \cdot p_2, p_1 \cdot k, \dots)$, where k, p_1 and p_2 are the four-momenta of the three particles. (16 pts)

The Feynman amplitude for this process is

$$i\mathcal{M} = -\frac{ig}{2\sqrt{2}}\varepsilon_\mu^{(r)}(k)[\bar{u}(p_2)\gamma^\mu(1-\gamma^5)v(p_1)] \quad (3)$$

and thus

$$|\mathcal{M}|^2 = \frac{g^2}{8}\varepsilon_\mu^{(r)}\varepsilon_\nu^{(r)}[\bar{u}(p_2)\gamma^\mu(1-\gamma^5)v(p_1)][\bar{u}(p_2)\gamma^\nu(1-\gamma^5)v(p_1)]^*. \quad (4)$$

We determine $[\dots]^* = [\dots]^\dagger$ either directly

$$[u(p_2)^\dagger\gamma^0\gamma^\nu(1-\gamma^5)v(p_1)]^\dagger = [v(p_1)^\dagger(1-\gamma^5)\gamma^{\nu\dagger}\gamma^0u(p_2)] = [v(p_1)^\dagger(1-\gamma^5)\gamma^0\gamma^\nu\gamma^0u(p_2)] \quad (5)$$

$$= [v(p_1)^\dagger\gamma^0(1+\gamma^5)\gamma^\nu u(p_2)] = [\bar{v}(p_1)^\dagger\gamma^\nu(1-\gamma^5)u(p_2)]. \quad (6)$$

or we use $\bar{\Gamma} \equiv \gamma^0\Gamma^\dagger\gamma^0$,

$$\bar{\gamma}^5 = \gamma^0\gamma^{5\dagger}\gamma^0 = \gamma^0\gamma^5\gamma^0 = -\gamma^5. \quad (7)$$

Next we sum over the spins of the fermions, setting $A^\mu = \gamma^\mu(1-\gamma^5)$,

$$\sum_{s_1, s_2} [\bar{u}_a(s_2, p_2)A_{ab}^\mu v_b(s_1, p_1)][\bar{v}_d(s_1, p_1)^\dagger A_{de}^\nu u_e(s_2, p_2)] = (\not{p}_2)_{ea}A_{ab}^\mu(\not{p}_1)_{bd}A_{de}^\nu = \text{tr}[\not{p}_2 A^\mu \not{p}_1 A^\nu] \quad (8)$$

We anti-commute the right factor $(1-\gamma^5)$ to the left (or the left to the right), and use $(1-\gamma^5)^2 = 2(1-\gamma^5)$,

$$\text{tr}[\not{p}_2\gamma^\mu(1-\gamma^5)\not{p}_1\gamma^\nu(1-\gamma^5)] = 2\text{tr}[\not{p}_2\gamma^\mu(1-\gamma^5)\not{p}_1\gamma^\nu]. \quad (9)$$

The factor with γ^5 will lead to term containing the completely anti-symmetric Levi-Civita tensor, which is contracted with the symmetric factor $\varepsilon_\mu^{(r)}\varepsilon_\nu^{(r)}$. Thus this term vanishes. The remaining term gives

$$\text{tr}[\not{p}_2\gamma^\mu(1-\gamma^5)\not{p}_1\gamma^\nu(1-\gamma^5)] = 2\text{tr}[\not{p}_2\gamma^\mu\not{p}_1\gamma^\nu] = 8\text{tr}[p_2^\mu p_1^\nu - (p_2 \cdot p_1)\eta^{\mu\nu} + p_2^\nu p_1^\mu] \quad (10)$$

We average next over the polarisations of the W ,

$$\frac{1}{3}\sum_r |\mathcal{M}|^2 = \frac{g^2}{3}(-\eta_{\mu\nu} + k_\mu k_\nu/M^2)[p_2^\mu p_1^\nu - (p_2 \cdot p_1)\eta^{\mu\nu} + p_2^\nu p_1^\mu] \quad (11)$$

$$= \frac{g^2}{3}[(p_2 \cdot p_1) + 2(k \cdot p_1)(k \cdot p_2)/M^2] \quad (12)$$

5. Neutrino scattering.

Let us assume that at the Galactic Center a source of high-energy neutrons exist. These neutrons escape from the source and produce neutrinos via beta-decay which we aim to

measure.

- a.) Neglect neutrino masses and oscillations. Draw the lowest order Feynman diagrams for the interaction of these neutrinos with matter (assumed to consist of e^- , u - and d -quarks). (6 pts)
- b.) In which of the diagrams can the virtual particle be on mass-shell? Find the required neutrino energy. (6 pts)
- c.) Estimate the maximal cross section (in SI or cgs units) using the Breit-Wigner formula. (6 pts)
- d.) Describe qualitatively, how neutrino mixing and oscillations would change this picture (maximal 100 words). (6 pts)

a.) s -channel Glashow resonance $\bar{\nu}_e e^- \rightarrow W^- \rightarrow \text{all}$, t -channel CC $\bar{\nu}_e u \rightarrow e^+ d$ and t -channel NC $\bar{\nu}_e X \rightarrow \bar{\nu}_e X$ with $X = \{e^-, u, d\}$ interactions.

b.) The Mandelstam variables t and u are always negative, while s is positive. Thus only the denominator in the propagator of the first diagram, $\propto 1/(s - M^2)$ can become zero. In the lab frame

$$s = M^2 = (p_1 + p_2)^2 = 0 + 2E_\nu m_e + m_e^2$$

or

$$E_\nu = \frac{M^2 - m_e^2}{2m_e} \simeq \frac{M^2}{2m_e} \simeq 6 \times 10^{15} \text{ eV}$$

c.) The prefactor of Breit-Wigner formula (13) becomes

$$\frac{4\pi s}{p_{\text{cms}}^2} \frac{(2s_r + 1)}{(2s_1 + 1)(2s_2 + 1)} = 16\pi \frac{3}{2} = 24\pi \quad (13)$$

where we use that only left-chiral neutrinos are produced. Thus

$$\sigma(\bar{\nu}_e e^- \rightarrow W \rightarrow \bar{f} f) = 24\pi \frac{\Gamma(W \rightarrow \bar{\nu}_e e^-) \Gamma(W \rightarrow \text{all})}{(s - M_W^2)^2 + (M_W \Gamma_W)^2} \quad (14)$$

$$= \frac{24\pi}{M_W^2} \text{BR}(W \rightarrow \bar{\nu}_e e^-) \text{BR}(\text{all}) \quad (15)$$

where BR denotes the branching ratio into the considered channel. We include all final states, and thus $\text{BR}(W \rightarrow \text{all}) = 1$. With $\text{BR}(W \rightarrow \bar{\nu}_e e^-) = 10.5\%$ it follows

$$\sigma(\bar{\nu}_e e^- \rightarrow W \rightarrow \bar{f} f) \simeq 24\pi \left(\frac{1}{80.4} \frac{\text{cm}}{5.06 \times 10^{13}} \right)^2 0.105 \simeq 5 \times 10^{-31} \text{ cm}^2 \quad (16)$$

For comparison, Fig. 1 shows the cross sections for neutrino interactions on electron targets as function of neutrino energy.

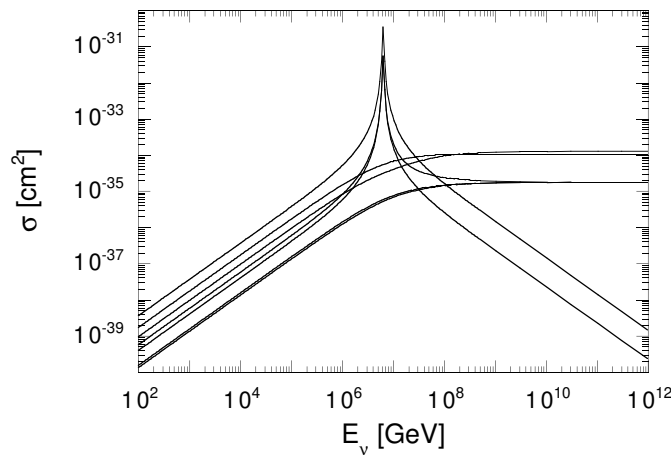


Figure 1: At low energies, from largest to smallest cross section, the processes are (i) $\bar{\nu}_e e \rightarrow$ hadrons, (ii) $\nu_\mu e \rightarrow \mu \nu_e$, (iii) $\nu_e e \rightarrow \nu_e e$, (iv) $\bar{\nu}_e e \rightarrow \bar{\nu}_\mu \mu$, (v) $\bar{\nu}_e e \rightarrow \bar{\nu}_e e$, (vi) $\nu_\mu e \rightarrow \nu_\mu e$, (vii) $\bar{\nu}_\mu e \rightarrow \bar{\nu}_\mu e$.

d.) The neutrino state ν_e created in beta-decay is a superposition of mass eigenstates ν_i ,

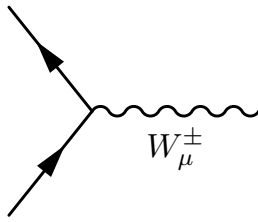
$$\nu_e(0) = U_{ei}\nu_i = U_{e1}\nu_1(0) + U_{e2}\nu_2(0) + U_{e3}\nu_3(0)$$

The phase of the mass eigenstates will evolve as

$$\nu_i(x) = \nu_i(0) \exp(ip_i x) \simeq \nu_i(0) e^{i\phi} \exp[-iEx/(2m_i^2)]$$

The last term implies that the states develop an oscillating phase difference as they propagate. At detection, we have to split $\nu_i(L)$ into flavor states, $\nu_i(L) = U_{i\alpha}^* \nu_\alpha$. Then $|\langle \nu_\alpha | \nu_i(L) \rangle|^2$ gives the probability to observe flavour alpha. As a result, the originally 100% pure ν_e becomes a superposition of all three flavors. In this case, the importance of the Glashow resonance would be thereby reduced.

Feynman rules and useful formulas



$$-\frac{ig}{2\sqrt{2}}\gamma^\mu(1-\gamma^5)$$

The gamma matrices form a Clifford algebra,

$$\{\gamma^\mu, \gamma^\nu\} = 2\eta^{\mu\nu}. \quad (17)$$

The matrix

$$\gamma^5 \equiv \gamma_5 \equiv i\gamma^0\gamma^1\gamma^2\gamma^3. \quad (18)$$

satisfies $(\gamma^5)^\dagger = \gamma^5$, $(\gamma^5)^2 = 1$, and $\{\gamma^\mu, \gamma^5\} = 0$.

$$\bar{\Gamma} \equiv \gamma^0 \Gamma^\dagger \gamma^0. \quad (19)$$

The trace of an odd number of γ^μ matrices vanishes, as well as

$$\text{tr}[\gamma^5] = \text{tr}[\gamma^\mu \gamma^5] = \text{tr}[\gamma^\mu \gamma^\nu \gamma^5] = 0. \quad (20)$$

Non-zero traces are

$$\text{tr}[\gamma^\mu \gamma^\nu] = 4\eta^{\mu\nu} \quad \text{and} \quad \text{tr}[\not{a}\not{b}] = 4a \cdot b \quad (21)$$

$$\text{tr}[\not{a}\not{b}\not{c}\not{d}] = 4[(a \cdot b)(c \cdot d) - 4(a \cdot c)(b \cdot d) + 4(a \cdot d)(b \cdot c)] \quad (22)$$

$$\text{tr}[\gamma^5 \gamma_\mu \gamma_\nu \gamma_\alpha \gamma_\beta] = 4i\varepsilon_{\mu\nu\alpha\beta} \quad (23)$$

Useful are also $\not{a}\not{b} + \not{b}\not{a} = 2a \cdot b$, $\not{a}\not{a} = a^2$ and the following contractions,

$$\gamma^\mu \gamma_\mu = 4, \quad \gamma^\mu \not{a} \gamma_\mu = -2\not{a}, \quad \gamma^\mu \not{a}\not{b} \gamma_\mu = 4a \cdot b, \quad \gamma^\mu \not{a}\not{b}\not{c} \gamma_\mu = -2\not{c}\not{b}\not{a}. \quad (24)$$

Completeness relations

$$\sum_s u_a(p, s) \bar{u}_b(p, s) = (\not{p} + m)_{ab}, \quad (25)$$

$$\sum_s v_a(p, s) \bar{v}_b(p, s) = (\not{p} - m)_{ab}. \quad (26)$$

Decay rate

$$d\Gamma_{fi} = \frac{1}{2E_i} |\mathcal{M}_{fi}|^2 d\Phi^{(n)}. \quad (27)$$

The two particle phase space $d\Phi^{(2)}$ in the rest frame of the decaying particle is

$$d\Phi^{(2)} = \frac{1}{16\pi^2} \frac{|\mathbf{p}'_{\text{cms}}|}{M} d\Omega, \quad (28)$$

Breit-Wigner formula

$$\sigma(12 \rightarrow R \rightarrow 34) = \frac{4\pi s}{p_{\text{cms}}^2} \frac{(2s_r + 1)}{(2s_1 + 1)(2s_2 + 1)} \frac{\Gamma(R \rightarrow 12)\Gamma(R \rightarrow 34)}{(s - M_R^2)^2 + (M_R\Gamma_R)^2} \quad (29)$$

where Γ_R is the total decay width of the resonance with mass m_R and spin s_R , while s_1 and s_2 are the spins of the particles in the initial state.

$$\mathcal{L} = \frac{1}{2}(\partial_\mu \phi)(\partial^\mu \phi) - \frac{1}{2}m_\phi^2 \phi^2 + \bar{\psi}(i\not{\partial} - m_\psi)\psi$$

	mass	energy	1/length	1/time	temperature
GeV	1.78×10^{-24} g	1.60×10^{-3} erg	5.06×10^{13} cm ⁻¹	1.52×10^{24} s ⁻¹	1.16×10^{13} K