NTNU – Trondheim
Norwegian University of
Science and Technology

Exam FY3452 Gravitation and Cosmology fall 2019

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December 5 2019
15.00-19.00

Permitted examination support material:
Rottmann: Matematisk Formelsamling
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Barnett & Cronin: Mathematical Formulae
Angell og Lian: Fysiske størrelser og enheter: navn og symboler

Read carefully. Good luck! Bonne chance! Viel Glück! Veel succes! Lykke til!

Note: Problems 1-3, write on paper and scan. Problem 4, write in box on screen

Problem 1

Consider a coordinate transformation from the usual coordinates x and t in two-dimensional Minkowski space to coordinates x' and t', where

$$t = \left(\frac{c}{g} + \frac{x'}{c}\right) \sinh \tfrac{gt'}{c} , \tag{1}$$

$$x = c\left(\frac{c}{g} + \frac{x'}{c}\right) \cosh \tfrac{gt'}{c} - \frac{c^2}{g} , \tag{2}$$

where c is the speed of light and g is a constant.

a) Find the engineering dimension of g. Express the line element $ds^2 = -c^2dt^2+dx^2$ in terms of the primed coordinates t' and x'.

b) The only nonzero Christoffel symbols in the primed coordinates x' and t' are Γ^1_{00} and $\Gamma^0_{01} = \Gamma^0_{10}$. Calculate the nonzero Christoffel symbols.

c) Calculate the diagonal elements of the Ricci curvature tensor $R_{\alpha\beta}$. Calculate the Ricci scalar R. How could you have obtained these results without doing any calculations?

Problem 2

a) State and explain briefly the two postulates or assumptions on which special relativity is based.

b) Define timelike, spacelike, and lightlike curves in special relativity. Sketch each type of curve in a spacetime diagram.

c) Consider the standard configuration in special relativity, where the frame S' moves along the x-axis in the frame S with speed v. The Lorentz transformations between the two frames are given by

$$x' = \gamma(x - vt) , \tag{3}$$

$$y' = y , \tag{4}$$

$$z' = z , \tag{5}$$

$$t' = \gamma\left(t - \frac{v}{c^2}x\right) . \tag{6}$$

Consider two events A and B in S whose separation is spacelike. Their coordinates are $(ct_A, x_A, 0, 0)$ and $(ct_B, x_B, 0, 0)$ with $t_B > t_A$ and $x_B > x_A$. Thus the event A is before B in the inertial frame S. Show that there exists a frame S' such that $t'_A > t'_B$. Explain why the event A cannot be the cause of event B. Is causality violated in special relativity?

Problem 3

The Lagrangian for a free electron-positron field is

$$\mathcal{L} = \bar{\psi}(i\gamma^\mu \partial_\mu - m)\psi , \tag{7}$$

where ψ is a four-component column vector, $\bar{\psi} = \psi^\dagger \gamma^0$ is four-component row vector, and γ^μ are 4×4 matrices, called the γ-matrices. The γ-matrices satisfy

$$\{\gamma^\mu, \gamma^\nu\} = 2\mathbb{I}_4 \eta^{\mu\nu} , \tag{8}$$

where $\eta^{\mu\nu} = \text{diag}(-1, 1, 1, 1)$ and $\mathbb{I}_4$ is the 4×4 unit matrix.

a) Consider a so-called *chiral* transformation

$$\psi \rightarrow e^{i\gamma^5 \alpha}\psi , \tag{9}$$

where α is constant and $\gamma^5 = (i\gamma^0\gamma^1\gamma^2\gamma^3)^\dagger$. Show that $\bar{\psi}$ transforms as

$$\bar{\psi} \rightarrow \bar{\psi} e^{i\gamma^5 \alpha} . \tag{10}$$

Hint: γ^5 is Hermitian.

b) For which values of m is the Lagrangian (7) invariant under chiral transformations? **Hint:** γ^5 anticommutes with γ^μ.

Useful formulas

$$\Gamma^\alpha{}_{\beta\gamma} = \frac{1}{2} g^{\alpha\mu} \left[\frac{\partial g_{\gamma\mu}}{\partial x^\beta} + \frac{\partial g_{\beta\mu}}{\partial x^\gamma} - \frac{\partial g_{\beta\gamma}}{\partial x^\mu} \right] , \tag{11}$$

$$R_{\mu\nu} = \partial_\gamma \Gamma^\gamma_{\mu\nu} - \partial_\nu \Gamma^\gamma_{\mu\gamma} + \Gamma^\gamma_{\mu\nu}\Gamma^\delta_{\gamma\delta} - \Gamma^\gamma_{\mu\delta}\Gamma^\delta_{\nu\gamma} . \tag{12}$$