NTNU Trondheim, Institutt for fysikk

Examination for FY3464 Quantum Field Theory I
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Allowed tools: mathematical tables

1. Procca equation. ~ 25 points

A massive spin-1 particle satisfies the Procca equation,

$$(\eta^{\mu\nu}\Box - \partial^\mu\partial^\nu)\, A_\nu + m^2 A^\mu = 0\,. \tag{1}$$

a.) "Derive" the Procca equation combining Lorentz invariance with your knowledge how many spin states a massive spin-1 particle contains.
b.) Derive the propagator $D_{\mu\nu}(k)$ of a massive spin-1 particle. [You don't have to care how the pole is handeled.]
c.) Why is the limit $m \to 0$ in your result for b.) ill-defined? [max. 50 words]
d.) Write down the generating functional $Z[J]$ for this theory.
e.) How does one obtain connected Green functions $G(x_1, \ldots, x_n)$ from the generating functional $Z[J]$?

2. Gauge invariance. ~ 17 points

Consider a local gauge transformation

$$U(x) = \exp[\mathrm{i}g \sum_{a=1}^{m} \vartheta^a(x) T^a] \tag{2}$$

which changes a vector of fermion fields $\boldsymbol{\psi}$ with components $\{\psi_1, \ldots, \psi_n\}$ as

$$\psi(x) \to \psi'(x) = U(x)\psi(x)\,. \tag{3}$$

Assume that U are elements of a non-abelian gauge group.
a.) Derive the transformation law of $A_\mu = A_\mu^a T^a$ under a gauge transformation. One way is to require that i) the covariant derivatives transform in the same way as ψ,

$$D_\mu\psi(x) \to [D_\mu\psi(x)]' = U(x)[D_\mu\psi(x)]\,. \tag{4}$$

and ii) that the gauge field should compensate the difference between the normal and the covariant derivative,

$$D_\mu\psi(x) = [\partial_\mu + \mathrm{i}gA_\mu(x)]\psi(x)\,. \tag{5}$$

b.) Writing down the generating functional $Z[J]$ for this theory in the same way as in 1.d) results in an ill-defined expression. Why? Which solution do you suggest? [max. 50

words]
c.) Draw the Feynman rules (only the diagrams, no specific rules like $(p^\mu - p'^\mu)\gamma_\mu \ldots$, group or other factors) for this theory. (The number of diagrams depends on your suggested solution in b.))

3. Scale invariance. ∼ 15 points
Consider a massless scalar field with ϕ^4 self-interaction,

$$\mathscr{L} = \frac{1}{2}(\partial_\mu \phi)^2 - \frac{\lambda}{4!}\phi^4 \,. \tag{6}$$

in $d = 4$ space-time dimensions.
a.) Find the equation of motion for $\phi(x)$.
b.) Assume that $\phi(x)$ solves the equation of motion and define a scaled field

$$\tilde{\phi}(x) \equiv \mathrm{e}^{Da}\phi(\mathrm{e}^a x) \,, \tag{7}$$

where D is a constant. Show that the scaled field $\tilde{\phi}(x)$ is also a solution of the equation of motion, provided that the constant D is choosen appropriately.
c.) Bonus question: Argue, if the classical symmetry (7) is (not) conserved on the quantum level. [max. 50 words]

4. Dirac (quiz). ∼ 10 points
a.) Helicity of a free massive particle is invariant under Lorentz transformations:
yes □, no □
Chirality of a free massive particle is invariant under Lorentz transformations
yes □, no □
b.) Helicity of a free massive particle is a conserved quantity
yes □, no □
Chirality of a free massive particle is a conserved quantity
yes □, no □
c.) Decompose a Dirac spinor ψ_D into Majorana spinors ψ_M.
d.) The bilinear $\phi_R^\dagger \sigma^\mu \phi_R$ transforms as ... under proper Lorentz transformations, as ... under parity (where ϕ_R is a Weyl spinor).