NTNU Trondheim, Institutt for fysikk

Examination for FY3464 Quantum Field Theory I
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Allowed tools: mathematical tables

1. The $\lambda\phi^3$ theory.
Consider the theory of a massive real scalar field ϕ and a $\lambda\phi^3$ self-interaction in $d = 6$ dimensions.
a.) Write down the Lagrange density $\mathscr{L}$ and explain your choice of signs and pre-factors.
. (6 pts)
b.) Write down the corresponding generating functional for disconnected and connected Green functions. How does one obtain connected Green functions? (3 pts)
c.) Determine the dimension of the field ϕ and of the coupling λ. (3 pts)
d.) Draw the Feynman diagram(s) and write down the analytical expression for the self-energy $\mathrm{i}\Sigma$ (i.e. the loop correction for the free propgator) at order $\mathcal{O}(\lambda^2)$ in momentum space. (4 pts)
e.) Determine the symmetry factor of $\mathrm{i}\Sigma$. (3 pts)
f.) Calculate the self-energy $\mathrm{i}\Sigma$ using dimensional regularisation, split the result into a divergent pole term and a finite remainder. (14 pts)

2. Fermions.
a.) Define left- and right-chiral fields ψ_L and ψ_R as eigenfunctions of γ^5. Express

$$\mathscr{L} = \bar{\psi}\mathrm{i}\partial\!\!\!/\psi - m\bar{\psi}\psi$$

in terms of ψ_L and ψ_R. (7 pts)
b.) Give an operator which commutes with the (free Dirac) Hamiltonian and can be used to classify the spin states of a fermion. Explain its meaning. (You don't have to calculate the commutator.) (3 pts)

3. Scattering.
Derive the optical theorem

$$2\Im T_{ii} = \sum_n T^*_{in} T_{ni}.$$

Give a physical interpretation of this relation (less than 100 words). (7 pts)

4. Gauge invariance.
Consider a local gauge transformation

$$U(x) = \exp[\mathrm{i}g \sum_{a=1}^{m} \vartheta^a(x) T^a]$$

which changes a vector of fermion fields $\boldsymbol{\psi}$ with components $\{\psi_1, \ldots, \psi_k\}$ as

$$\psi(x) \to \psi'(x) = U(x)\psi(x) .$$

a.) Assume that U are elements of the non-abelian gauge group $\mathrm{SU}(n)$ and that $\{\psi_1, \ldots, \psi_5\}$ transform with the fundamental representation. What are then the values of n and m? What is the physical interpretation of m? (5 pts)

b.) Derive the transformation law of $A_\mu = A_\mu^a T^a$ under a gauge transformation. One way to do this is to require that i) the covariant derivatives transform in the same way as ψ,

$$D_\mu \psi(x) \to [D_\mu \psi(x)]' = U(x)[D_\mu \psi(x)] .$$

and ii) that the gauge field should compensate the difference between the normal and the covariant derivative, (8 pts)

$$D_\mu \psi(x) = [\partial_\mu + \mathrm{i}g A_\mu(x)]\psi(x) .$$

c.) The non-abelian field-strength $F_{\mu\nu} = F_{\mu\nu}^a T^a$ transforms under a local gauge transformation $U(x)$ as (2 pts)

☐ $F_{\mu\nu} \to F'_{\mu\nu} = F_{\mu\nu}$
☐ $F_{\mu\nu} \to F'_{\mu\nu} = U(x)F_{\mu\nu}U^\dagger(x)$
☐ $F_{\mu\nu} \to F'_{\mu\nu} = U(x)F_{\mu\nu}U^\dagger(x) + \frac{\mathrm{i}}{g}(\partial_\mu U(x))\partial_\nu U^\dagger(x)$
☐ $F_{\mu\nu} \to F'_{\mu\nu} = F_{\mu\nu} + [D_\mu, A_\nu]$

Some formulas

$$\{\gamma^\mu, \gamma^\nu\} = 2\eta^{\mu\nu} \,. \tag{1}$$

$$\gamma^5 \equiv \gamma_5 \equiv \mathrm{i}\gamma^0\gamma^1\gamma^2\gamma^3 \tag{2}$$

$$\sigma^{\mu\nu} = \frac{\mathrm{i}}{2}[\gamma^\mu, \gamma^\nu] \tag{3}$$

$$\overline{\Gamma} = \gamma^0\Gamma^\dagger\gamma^0 \tag{4}$$

$$\frac{1}{ab} = \int_0^1 \frac{\mathrm{d}z}{[az + b(1-z)]^2} \,. \tag{5}$$

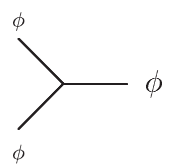
$$\begin{aligned} I(\omega, \alpha) &= \int \frac{\mathrm{d}^{2\omega}k}{(2\pi)^{2\omega}} \frac{1}{[k^2 + 2pk + M^2 + \mathrm{i}\varepsilon]^\alpha} \\ &= \mathrm{i}\, \frac{(-\pi)^\omega}{(2\pi)^{2\omega}} \frac{\Gamma(\alpha - \omega)}{\Gamma(\alpha)} \frac{1}{[M^2 - p^2 + \mathrm{i}\varepsilon]^{\alpha-\omega}} \,. \end{aligned} \tag{6}$$

$$f^{-\varepsilon/2} = 1 - \frac{\varepsilon}{2} \ln f + \mathcal{O}(\varepsilon^2) \,. \tag{7}$$

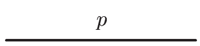
$$\Gamma(n+1) = n! \tag{8}$$

$$\Gamma(-n+\varepsilon) = \frac{(-1)^n}{n!} \left[\frac{1}{\varepsilon} + \psi(n+1) + \mathcal{O}(\varepsilon)\right] \,, \tag{9}$$

$$\psi(n+1) = 1 + \frac{1}{2} + \ldots + \frac{1}{n} - \gamma \,, \tag{10}$$



$-\,\mathrm{i}\lambda$



$$\frac{\mathrm{i}}{p^2 - m^2 + \mathrm{i}\varepsilon}$$