NTNU Trondheim, Institutt for fysikk

Examination for FY3464 Quantum Field Theory I
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Allowed tools: mathematical tables

1. Miscellaneous and quiz

(Several answers could be correct.)

a.) Write down A^* for (4 pts)

$$A = \bar{u}(p_2)\gamma^\mu u(p_1)$$

b.) Calculate (4 pts)

$$\mathrm{tr}[\gamma^\mu\gamma^\nu\gamma_\mu\gamma_\nu].$$

c.) The covariant derivative of a Yang-Mills theory transforms under a local gauge transformation $U(x)$ as: (1 pt)

- ☐ $D \to D' = D$
- ☐ $D \to D' = U(x)D$
- ☐ $D \to D' = U(x)DU^\dagger(x)$
- ☐ $D \to D' = U(x)DU^\dagger(x) + \frac{\mathrm{i}}{g}(\partial_\mu U(x))U^\dagger(x)$

d.) The field strength of a Yang-Mills theory transforms under a local gauge transformation $U(x)$ as: (1 pt)

- ☐ $\mathbf{F}(x) \to \mathbf{F}'(x) = \mathbf{F}(x)$
- ☐ $\mathbf{F}(x) \to \mathbf{F}'(x) = U(x)\mathbf{F}(x)$
- ☐ $\mathbf{F}(x) \to \mathbf{F}'(x) = U(x)\mathbf{F}(x)U^\dagger(x)$
- ☐ $\mathbf{F}(x) \to \mathbf{F}'(x) = U(x)\mathbf{F}(x)U^\dagger(x) + \frac{\mathrm{i}}{g}(\partial_\mu U(x))U^\dagger(x)$

2. Scalar field.

Consider a real, scalar field ϕ with mass m and self-interaction $g\phi^3$.

a.) Write down the Lagrange density $\mathscr{L}$, explain your choice of signs and pre-factors (when physically relevant). (6 pts)

b.) Write down the generating functional for connected Green function. (4 pts)

c.) Determine the mass dimension in $d = 4$ space-time dimensions of all quantities in the Lagrange density $\mathscr{L}$. (6 pts)

d.) Draw the divergent one-loop diagrams and determine their superficial degree of divergence D (in $d = 4$ space-time dimensions). (6 pts) e.) Determine the number d of space-time dimension for which the theory is renormalisable. (6 pts)

3. Fermion with Yukawa interaction.
Consider a fermion ψ with mass m interacting with real scalar field ϕ with mass M through a Yukawa interaction,

$$\mathscr{L} = -\mathrm{i}g\bar{\psi}\gamma^5\psi\phi\,.$$

a.) Determine the global symmetries of this Lagrangian for $m = M = 0$ and the resulting Noether charges.
b.) Calculate the self-energy $\Sigma(\not{p})$ of a fermion with momentum p using dimensional regularisation. Express $\Sigma(\not{p})$ as

$$\Sigma(\not{p}) = \frac{A}{\varepsilon} + B\ln(D/\mu^2)]\,. \tag{1}$$

d.) What is your interpretation of the functional form of A?
s.) What is your interpretation on the dependence of the self-energy the parameter μ? [max. 50 words or one formula without explicit calculation is enough] (6 pts)

4. Spin-1 fields.

a.) Use the tensor method to determine the propagator $D_{\mu\nu}(k)$ of a massive spin-1 field described by the Proca equation (6 pts)

$$(\eta^{\mu\nu}\Box - \partial^\mu\partial^\nu)\,A_\nu + m^2A^\mu = 0\,.$$

b) Give one argument why this method does not work for $m = 0$. (3 pts)

Feynman rules and useful formulas

$$\{\gamma^\mu, \gamma^\nu\} = 2\eta^{\mu\nu}\,. \tag{2}$$

$$\{\gamma^\mu, \gamma^5\} = 0 \quad \text{and} \quad (\gamma^5)^2 = 1\,. \tag{3}$$

$$\sigma^{\mu\nu} = \frac{\mathrm{i}}{2}[\gamma^\mu, \gamma^\nu] \tag{4}$$

$$\overline{\Gamma} = \gamma^0 \Gamma^\dagger \gamma^0 \tag{5}$$

$$\frac{1}{ab} = \int_0^1 \frac{\mathrm{d}z}{[az + b(1-z)]^2}\,. \tag{6}$$

$$\frac{1}{k^2 + m^2} = \int_0^\infty \mathrm{d}s\, \mathrm{e}^{-s(k^2+m^2)} \tag{7}$$

$$I_0(\omega, \alpha) = \int \frac{\mathrm{d}^{2\omega}k}{(2\pi)^{2\omega}} \frac{1}{[k^2 - m^2 + \mathrm{i}\varepsilon]^\alpha} = \mathrm{i}\, \frac{(-1)^\alpha}{(4\pi)^\omega} \frac{\Gamma(\alpha - \omega)}{\Gamma(\alpha)} [m^2 - \mathrm{i}\varepsilon]^{\omega - \alpha}\,. \tag{8}$$

$$I(\omega, 2) = \mathrm{i}\, \frac{1}{(4\pi)^\omega} \frac{\Gamma(2-\omega)}{\Gamma(2)} \frac{1}{[m^2 - q^2 z(1-z)]^{2-\omega}}\,. \tag{9}$$

$$\int_{-\infty}^\infty \mathrm{d}x \exp(-x^2/2) = \sqrt{2\pi} \tag{10}$$

$$f^{-\varepsilon/2} = 1 - \frac{\varepsilon}{2} \ln f + \mathcal{O}(\varepsilon^2)\,. \tag{11}$$

$$\Gamma(z) = \int_0^\infty \mathrm{d}t\, \mathrm{e}^{-t} t^{z-1} \tag{12}$$

$$\Gamma(n+1) = n! \tag{13}$$

$$\Gamma(-n + \varepsilon) = \frac{(-1)^n}{n!} \left[\frac{1}{\varepsilon} + \psi_1(n+1) + \mathcal{O}(\varepsilon)\right]\,, \tag{14}$$

$$\psi_1(n+1) = 1 + \frac{1}{2} + \ldots + \frac{1}{n} - \gamma\,, \tag{15}$$