NTNU

Department of Physics

Examination paper for FY3464 Quantum Field Theory I

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Examination for FY3464 Quantum Field Theory I
Contact: Michael Kachelrieß, tel. 99890701
Allowed tools: mathematical tables

1. Scalar field and scale invariance.
Consider a complex, scalar field ϕ with mass m and self-interaction $g\phi^n$.
a.) Write down the Lagrange density $\mathscr{L}$, explain your choice of signs and pre-factors (when physically relevant). (5 pts)
b.) Determine the mass dimension in $d = 4$ space-time dimensions of all quantities in the Lagrange density $\mathscr{L}$. Choose n such that the coupling g is dimensionless. (5 pts)
c.) Set now $m = 0$ and consider a real scalar field ϕ. Find the equation of motion for $\phi(x)$. (4 pts)
d.) Assume that $\phi(x)$ solves the equation of motion and define a scaled field

$$\tilde{\phi}(x) \equiv \mathrm{e}^{Da}\phi(\mathrm{e}^{a}x)\,, \tag{1}$$

where D and a are constants. Show that the scaled field $\tilde{\phi}(x)$ is also a solution of the equation of motion, provided that the constant D is choosen appropriately. (6 pts)
e.) Bonus question: Argue, if the classical symmetry (1) is (not) conserved on the quantum level. [max. 50 words] (2 pts)

2. Fermion field.
Consider a massless Dirac field ψ with Lagrangian

$$\mathscr{L} = \bar{\psi}(\mathrm{i}\partial\!\!\!/)\psi.$$

a.) Derive the propagator $S_F(p)$ of the field ψ. [You do not have to discuss how the poles of $S_F(p)$ are treated.] (4 pts)
b.) Write down the generating functional for disconnected Green functions for this theory. (4 pts)
c.) Show that the Lagrange density $\mathscr{L}$ is invariant under global vector phase transformations $\mathrm{U_V}(1)$, $\psi \to \psi' = \mathrm{e}^{\mathrm{i}\vartheta}\psi$, and under global axial phase transformations $\mathrm{U_A}(1)$, $\psi \to \psi' = \mathrm{e}^{\mathrm{i}\vartheta\gamma^5}\psi$. (6 pts)
d.) Show that global symmetry under vector phase transformation $\mathrm{U_V}(1)$ can be made local, if a coupling to a gauge boson is added. (5 pts)
e.) Draw the divergent one-loop diagrams and determine their superficial degree of divergence D (in $d = 4$ space-time dimensions) for the theory coupled to a gauge boson. (8 pts)

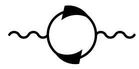
3. Unitarity.

a.) Derive the optical theorem

$$2\Im T_{ii} = \sum_n T^*_{in} T_{ni}.$$

Give a physical interpretation of this relation (less than 50 words). (6 pts)

b.) The vacuum polarisation of a photon,

q  q $= \Pi^{\mu\nu}(q^2) = (q^2\eta^{\mu\nu} - q^\mu q^\nu)\Pi(q^2)$

is given in dimensional regularisation by

$$\Pi(q^2) = -\frac{e^2}{12\pi^2}\left\{\frac{1}{\varepsilon} - \gamma + \ln(4\pi) - 6\int_0^1 \mathrm{d}x\, x(1-x)\ln\left[\frac{m^2 - q^2 x(1-x)}{\mu^2}\right]\right\}.$$

Show that gauge invariance, $q_\mu \Pi^{\mu\nu}(q) = 0$, implies as tensor structure of the vacuum polarisation tensor $\Pi^{\mu\nu}(q) = (q^2\eta^{\mu\nu} - q^\mu q^\nu)\Pi(q^2)$. (4 pts)

c.) Derive the imaginary part of the vacuum polarisation, $\Im[\Pi(q^2)]$. (6 pts)

d.) How does the imaginary part of the vacuum polarisation changes, if the renormalisation scheme is changed? (4 pts)

Useful formulas

$$\{\gamma^\mu, \gamma^\nu\} = 2\eta^{\mu\nu}\,. \tag{2}$$

$$\{\gamma^\mu, \gamma^5\} = 0 \quad \text{and} \quad (\gamma^5)^2 = 1\,. \tag{3}$$

$$\sigma^{\mu\nu} = \frac{\mathrm{i}}{2}[\gamma^\mu, \gamma^\nu] \tag{4}$$

$$\overline{\Gamma} = \gamma^0 \Gamma^\dagger \gamma^0 \tag{5}$$

$$\frac{1}{ab} = \int_0^1 \frac{\mathrm{d}z}{[az + b(1-z)]^2}\,. \tag{6}$$

$$\frac{1}{k^2 + m^2} = \int_0^\infty \mathrm{d}s\, \mathrm{e}^{-s(k^2+m^2)} \tag{7}$$

$$I_0(\omega, \alpha) = \int \frac{\mathrm{d}^{2\omega}k}{(2\pi)^{2\omega}} \frac{1}{[k^2 - m^2 + \mathrm{i}\varepsilon]^\alpha} = \mathrm{i}\,\frac{(-1)^\alpha}{(4\pi)^\omega}\frac{\Gamma(\alpha - \omega)}{\Gamma(\alpha)}\,[m^2 - \mathrm{i}\varepsilon]^{\omega - \alpha}\,. \tag{8}$$

$$I(\omega, 2) = \mathrm{i}\,\frac{1}{(4\pi)^\omega}\frac{\Gamma(2-\omega)}{\Gamma(2)}\frac{1}{[m^2 - q^2 z(1-z)]^{2-\omega}}\,. \tag{9}$$

$$\Im \ln(x + \mathrm{i}\varepsilon) = -\pi \tag{10}$$