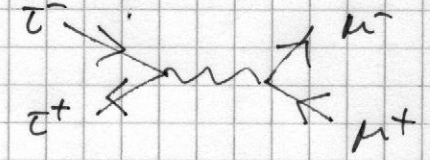
FY3464 QUANTUM FIELD THEORY I

Soln exam Nov. 30 2007

Problem 1

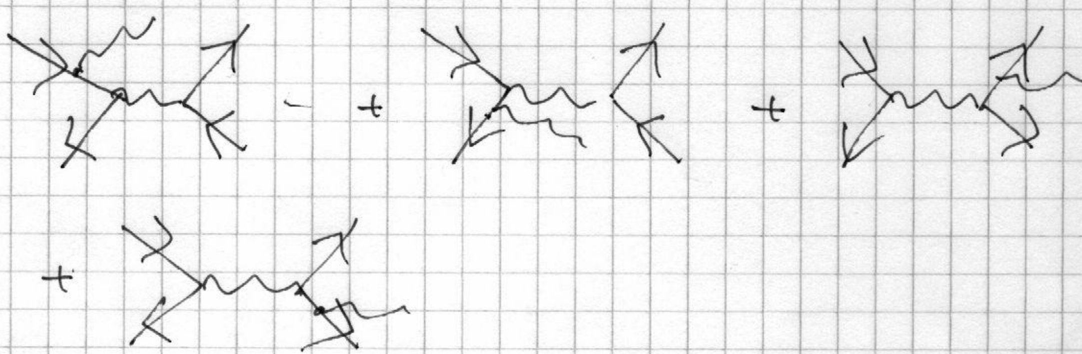
a) $\tau^+ \to \mu^+ e^+ e^-$ Impossible. Violates conservation of μ- and τ-numbers.

b) $\tau^+ \tau^- \to \mu^+ \mu^-$

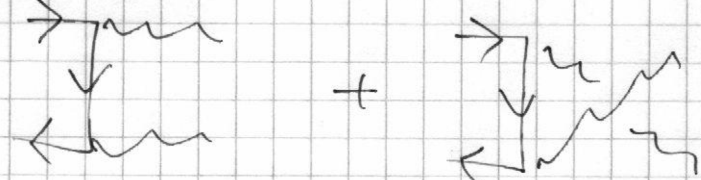


c) $\tau^+ \mu^- \to \mu^+ e^-$ Impossible. Violates conservation of e-, μ- and τ-numbers.

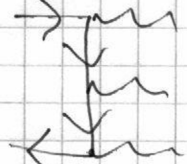
d) $\tau^+ \tau^- \to \mu^+ \mu^- \gamma$



e) $\tau^+ \tau^- \to \gamma\gamma$

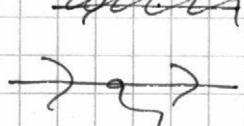


f) $\tau^+ \tau \to \gamma\gamma\gamma$



+ 5 permutation of γ-lines

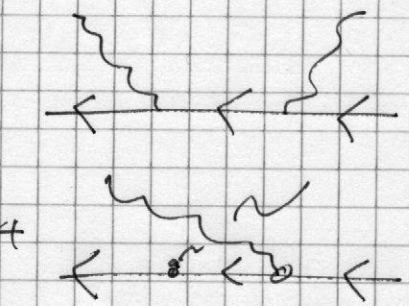
g) ~~[illegible]~~ $\tau^+ \to \tau^+ \gamma$



Impossible. Violates conservation of 4-momentum.

Problem 1

f) $\tau^+\gamma \to \tau^+\gamma$

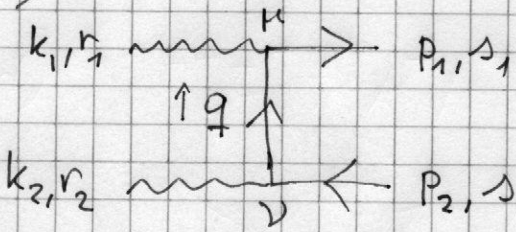


(Compton process)

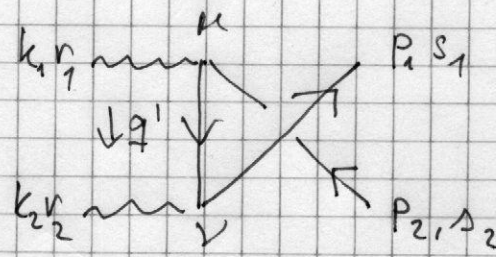
Problem 2.

$\gamma\gamma \to \tau^+\tau^-$

a)



(a)



(b)

b)

$$\mathcal{M}_{fi} = \mathcal{M}_{fi}^{(a)} + \mathcal{M}_{fi}^{(b)}$$

$$-i\mathcal{M}_{fi}^{(a)} = (ie)^2 i * \left[\bar{u}(p_1,s_1)\gamma^\mu(\not{q}+m_\tau)\gamma^\nu v(p_2,s_2)\right] * e_\mu(k_1,r_1)\, e_\nu(k_2,r_2) * \frac{1}{q^2-m_\tau^2}$$

$$-i\mathcal{M}_{fi}^{(b)} = (ie)^2 i * \left[\bar{u}(p_1,s_1)\gamma^\nu(\not{q}'+m_\tau)\gamma^\mu v(p_2,s_2)\right] * e_\mu(k_1,r_1)\, e_\nu(k_2,r_2) * \frac{1}{q'^2-m_\tau^2}$$

c)

$\sigma \propto e^4 \propto \alpha^2$. Scale set by m_τ.

I.e.

$$\sigma(E=2m_\tau) \simeq \alpha^2\left(\frac{\hbar}{m_\tau c}\right)^2 = \cdots$$

Problem 2

d) If $p_{2\mu} e_1^{\mu} = p_{2\mu} e_2^{\mu} = 0$ we would have $\not p_2 \not e_1 = -\not e_1 \not p_2$ and $\not p_2 \not e_2 = -\not e_2 \not p_2$. I.e.

$$(-\not p_2 + m_\tau)\not e_1 v(p_2,s_2) = \not e_1 (\not p_2 + m_\tau) v(p_2,s_2) = 0$$

and

$$(-\not p_2 + m_\tau)\not e_2 v(p_2,s_2) = \not e_2 (\not p_2 + m_\tau) v(p_2,s_2) = 0$$

since $(\not p + m) v(p,s) = 0$.

Now. Assume we have polarization vectors such that

$$(p_2 e_1) \neq 0 \quad , (p_2 e_2) \neq 0$$

Then we may choose new vectors

$$e_1'^{\mu} = e_1^{\mu} - c_1 k^{\mu}, \quad e_2'^{\mu} = e_2^{\mu} - c_2 k^{\mu}$$

with

$$c_1 = \frac{(p_2 k)}{(p_2 e_1)} \quad , \quad c_2 = \frac{(p_2 k)}{(p_2 e_2)} .$$

e) ~~With $q = p_1 - k_1$ we m~~

With $\not q = \not k_2 - \not p_2$ we may simplify

$$\bar u(p_1,s_1) \not e_1 (\not q + m_\tau) \not e_2 v(p_2,s_2) =$$

$$\bar u(p_1 s_1) \not e_1 (\not k_2 - \not p_2 + m_\tau) \not e_2 v(p_2,s_2)$$

$$= \bar u(p_1 s_1) \not e_1 \not k_2 \not e_2 v(p_2 s_2) + \bar u(p_1,s_1) \not e_1 \not e_2 \underbrace{(\not p_2 + m_\tau) v(p_2,s_2)}_{=0}$$

$$= \bar u(p_1 s_1) \not e_1 \not k_2 \not e_2 v(p_2,s_2).$$

Likewise

$$q^2 - m_\tau^2 = (k_2 - p_2)^2 - m_\tau^2 = -2k_2 p_2 + \underbrace{k_2^2}_{=0} + \underbrace{p_2^2 - m_\tau^2}_{=0}$$

$$= -2k_2 p_2$$

Hence: $\underline{-i\mathcal{M}_{fi}^{(a)} = \frac{-ie^2}{(-2k_2 p_2)} \bar u(p_1 s_1) \not e_1 \not k_2 \not e_2 v(p_2 s_2)}$

Problem 3

a) $[\varphi] = \text{mass} \qquad [\psi] = \text{mass}^{3/2}$

$[\lambda] = 1 \quad (\text{dimensionless})$

b)
$$\partial_\mu \frac{\partial \mathcal{L}}{\partial \partial_\mu \varphi^*} = \frac{\partial \mathcal{L}}{\partial \varphi^*}$$
$$\Updownarrow$$
$$\underline{\underline{\Box \varphi + \frac{\lambda}{2} |\varphi|^2 \varphi = 0}}$$

$$\partial_\mu \frac{\partial \mathcal{L}}{\partial \partial_\mu \overline{\psi}} = \frac{\partial \mathcal{L}}{\partial \overline{\psi}}$$
$$\Updownarrow$$
$$\underline{\underline{i \not{\partial} \psi = 0}}$$

c) Only combinations $\varphi^* \varphi$ occur

$\Rightarrow$ $\varphi \to e^{i\alpha} \varphi$ is ~~not~~ a symmetry ~~for~~ $\mathcal{L}$. $\delta \varphi = i\alpha\varphi$, $\delta \varphi^* = -i\alpha\varphi^*$

$\Rightarrow$
$$\underline{\underline{j^\mu_\varphi}} = \frac{\partial \mathcal{L}}{\partial_\mu \varphi} \delta \varphi + \frac{\partial \mathcal{L}}{\partial \partial_\mu \varphi^*} \delta \varphi^*$$
$$= \underline{\underline{i \left[(\partial^\mu \varphi^*) \varphi - \varphi^* (\partial^\mu \varphi) \right]}}$$

d) Likewise
$$\underline{\underline{j^\mu_\psi = \overline{\psi} \gamma^\mu \psi}}$$

e)
$$\underline{\underline{j^\mu_5 = \overline{\psi} \gamma^\mu \gamma^5 \psi}}$$

f) See soln. ~~problem 4?~~ Problemset 4.