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FY3464

KVANTEFELTTEORI I

①

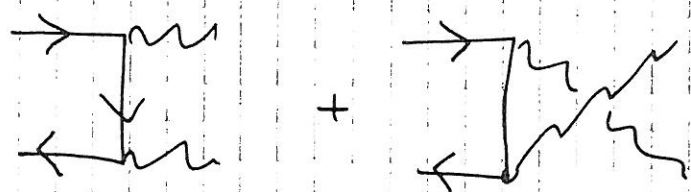
Løsning eksamen 3.12.2008

Oppgave 1.

a) $\mu^- \rightarrow e^- \gamma$

i QED)
Umulig. Bytter μ - og e -tall.
(Forventes å skje i naturen,
men aldri observert)

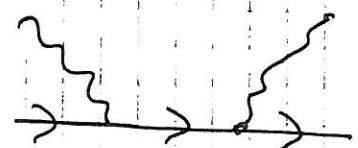
b) $e^- e^+ \rightarrow \gamma \gamma$



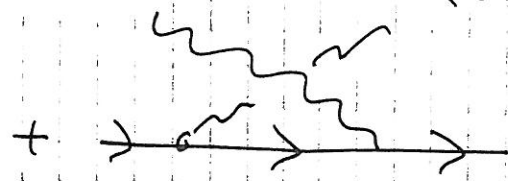
c) $e^- e^+ \rightarrow \gamma$

Men kinematisk umulig pga
bevaring av 4-impuls.

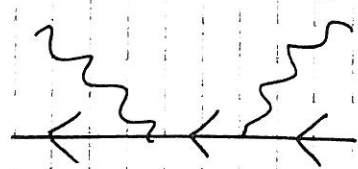
d) $e^- \gamma \rightarrow e^- \gamma$



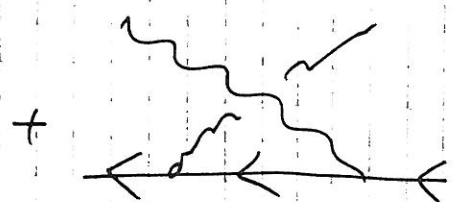
(Compton)



e) $e^+ \gamma \rightarrow e^+ \gamma$

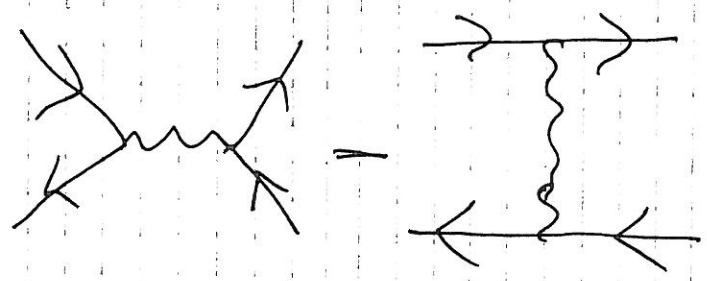
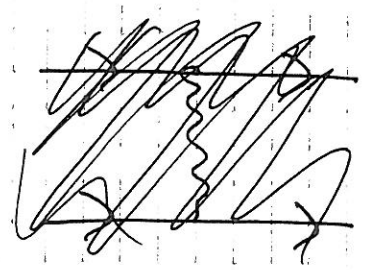


(Compton)



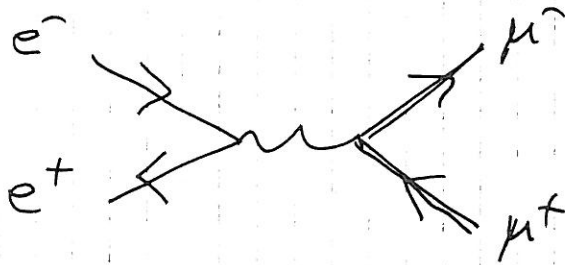
f) $e^- e^+ \rightarrow e^- e^+$

~~Umulig~~ (Bhabha)



(2)

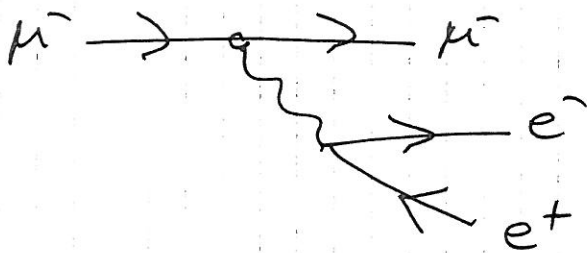
g) $e^- e^+ \rightarrow \mu^- \mu^+$



h) $e^- \mu^+ \rightarrow e^+ \mu^-$

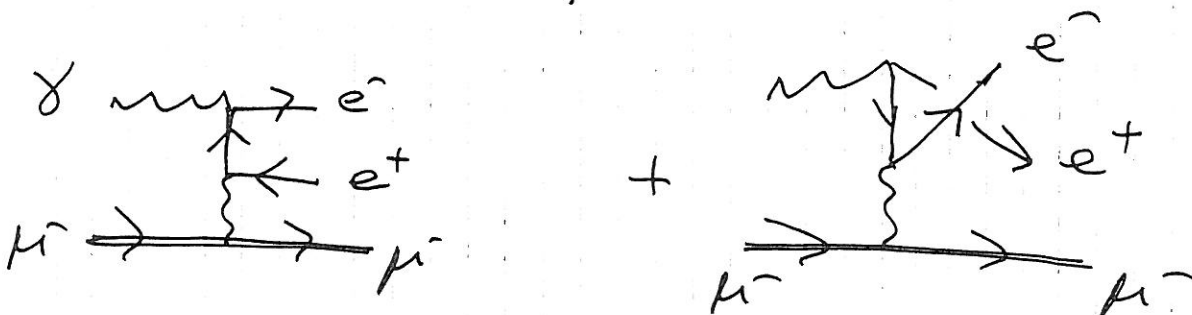
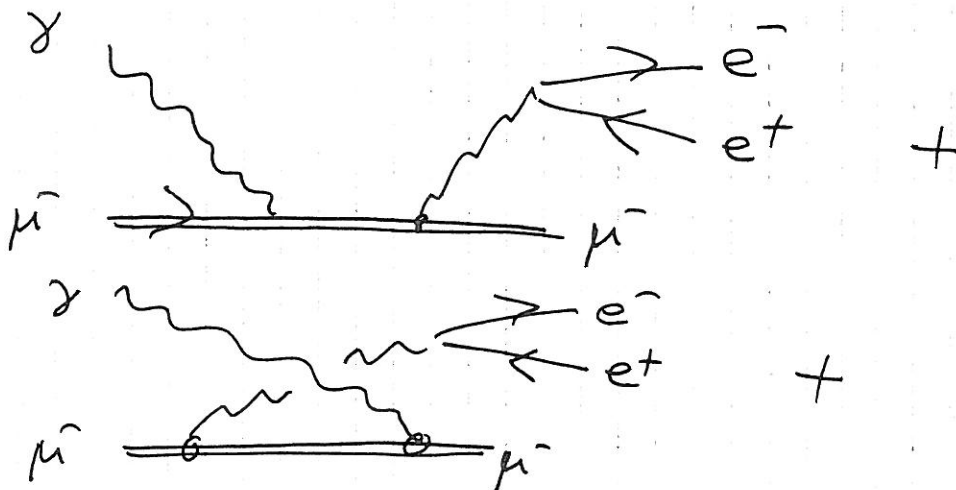
Umulig i QED.
Bryklen e^- og μ^- -tall.

i) $\mu^- \rightarrow \mu^- e^- e^+$



Men kinematisk
umulig pga bevægelse
av 4-impuls.

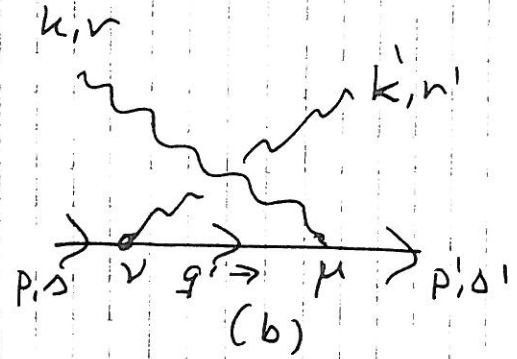
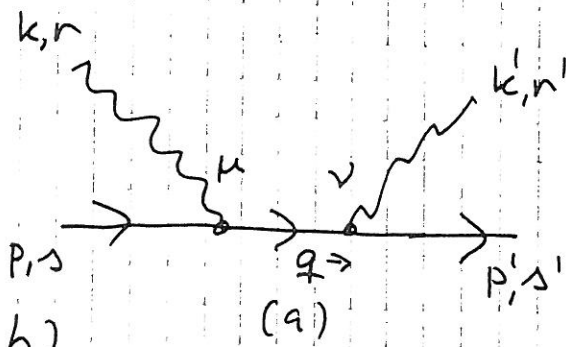
j) $\mu^- \gamma \rightarrow \mu^- e^- e^+$



Oppgave 2

3

a) $e^- \gamma \rightarrow e^- \gamma$



b)

$$\mathcal{M}_{fi} = \mathcal{M}_{fi}^{(a)} + \mathcal{M}_{fi}^{(b)}$$

$$-i\mathcal{M}_{fi}^{(a)} = (ie)^2 \frac{i}{q^2 - m^2} \bar{u}(p', s') \not{\epsilon}' (\not{q} + m) \not{\epsilon} u(p, s)$$

$$-i\mathcal{M}_{fi}^{(b)} = (ie)^2 \frac{i}{q'^2 - m^2} \bar{u}(p', s') \not{\epsilon} (\not{q}' + m) \not{\epsilon}' u(p, s)$$

Her er $\not{\epsilon} = \gamma_\mu \epsilon^\mu(k)$, $\not{\epsilon}' = \gamma_\nu \epsilon'^\nu(k')$,
 $q = p + k$, $q' = p - k'$.

c)

$$p = (E, 0, 0, -w)$$

$$E = \sqrt{m^2 + w^2}$$

$$p' = (E, -w \sin \vartheta, 0, -w \cos \vartheta)$$

(den totale)

I masseløst systemet er 3-impulsen lik null. Dette bestemmer $\vec{p}$ og $\vec{p}'$; p^0 og p'^0 finnes så av energi-impuls relasjonen.

d)

$$e_\perp = (0, 0, 1, 0)$$

$$e_\parallel = (0, 1, 0, 0)$$

$$e'_\perp = (0, 0, 1, 0)$$

finnes ved trivial

Sam første forsøk setter inspeksjon.

$$\tilde{e}'_\parallel = (0, \cos \vartheta, 0, -\sin \vartheta) \quad (\text{som oppfyller } k^\mu \tilde{e}'_\mu = 0)$$

Oppgave 2 forts.

(4)

d forts)

Men $(\tilde{e}'_{||} p) = \tilde{e}'_{||}{}^{\mu} p_{\mu} = -\omega \sin \vartheta$, så dette er ikke helt rett.
Velger derfor

$$e'_{||}{}^{\mu} = \tilde{e}'_{||}{}^{\mu} - c k'^{\mu}$$

med

$$c = \frac{(\tilde{e}' p)}{(k' p)} = \frac{-\omega \sin \vartheta}{\omega E + \omega^2 \cos \vartheta} \\ = - \frac{\sin \vartheta}{E + \omega \cos \vartheta}$$

Fordi $(k' e'_{||}) = 0$ og $k'^2 = 0$ kan vi at

$$e'_{||} e'_{||} = \tilde{e}'_{||} \tilde{e}'_{||} = -1.$$

Videre er

$e'_{||} e'_{\perp} = 0$ fordi verken $\tilde{e}'_{||}$ eller k' har noen komponent i y-retningen.

e) I $\mathcal{M}_{fi}^{(a)}$ kan vi skrive

$$T^{(a)} \equiv \bar{u}(p', s') \not{\epsilon}' (\not{q} + m) \not{\epsilon} u(p, s) \\ = \bar{u}(p', s') \not{\epsilon}' (\not{p} + \not{k} + m) \not{\epsilon} u(p, s) \\ = \bar{u}(p', s') \not{\epsilon}' \not{k} \not{\epsilon} u(p, s) - \bar{u}(p', s') \not{\epsilon}' \not{\epsilon} (\not{p} - m) u(p, s)$$

fordi $\not{p} \not{\epsilon} = -\not{\epsilon} \not{p}$ når $(\epsilon p) = 0$.

Videre oppfyller $u(p, s)$ Dirac-ligningen, $(\not{p} - m)u(p, s) = 0$, så vi får

$$T^{(a)} = \bar{u}(p', s') \not{\epsilon}' \not{k} \not{\epsilon} u(p, s)$$

$$\text{Tilsvarende fås } T^{(b)} \equiv \bar{u}(p', s') \not{\epsilon} (\not{p} - \not{k} + m) \not{\epsilon}' u(p, s) \\ = -\bar{u}(p', s') \not{\epsilon} \not{k}' \not{\epsilon}' u(p, s) - \bar{u}(p', s') \not{\epsilon} \not{\epsilon}' (\not{p} - m) u(p, s) \\ = 0$$

Oppgave 2 forts.

(5)

e forts)

Vi kan også skrive

$$q^2 - m^2 = (p+k)^2 - m^2 = 2pk + \underbrace{k^2}_{=0} + \underbrace{p^2 - m^2}_{=0} = 2pk$$

og

$$q'^2 - m^2 = (p-k')^2 - m^2 = -2pk' + \underbrace{k'^2}_{=0} + \underbrace{p^2 - m^2}_{=0} = -2pk'$$

Vi får derfor

$$M_{fi}^{(a)} + M_{fi}^{(b)} = \frac{e^2}{2} \left[\frac{\bar{u}(p',s') \not{\epsilon}' \not{k} \not{\epsilon} u(p,s)}{(pk)} + \frac{\bar{u}(p',s') \not{\epsilon} \not{k}' \not{\epsilon}' u(p,s)}{(pk')} \right]$$

f) Vi har $|M_{fi}|^2 \propto e^4 \propto d^2$.

Når $\omega \rightarrow 0$ er det bare elektronmassen m som kan brukes til å konstruere en lengde-dimensjon,

$$\lambda_e = \frac{h}{mc}$$

Så vi må ha

$$\sigma_{\text{tot}} \simeq (\alpha \lambda_e)^2 = r_e^2 =$$

r_e kalles den klassiske elektronradius.

[Det virkelige tverrsnittet i denne grensen kalles Thomson tverrsnittet, og er

$$\sigma_{\text{Th}} = \frac{8\pi}{3} r_e^2$$

Så overlagsvurderingen gir ^{ca.} en faktor 8 for lite.]

$$7.92 \cdot 10^{-30} \text{ m}^2$$

g)

$$\begin{aligned} |M_{fi}|^2 &= |M_{fi}^{(a)} + M_{fi}^{(b)}|^2 \\ &= |M_{fi}^{(a)}|^2 + |M_{fi}^{(b)}|^2 + M_{fi}^{(a)} M_{fi}^{(b)*} \\ &\quad + M_{fi}^{(b)*} M_{fi}^{(a)} \end{aligned}$$

Oppgave 2 forts.

⑥

$$g) \quad M_{fi} = \frac{e^2}{2} \left[\frac{\bar{u}' \not{\epsilon}' \not{k} \not{\epsilon} u}{(pk)} + \frac{\bar{u}' \not{\epsilon} \not{k}' \not{\epsilon}' u}{(pk')} \right] \Rightarrow$$

$$\begin{aligned} |M_{fi}|^2 &= \frac{e^4}{4} \cdot \frac{1}{2} \left\{ \frac{\text{Tr} \{ \not{\epsilon}' \not{k} \not{\epsilon} (\not{p} + m_e) \not{\epsilon} \not{k} \not{\epsilon}' (\not{p}' + m_e) \}}{(pk)^2} \right. \\ &+ \frac{\text{Tr} [\not{\epsilon} \not{k}' \not{\epsilon}' (\not{p} + m_e) \not{\epsilon}' \not{k}' \not{\epsilon} (\not{p}' + m_e)]}{(pk')^2} \\ &+ \frac{\text{Tr} [\not{\epsilon}' \not{k} \not{\epsilon} (\not{p} + m_e) \not{\epsilon} \not{k}' \not{\epsilon}' (\not{p}' + m_e)]}{(pk)(pk')} \\ &\left. + \frac{\text{Tr} [\not{\epsilon} \not{k}' \not{\epsilon}' (\not{p} + m_e) \not{\epsilon} \not{k} \not{\epsilon}' (\not{p}' + m_e)]}{(pk)(pk')} \right\} \\ &= \frac{e^4}{8} \left\{ \frac{\text{Tr}^{aa}}{(pk)^2} + \frac{\text{Tr}^{bb}}{(pk')^2} + \frac{\text{Tr}^{ab} + \text{Tr}^{ba}}{(pk)(pk')} \right\} \end{aligned}$$

$$\begin{aligned} h) \quad \text{Tr}^{aa} &= \text{Tr} \{ \not{\epsilon}' \not{k} \not{\epsilon} \not{p} \not{\epsilon} \not{k} \not{\epsilon}' \not{p}' \} \\ &= -\text{Tr} \{ \not{\epsilon}' \not{k} \not{p} \not{\epsilon} \not{\epsilon} \not{k} \not{\epsilon}' \not{p}' \} \quad \not{\epsilon} \not{p} = -\not{p} \not{\epsilon} \\ &= \text{Tr} \{ \not{\epsilon}' \not{k} \not{p} \not{k} \not{\epsilon}' \not{p}' \} \quad \text{because } (pe) = 0 \\ &\quad -\not{p} \not{k} + 2(pk) \quad \not{\epsilon} \not{\epsilon} = e^2 = -1. \\ &= -\text{Tr} \{ \not{\epsilon}' \not{p} \underbrace{\not{k} \not{k}}_{=k^2=0} \not{\epsilon}' \not{p}' \} + 2(pk) \text{Tr} \{ \not{\epsilon}' \not{k} \not{\epsilon}' \not{p}' \} \\ &= 2(pk) * 4 \left[2(ke')(\not{p}'e') - \underbrace{(e'e')}_{=-1} (\not{p}'k) \right] \\ &= 8(pk) [(p'k) + 2(p'e')(ke')] \\ &= \underline{8(pk)(p'k) + 16(ke')^2} \quad p'e' = \underbrace{pe'}_{=0} + \underbrace{k'e'}_{=0} - ke' \end{aligned}$$