Solutions 4107 6_2020

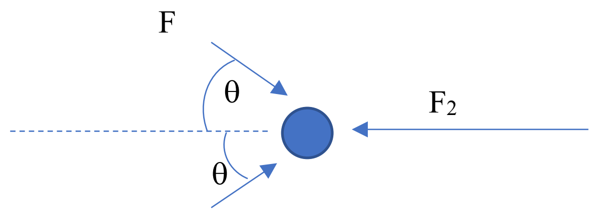
1) An airplane getting ready for takeoff accelerates at *a* m/s^2 with a takeoff roll that lasts *t* seconds. What is the velocity at takeoff in km/hr? [(288/80)*speed in m/s –>> 1m/s = 3.6km/h]
v=a*t, with a units conversion
v_f=a*t*3.6; %km/h

2) What is the average speed of the plane during the time it is on the runway?
$v_{avg} = ½ at = v_f/2$

3) An airplane getting ready for takeoff accelerates at ***a*** m/s^2 with a takeoff roll that lasts ***t*** seconds. What is the length of the runway used, in meters?
d=0.5*a*t^2

4) Forces are applied to a disk on a surface (only 2-dimensional motion is possible) as shown in the figure. The disk is not moving. What is θ_3, given (with units degrees and newton) θ_1, f_1, f_2

Forces on disk; $\Sigma F_x = 0$,
$F_1 \cos\theta_1 + F_3 \cos\theta_3 \quad = F_2$
$F_1 \sin\theta_1 = F_3 \sin\theta_3$
θ_3=atan((f1*sin(t1))/(f2-f1*cos(t1))



5) An asteroid is traveling with a velocity of v0 in the positive x direction. A spaceship energy beam catches it and changes the velocity at a along the x axis . What is the velocity of the asteroid when it is at a displacement of d relative to where it was first trapped by the beam? Note that answers have been rounded. Given v0= 3250 m/s a= -10 m/s2 d= 215 km
$v = \pm \text{sqrt}(v_0^2 + 2ax)$
Note that there are two solutions; one when the asteroid has moved to the positive x coordinate from the tractor beam, and one after it reaches zero velocity and doubles back to reach the same x coordinate. The motion will be symmetrical, with the velocity component will be reversed.

6) A projectile is launched at a target that is located x_t meters away. Given that the initial velocity of the projectile is v_0, at what angle, θ, in degrees) must the projectile be aimed, relative to the horizon, to hit the target (assume that the target is at the same height as the launcher).
$(v_0\cos\theta)t$= distance to target= x_t
vertical:
$(v_0\sin\theta)t - g/2\ t^2 = 0$; g is gravitational acceleration; divide by t, then $t = 2v_0\sin\theta/g$
substitute this t into horizontal

$(v_0)^2/g\ 2\cos\theta\sin\theta = x_t$ use trig identity
$(v_0)^2/g \sin 2\theta = x_t$ Note that sin(2θ)=sin (180-2θ), so there are two solutions
then $\theta = 0.5 \sin^{-1}(x_t{*}g/(v_0)^2)$
thet=0.5*asin((g*xt)/(v0*v0));

7) In the problem where a projectile is launched at a target x_t meters away, is there another angle that can be used to hit the target? If not, why not, and if there is another angle, what is it?

$(v_0)^2/g \sin 2\theta = x_t$ Note that $\sin(2\theta)=\sin(180-2\theta)$, so there are two solutions, θ and $90-\theta$

8) A ball is hit to a recipient who is on a hill, yf meters above the launch height. The ball starts with velocity v0 at an angle of θ degrees relative to the horizontal, and is descending when caught.
What is the maximum height of the ball, relative to the launch point?
Energy conservation;
$0.5\ m\ v^2 = mgh$;
vy=v0*sin(pi*thet/180) %radians
h=0.5*vy*vy/9.8

9) A ball is hit to a recipient who is on a hill, yf meters above the launch height. The ball starts with velocity v0, at an angle of θ degrees relative to the horizontal, and is descending when caught. How many seconds is the ball in the air?
> calculate time to reach peak, then add time to fall from peak to recipient. $d= 0.5\ at^2$
h=0.5*vy*vy/9.8
tu=sqrt(h/4.9)
td=sqrt((h-yf)/4.9)
tt=tu+td % total time

10)A pulley system is attached to the ceiling as shown. The pulling force is F and the combined mass of the person and the platform is M. Assume that the force applied by the person on the rope is directed vertically downwards. The rope is oriented vertically everywhere (except around the pulleys, of course). The person is attached to the platform. What is the acceleration of the platform and person? (Positive if up, negative if down.)

The pulleys make the platform rise three times as slowly, with three times the force; add in gravity
f=540 (tension force)
M=155
g=9.8;
a=(3*f-(M*g))/M

11) Block A (mass M_A) is on a frictionless incline with an angle of θ to the horizontal. It is connected to block B, (mass M_B) with a massless string that passes over a perfect, frictionless pulley. Find the magnitude of the acceleration of the blocks.
Add the forces, and recognize that the blocks move together, so divide by the sum of the masses
Ma=8
Mb=22
thet=30
theta=pi*thet/180 % radians
g=-9.8;
a=(-Ma*g*sin(theta)+(Mb*g))/(Ma+Mb)

12) A frictionless marble of mass M is on a track with an elliptical element (height a, horizontal axis b) after a downward slope (see figure). What is the speed of the marble when it is at the high point of the ellipse, given the starting height, h. The initial speed is zero.
Energy conservation; the width of the ellipse makes no difference
v=sqrt(2*g*(h-a))

13) Sliding down the inside of a vertical tube with your knees and elbows as brakes, friction controls the speed (and the discomfort of the landing) that you will have at the end of the ride. For a starting height h, starting velocity zero, and a mass of M, you push hard enough to have a final velocity v. What (constant) frictional force will be required to achieve this? (Answers may be rounded.)
Conservation of energy, yet again, but here there is some residual kinetic energy
U=M*g*h; % initial energy
KE=0.5*M*v^2; % residual kinetic
F=(U-KE)/h % work done by friction is the difference in energy

14) A pendulum (point mass at end of weightless string) has a mass of 2M and length 2R. What is the moment of inertia of the pendulum? (With respect to an axis perpendicular to the pendulum, through the end point where the pendulum is fixed.)
$I = 2M*(2R)^2 = 8\ MR^2$

15) A pendulum consists of a barbell with total mass 2M, attached to a stiff, negligible mass rod, so that half the weight is at a distance R from the rotation axis and the other half is located at the distance 2R. You can neglect the weight of the barbell rod, and approximate the weights as point masses. For small amplitude oscillations, what is the period of oscillation?
$I= MR^2+ M(2R)^2 = 5MR^2$
$\tau = -MgR\sin\theta - Mg2R\sin\theta = I\,(d^2\theta /dt^2)= -\omega^2\theta$
$\sin\theta\sim\theta$, $T=2\pi/\omega$
$T= 2\pi$ sqrt(5R/3g)

16) A spring with force constant k (N/cm) is compressed downward to a distance y0 by a projectile of mass m that is loaded onto it, as it sits in a frictionless tube. How far does the spring compress (in cm) when loaded?
y0=mg/k (definition of spring force constant)

17) A spring with force constant *k* (N/cm) is compressed downward to a distance y_0 by a projectile of mass *m* that is loaded onto it, as it sits in a frictionless tube. The spring is then compressed an additional distance *d* (in cm). Assuming that the projectile stays perfectly centered, so that it goes directly upward when released, what is the maximum height, *h,* reached, in meters? (Neglect that the spring may continue beyond its equilibrium position.)
Energy conservation: $\frac{1}{2} kd^2=mgh$

18) A spring with force constant *k* (N/cm) is in a frictionless tube under a projectile. If two additional springs (each with force constant *k*) are placed under the first spring (total of three),

and compressed a distance *d* (in cm) beyond the equilibrium position what will the velocity of the projectile be (in m/s) after release, as the combined spring reaches its equilibrium position?
Energy conservation: $\frac{1}{2} kd^2 = \frac{1}{2} mv^2$. Adding springs reduces the force constant (the displacement is divided over three of them) so $k_{eff} = k/3$
v=sqrt(kd^2/3m)

19) A soup spoon with a small ball (point mass) in it is being swung/spun around a pivot by a child; you can simplify by assuming that the handle is a rod of evenly distributed mass, and the cup has no mass. The parents have tied the ball with a string to the end of the handle because they got tired of fetching the ball. The masses of the rod and the ball are each M, the length of the rod is L and the length of the string is L. What is the new angular velocity (in degrees per second) when the ball falls out of the cup as shown, if there are no torques applied, and the angular velocity is ω before the ball falls out? (ignore gravity)
Conservation of angular momentum, combine moments of inertia
$I_0 = 1/3\ ML^2 + ML^2 = 4/3\ ML^2$
$I_1 = 1/3\ ML^2 + M(2L)^2 = 13/3\ ML^2$
$\omega_1 = 4/13\ \omega_0$

20) When a cylinder of radius R and mass M on a slope with an angle β to the horizontal undergoes pure rolling (no slip), what is the relation between the magnitudes of the center of mass velocity v_{CM} (as measured in the lab system) and the tangential velocity v_t at the surface of the cylinder (as measured in the CM system of the cylinder)?
In the lab frame, the velocity of the point of contact is zero, so the CM and tangential velocity magnitudes must be equal.

21) For a solid homogeneous cylinder with mass M, radius R, rolling (without slipping) on a frictional slope, what is the torque with respect to the center of mass provided by the frictional force (f), if the center of mass has a measured acceleration *a*?
$\tau = Rf = I\alpha = I\ a/R$ so $f = Ia/R^2$

22) For a solid homogeneous cylinder with mass M, radius R, rolling (without slipping) on a frictional slope, what is the value of the center of mass acceleration, a?

$Ma = Mg\sin\beta - f$; substitute frictional force
$Ma = Mg\sin\beta - Ia/R^2$
$a = MR^2 g\sin\beta/(MR^2+I)$
$I = \frac{1}{2} MR^2$
$a = 2/3\ g\ \sin\beta$

23) A mass and spring system is damped due to a fluid surrounding the mass. For a mass of M, a spring constant of k and a damping constant of b, how many oscillations will be completed before the amplitude drops to half of its initial value, A?
The solution will be a harmonic term multiplied by a decreasing exponential; take exponential damping term, set = to ½, take natural log of both sides. t=2m/bln2, then see how many complete periods (T) occur before this value of t
t=(2*M/b)*log(2)
T=2*pi*(sqrt(M/k))
ans = t/T % number of oscillations

24) A ship is sailing in deep water, where the (phase) velocity of the waves is given by sqrt($g\lambda/2\pi$). What is the period of these waves?

$v=\lambda/T$
$v=$ sqrt($g\lambda/2\pi$)
$T= 2\pi\ v/g$

25) A thin and hollow cylindrical metal tube has a lowest resonant frequency of f_l Hz and the first overtone of f_2 Hz. Assuming the velocity of sound in air is v_s m/s, what is the length of the cylinder, and is it open at one end or closed at both ends?
If the ratio of the lowest two frequencies is an odd number, it is open at one end (e.g. $\lambda/4$ and $3\lambda/4$)
If it is closed at both ends, it's an even number (2 for the lowest frequencies)
In this problem the ratio of frequencies is 3, so it's open at one end.
$f_l = v/4L$; $L=0.25(v/f_l)$