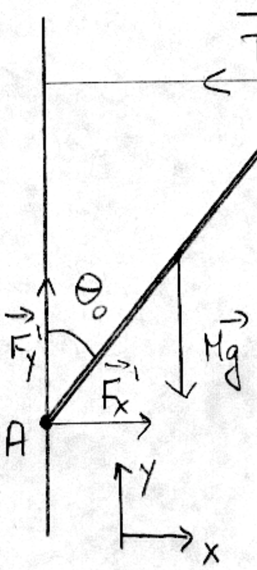
LØSNINGSFORSLAG

KONT AUG 2005 TFY4115 FYSIKK

Oppgave 1



a) Statisk likevekt:

$$\Sigma F_x = 0 \qquad \Sigma F_y = 0 \qquad \Sigma \tau_z = 0$$

$$\Rightarrow \underline{F_y' = Mg}$$

Dreiemoment om A:

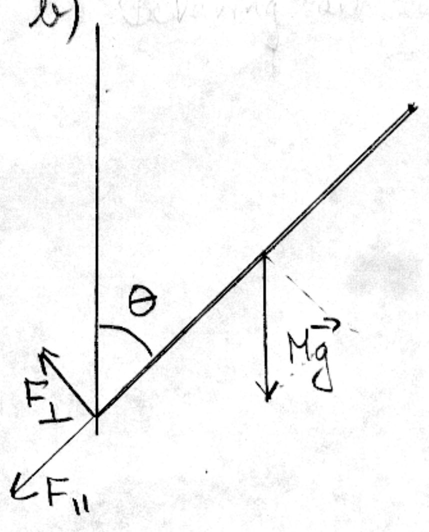
$$L \cdot T\cos\theta_0 - Mg\frac{L}{2}\sin\theta_0 = 0$$

$$\underline{\underline{T = \frac{Mg}{2}\tan\theta_0}}$$

$$F_x = T = \frac{Mg}{2}\tan\theta_0$$

$$\underline{\underline{\vec{F} = \frac{Mg}{2}\tan\theta_0\,\vec{i} + Mg\,\vec{j}}}$$

b)



Bevaring av energi:

$$E_{ki} + U_i = E_{kf} + U_f$$

$$0 + Mg\frac{L}{2}\cos\theta_0 = \frac{1}{2}I\omega^2 + \frac{MgL}{2}\cos\theta$$

$$\omega^2 = \frac{MgL}{I}\left[\cos\theta_0 - \cos\theta\right]$$

$$I_A = \frac{1}{3}ML^2 \quad \Rightarrow \omega^2 = \frac{3g}{L}\left[\cos\theta_0 - \cos\theta\right]$$

Sentripetalkrafta

$$F_c = M\frac{L}{2}\omega^2 = F_{\parallel} + (Mg)_{radiell}$$

$$F_c = \frac{3}{2}Mg(\cos\theta_0 - \cos\theta)$$

$$\Rightarrow F_{\parallel} = F_c - Mg\cos\theta = \frac{3}{2}Mg(\cos\theta_0 - \cos\theta) - Mg\cos\theta$$

$$\underline{\underline{F_{\parallel} = \frac{1}{2}Mg(3\cos\theta_0 - 5\cos\theta)}}$$

Tangensialakselerasjonen til CM:

$$a_\perp = \frac{1}{2}L\alpha$$

Komponenten av Mg $\perp$ stanga gir dreiemoment om A.
Newtons 2. lov gir da:

$$\tau_{net,A} = I\alpha = Mg\frac{L}{2}\sin\theta = \frac{1}{3}ML\alpha$$

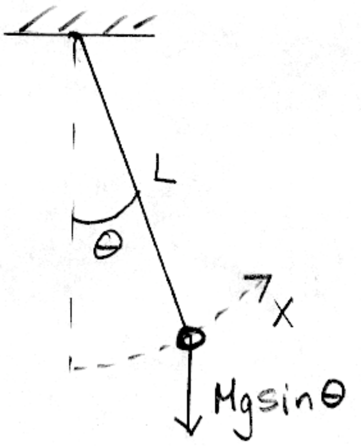
$$\Rightarrow \alpha = \frac{3}{2}\frac{g\sin\theta}{L}$$

$$\Rightarrow a_\perp = \frac{1}{2}L\alpha = \frac{3}{4}g\sin\theta = g\sin\theta - F_\perp/M$$

$$\Rightarrow \underline{\underline{F_\perp = \frac{1}{4}Mg\sin\theta}}$$

Rettet motsatt av tangensialkomp av $\vec{Mg}$.

Oppgave 2



a) Bevegelsesligningen:

$$\Sigma\vec{F} = m\cdot\vec{a}$$

$$Ma_{\parallel} = -F_{V,\parallel} - (Mg)_{\parallel}$$

$\parallel$ x-retningen.

$$\Rightarrow Ma_{\parallel} = -bv_{\parallel} - Mg\sin\theta$$

Med $\sin\theta \simeq \frac{x}{L}$ får vi:

$$M\ddot{x} + b\dot{x} + \frac{Mg}{L}x = 0$$

Med $\gamma = \frac{b}{2M}$ $\Rightarrow \underline{\underline{\ddot{x} + 2\gamma\dot{x} + \frac{g}{L} + x = 0}}$ (1)

Vise at $x(t) = Ae^{-\gamma t}\cos(\omega' t+\delta)$ er en mulig løsning:

$$\dot{x} = -A\gamma e^{-\gamma t}\cos(\omega' t+\delta) - A\omega' e^{-\gamma t}\sin(\omega' t+\delta)$$

$$\ddot{x} = \gamma^2 Ae^{-\gamma t}\cos(\omega' t+\delta) + A\omega'\gamma e^{-\gamma t}\sin(\omega' t+\delta) + A\omega'\gamma e^{-\gamma t}\sin(\omega' t+\delta)$$
$$- A\omega'^2 e^{-\gamma t}\cos(\omega' t+\delta)$$

Innsetting i (1) gir:

$$(\gamma^2-\omega'^2)Ae^{-\gamma t}\cos(\omega' t+\delta)+2\omega'\gamma Ae^{-\gamma t}\sin(\omega' t+\delta)$$
$$-2\gamma^2 Ae^{-\gamma t}\cos(\omega' t+\delta)-2\gamma\omega' Ae^{-\gamma t}\sin(\omega' t+\delta)$$
$$+\omega_0^2 Ae^{-\gamma t}\cos(\omega' t+\delta)=0 \quad \text{med} \quad \omega_0^2=\sqrt{\frac{g}{L}}$$

$$\Rightarrow \quad -\gamma^2-\omega'^2+\omega_0^2=0$$
$$\omega'^2=\omega_0^2-\gamma^2$$

$x(t)=Ae^{-\gamma t}\cos(\omega' t+\delta)$ er en løsn. dersom
$\omega'=\sqrt{\omega_0^2-\gamma^2}$

$\gamma=\frac{b}{2M}$: Dempningsfaktoren
δ = fasekonstanten
A = Amplituden ved $t=0$

$$\omega'=\sqrt{\omega_0^2-\gamma^2}=\sqrt{\frac{9,8}{4,0}s^{-2}-\frac{8,0}{2\cdot 25}s^{-2}}=1,5\,s^{-1}$$

Perioden $T=\frac{2\pi}{\omega'}=4,2\,s$

b) Startbetingelser:
$x_0=A\cos\delta$
$v_0=\dot{x}(t=0)=-\gamma A\cos\delta-\omega' A\sin\delta=-\gamma x_0-\omega' A\sin\delta$

$$\Rightarrow \quad x_0=A\cos\delta$$
$$\frac{v_0+\gamma x_0}{\omega'}=-A\sin\delta$$
Kvadrerer og summerer

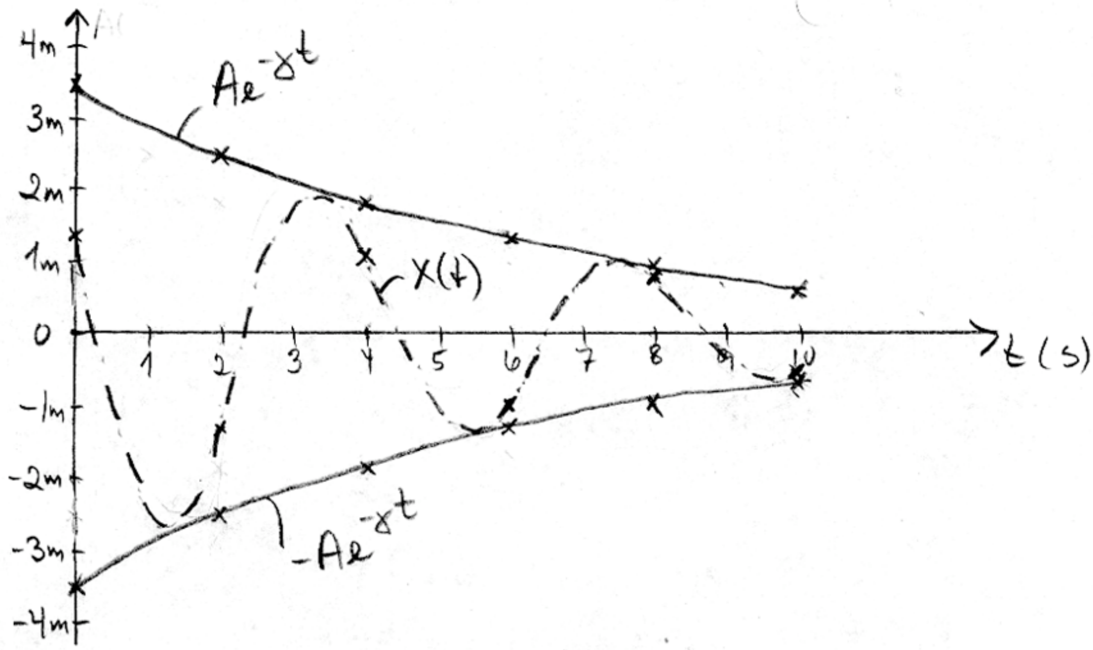
$$\Rightarrow \left(\frac{v_0+\gamma x_0}{\omega'}\right)^2+x_0^2=A^2 \quad \text{q.e.d}$$

$$\frac{\sin\delta}{\cos\delta}=-\frac{v_0+\gamma x_0}{\omega' x_0} \Rightarrow \delta=\arctan\left(-\frac{v_0+\gamma x_0}{\omega' x_0}\right) \quad \text{q.e.d}$$

$$A^2 = \left[\left(\frac{-5{,}0 + 0{,}16 \cdot 1{,}4}{1{,}51}\right)^2 + (1{,}4)^2\right] m^2$$

$A = 3.46m \simeq 3.5m$ $\qquad \delta = 66°$

t	0s	2,0s	4,0s	6,0s	8,0s	10,0s
$Ae^{-\gamma t}$	3,5m	2.5m	1.8m	1.3m	0.96m	0.70m
x	1,4m	-1.3	1.1m	-0,97m	0,89m	-0,63 m



c) Tilfellet 1 : Utsvinget er uavh. av massen

$$\Rightarrow \quad \underline{\underline{\frac{A_{etter}}{A_{før}} = 1}}$$

Tilfelle 2: Bevaring av bevegelsesmengde gir:

$$m \cdot 0 + M \cdot v = (m + M) V$$

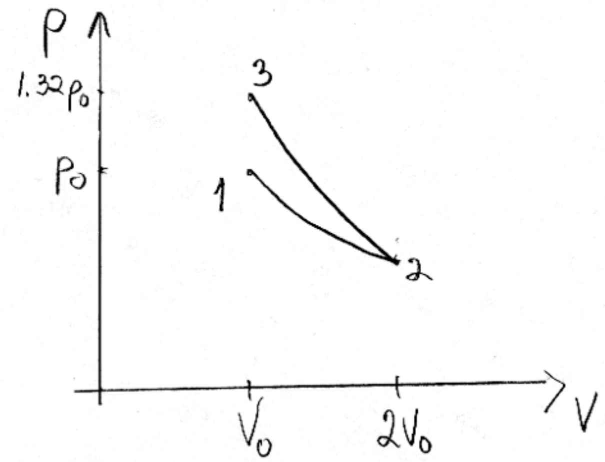
$$V = \frac{M}{(M+m)} v = \text{Ny max hastighet.}$$

$\frac{1}{2}(M+m)V^2 = \frac{1}{2} k A_{etter}^2$ ved analogi til masse-fjær system.

$$\frac{1}{2} M v^2 = \frac{1}{2} k A_{før}^2$$

$$\Rightarrow \frac{A_{etter}^2}{A_{før}^2} = \frac{(M+m)\frac{M^2}{(M+m)^2} v^2}{M v^2} \Rightarrow \underline{\underline{\frac{A_{etter}}{A_{før}} = \left(\frac{M}{M+m}\right)^{1/2}}}$$

Oppgave 3



a) i. Prosess 1 → 2 isoterm:

$$p_1 V_1 = p_2 V_2 = p_2\, 2V_0 = p_0 V_0$$

$$p_2 = \frac{1}{2} p_0$$

Prosess 2 → 3 adiabat:

$$p_2 V_2^{\gamma} = p_3 V_3^{\gamma}$$

$$\frac{1}{2} p_0 (2V_0)^{\gamma} = 1{,}32\, p_0 V_0^{\gamma}$$

$$\Rightarrow \quad 2^{\gamma} = 2{,}64$$

$$\Rightarrow \quad \gamma \ln 2 = \ln 2{,}64$$

$$\underline{\underline{\gamma}} = \frac{\ln 2{,}64}{\ln 2} = \underline{\underline{1{,}40}}$$

ii. For en toatomig gass har vi 5 frihetsgrader

$$C_V = \frac{5}{2} nRT \quad \text{og} \quad C_p = \frac{7}{2} nRT \Rightarrow \gamma = \frac{C_p}{C_V} = 1{,}40$$

⇒ Gassen i oppgaven er en toatomig gass.

iii. Frihetsgradene er translasjon av molekylet i x, y og z retning og 2 rotasjonsfrihetsgrader

og ⊥ papirplanet.

b) Arbeid i prosess 1 → 2:

$$\underline{\underline{W_{12}}} = \int_{V_1}^{V_2} p\,dV = RT_0 \int_{V_0}^{2V_0} \frac{dV}{V} = \underline{\underline{RT_0 \ln 2}}$$

Arbeidet utføres av gassen på omgivelsene.

Arbeid i prosess 2 → 3:

$$W_{23} = \Delta U \quad \text{da } \Delta Q = 0 \text{ i en adiabatisk prosess.}$$

$$RT_3 = p_3V_3 = 1{,}32\,p_0V_0 = 1{,}32\,RT_0$$
$$T_3 = 1{,}32\,T_0$$

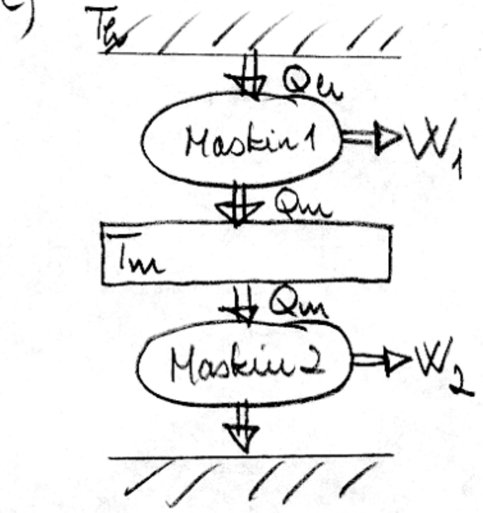
$$\Rightarrow \underline{\underline{W_{23}}} = \frac{5}{2}R\left(1{,}32T_0 - T_0\right) = \underline{\underline{0{,}32\cdot\frac{5}{2}RT_0}}$$

Arbeid utført på gassen av omgivelsene

Endring i indre energi fra punkt 1 til 3:

$$\underline{\underline{\Delta U_{13}}} = \Delta U_{23} = W_{23} = \underline{\underline{0{,}32\cdot\frac{5}{2}RT_0}}$$

c)



Virkningsgraden er definert:

$$\varepsilon_{net} = \frac{W_1 + W_2}{Q_h}$$

$$\varepsilon_1 = \frac{W_1}{Q_h} \quad ; \quad \varepsilon_2 = \frac{W_2}{Q_m}$$

$$\varepsilon_{net} = \frac{\varepsilon_1 Q_h + \varepsilon_2 Q_m}{Q_h}$$

$$\varepsilon_{net} = \varepsilon_1 + \varepsilon_2\frac{Q_m}{Q_h}$$

$$\varepsilon_1 = \frac{Q_h - Q_m}{Q_h} = 1 - \frac{Q_m}{Q_h} \Rightarrow \frac{Q_m}{Q_h} = 1 - \varepsilon_1$$

$$\Rightarrow \underline{\underline{\varepsilon_{net} = \varepsilon_1 + \varepsilon_2(1-\varepsilon_1)}} \quad q.e.d$$

For Carnotprosesser:

$$\varepsilon_1 = 1 - T_m/T_h \quad ; \quad \varepsilon_2 = 1 - \frac{T_c}{T_m}$$

$$\varepsilon_{net} = \left(1 - \frac{T_m}{T_h}\right) + \left(1 - \frac{T_c}{T_m}\right)\left(1 - 1 + \frac{T_m}{T_h}\right) = 1 - \frac{T_m}{T_h} + \frac{T_m}{T_h} - \frac{T_c}{T_h}$$

$$\Rightarrow \underline{\underline{\varepsilon_{net} = 1 - T_c/T_h}}$$

Kommentar: Resultatet er det samme som for en enkel Carnotmaskin som opererer mellom T_h og T_c.

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d) i. Netto effekt til omgivelsene:

$$P_{net} = \varepsilon\sigma A(T^4 - T_0^4) = \varepsilon\sigma A T^4\left[1-\left(\frac{T_0}{T}\right)^4\right]$$

For $T_0 = 300K$ og $T = 1673K$ blir

$$\left(\frac{T_0}{T}\right)^4 = \left(\frac{300}{1673}\right)^4 \simeq 1\cdot 10^{-3} << 1$$

$\Rightarrow$ Vi kan se bort fra innstrålingen fra omgivelsene.

$\Rightarrow$ $P_{net} \simeq \varepsilon\sigma A T^4$

ii. $$T = \left(\frac{P_{net}}{\varepsilon\sigma A}\right)^{1/4}$$

For $P_{net}' = 2P_{net}$ får vi:

$$T' = \left(\frac{2P_{net}}{\varepsilon\sigma A}\right)^{1/4}$$

$$\Rightarrow \frac{T'}{T} = (2)^{1/4}$$

$$T' = (2)^{1/4}\cdot 1673K = 1989K = 1716K$$

$\Rightarrow$ $\underline{T' \simeq 1700°C}$