

Løsningsforslag Eksamen 19. mai 2009. ①

Oppgave 1.

a) Horisontal bevegelse: $v_x = v \cdot \cos \theta$ $x(t) = v_x \cdot t$

Vertikal bevegelse: $v_y = v \cdot \sin \theta - g \cdot t$

$$y(t) = v_y \cdot t - \frac{1}{2} g t^2$$

Varighet av flykt: t_1

Fra horisontal bevegelse: $\underline{t_1 = \frac{x(t_1)}{v_x} = \frac{s_0}{v \cdot \cos \theta}}$

Fra vertikal bevegelse: $y(t_1) = h_0$

$$\Rightarrow h_0 = v_y \cdot t_1 - \frac{1}{2} g t_1^2$$

$$h_0 = v \cdot \sin \theta \cdot \frac{s_0}{v \cdot \cos \theta} - \frac{1}{2} g \frac{s_0^2}{v^2 \cos^2 \theta} \quad \left| v^2 \cos^2 \theta \cdot 2 \right.$$

$$2 v^2 \cdot h_0 \cos^2 \theta = 2 v^2 \cdot s_0 \sin \theta \cdot \cos \theta - g s_0^2$$

$$\Rightarrow v^2 = \frac{s_0^2 \cdot g}{2 \cos \theta (s_0 \sin \theta - h_0 \cos \theta)}$$

$$\Rightarrow \underline{v = s_0 \sqrt{\frac{g}{2 \cos \theta (s_0 \sin \theta - h_0 \cos \theta)}}} \quad \text{g.e.d.}$$

(2)

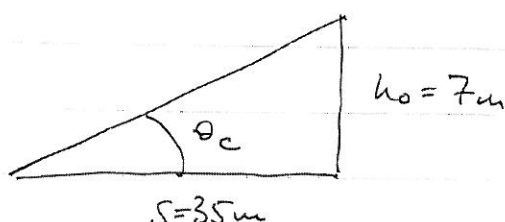
b) $v(\theta) \rightarrow \infty$ for $\theta \rightarrow \theta_c$

θ_c gitt ved at $S_0 \sin \theta_c - h_0 \cos \theta_c = 0$

$$\Rightarrow \tan \theta_c = \frac{h_0}{S_0} \Rightarrow \theta_c = \arctan\left(\frac{h_0}{S_0}\right)$$

Tallverdi: $\theta_c = \arctan\left(\frac{7.0}{35.0}\right) = 11.3^\circ$

Tolking:



For $\theta = \theta_c$ må bilen "fly" rettlinjet fra o' må treffpunktet, dvs. det kan ikke tillates noen avbøying pga. tyngdens akselerasjon, dvs. "flytiden" må gå mot null, og tilsvarende $v \rightarrow \infty$.

Hvis $\theta < \theta_c$ vil ~~ikke~~ bilen treffe $y < h_0$ selv om $v \rightarrow \infty$. Matematisk blir v da gitt som $\sqrt{-\text{tall}^2}$ dvs. et imaginært tall. Begge deler angir at problemet ikke har løsning.

c) Fra grafen estimeres $v(\theta)$ å ha minimalverdi for $\theta \approx 50^\circ$

$$v_{\min} = 35 \text{ m} \sqrt{\frac{9.81 \text{ m/s}^2}{2 \cdot \cos 50^\circ [35 \text{ m} \cdot \sin 50^\circ - 7 \text{ m} \cdot \cos 50^\circ]}} = 35 \sqrt{\frac{9.81}{28.68}} \text{ m/s}$$

(3)

$$\underline{v_{min} = 35.0 \cdot 0,585 \text{ m/s} = 20,5 \text{ m/s} = 74 \text{ km/t}}$$

Dette synes ikke å være en "svært høy fart", men er allikevel langt over fartsgransen for fellingsskade. Fartsgransen på stedet er ukjent.

- d) Kollisjonskraften F stammer bilen over strekkningen $s = 2 \text{ m}$.
Hastighet like før kollisjonen setter til v_1 .

$$\Rightarrow F \cdot s = \frac{1}{2} m v_1^2 \quad \text{fra energi betraktning.}$$

Finnes v_1 fra energi betraktning:

$$\frac{1}{2} m v_{min}^2 = \frac{1}{2} m v_1^2 + m g h_0$$

$$\Rightarrow \frac{1}{2} m v_1^2 = \frac{1}{2} m v_{min}^2 - m g h_0$$

$$\Rightarrow F = \frac{1}{2} m v_1^2 / s = \frac{m}{s} \left[\frac{v_{min}^2}{2} - g h_0 \right] = \frac{1400 \text{ kg}}{2 \text{ m}} \left[\frac{1}{2} \cdot 20,5^2 - 9,81 \cdot 7 \right] \frac{\text{m}^2}{\text{s}^2}$$

$$\underline{F = 99 \text{ kN}}$$

Alternativt: $t_1 = \frac{s_0}{v_{min} \cos \theta} = \frac{35 \text{ m}}{20,5 \text{ m/s} \cdot \cos 50^\circ} = \frac{35 \text{ m}}{12,8 \text{ m/s}} = 2,72 \text{ s}$

$$v_x = v_{min} \cos \theta = 20,5 \cdot \cos 50^\circ = 12,8 \text{ m/s}$$

$$v_y = v \cdot \sin \theta - g \cdot t_1 = (20,5 \cdot \sin 50^\circ - 9,81 \cdot 2,72) \text{ m/s} = -11,0 \text{ m/s}$$

$$v_1 = \sqrt{v_x^2 + v_y^2} = \sqrt{164 + 121} \text{ m/s} = \sqrt{285} \text{ m/s} = 16,9 \text{ m/s}$$

④

Energiebetrachtung son. für:

$$F \cdot s = \frac{1}{2} m v_1^2$$

$$\Rightarrow \underline{F} = \frac{m}{2s} \cdot v_1^2 = \frac{1400 \text{ kg} \cdot 12,85^2 \frac{\text{m}^2}{\text{s}^2}}{2 \cdot 2 \text{ m}} = \underline{99750 \text{ N}}$$

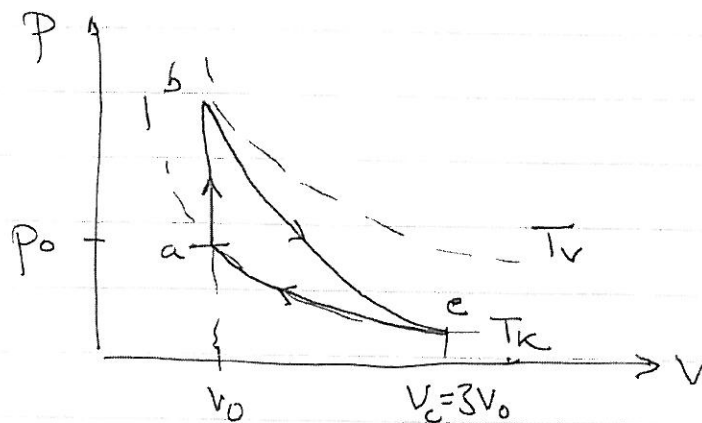
$$\underline{F \approx 100 \text{ kN}} \quad (\text{mit Faktorenreduktion})$$

Kollisionszeit t_2 bestimmen ^{über} Impuls

$$F \cdot t_2 = |\Delta p| = m v_1$$

$$\underline{t_2} = \frac{m v_1}{F} = \frac{1400 \text{ kg} \cdot 16,9 \frac{\text{m}}{\text{s}}}{99750} = \underline{0,24 \text{ s}}$$

Oppgave 2.



Diatomus gas, $\gamma = \frac{C_p}{C_v} = \frac{7}{5} = 1.4$.

- a) Gitt $P_0, V_0, V_c = 3V_0, T_k = T_0$
 Berett T_v, P_b, P_c

Isoterm c-a: $\frac{P_a V_a}{T_a} = \frac{P_c V_c}{T_c}$

Innsatt, kjente verdier: $\frac{P_0 V_0}{T_0} = \frac{P_c \cdot 3V_0}{T_0}$

$$\Rightarrow \underline{\underline{P_c = \frac{1}{3} P_0}}$$

Adiabatt b-c: $T_b V_b^{(\gamma-1)} = T_c V_c^{(\gamma-1)}$

Innsatt kjente verdier $T_b \cdot V_0^{(\gamma-1)} = T_0 \cdot V_0^{(\gamma-1)} \cdot 3^{(\gamma-1)}$

$$\Rightarrow \underline{\underline{T_b \equiv T_v = T_0 \cdot 3^{(\gamma-1)}}}$$

Tilstandsligning: $\frac{P_b \cdot V_b}{T_b} = \frac{P_c V_c}{T_c}$

(6)

Ansatt kvote verdi: $\frac{p_b \cdot V_0}{T_0 \cdot 3^{(\gamma-1)}} = \frac{\frac{1}{3} p_0 \cdot 3 \cdot V_0}{T_0}$

$$\Rightarrow \underline{\underline{p_b = p_0 \cdot 3^{(\gamma-1)}}}$$

b) Arbeid utført i hver av delprosessene:

Isokor a → b: $\underline{\underline{W_{ab} = \int_a^b p \cdot dV = 0}}$ fordi $V = \text{konstant}$

Adiabatt b → c: $Q_{bc} = \Delta U + W_{bc} = 0$

$$\Rightarrow W_{bc} = -\Delta U = -[nC_V \cdot \Delta T] = nC_V [T_b - T_c]$$

Fra hint: $C_V = \frac{R}{(\gamma-1)}$

$$\Rightarrow \underline{\underline{W_{bc} = \frac{nRT_b - nRT_c}{(\gamma-1)} = \frac{1}{(\gamma-1)} [p_b V_b - p_c V_c]}}$$

$$W_{bc} = \frac{1}{(\gamma-1)} [p_0 \cdot 3^{(\gamma-1)} \cdot V_0 - \frac{1}{3} p_0 \cdot 3 V_0]$$

$$\underline{\underline{W_{bc} = \frac{p_0 V_0}{(\gamma-1)} [3^{(\gamma-1)} - 1]}}$$

Isoterm c → a: $W_{ca} = \int_c^a p \cdot dV$

$$p \cdot V = nRT = \text{konstant} \Rightarrow p = \frac{nRT_0}{V}$$

(7)

$$\underline{\underline{W_{ca} = \int_c^a nRT_0 \cdot \frac{dV}{V} = nRT_0 \cdot \ln \frac{V_a}{V_c} = -nRT_0 \cdot \ln 3}}$$

$$p_0 \cdot V_0 = nRT_0$$

$$\Rightarrow \underline{\underline{W_{ca} = -p_0 V_0 \ln 3}}$$

c) Totalarbeld for et helt rullep:

$$W_{\text{net}} = W_{ab} + W_{bc} + W_{ca} = 0 + \frac{p_0 V_0}{(\gamma-1)} [3^{\gamma-1} - 1] - p_0 V_0 \ln 3$$

$$\underline{\underline{W_{\text{net}} = p_0 V_0 \left[\frac{3^{\gamma-1} - 1}{(\gamma-1)} - \ln 3 \right]}} \quad \text{f.z.d.}$$

Tallver:

$$\left\{ \frac{3^{\gamma-1} - 1}{\gamma-1} - \ln 3 \right\} = \left[\frac{3^{0,4} - 1}{0,4} - 1,0986 \right] = 0,28$$

$$\Rightarrow \underline{\underline{W_{\text{net}} = 2,0 \cdot 10^5 \frac{\text{N}}{\text{m}^2} \cdot 2,0 \cdot 10^{-3} \text{m}^3 \cdot 0,28 = 1,12 \cdot 10^2 \text{Nm} = 112 \text{J}}}}$$

$$P = \frac{W_{\text{net}}}{t} = \frac{W_{\text{net}}}{1/t} = f \cdot W_{\text{net}} = \frac{1000 \frac{\text{sykl}}{\text{min}}}{60 \frac{\text{s}}{\text{min}}} \cdot 112 \text{J} = \underline{\underline{1870 \text{W}}}$$

d) Tilførte varmemengder.

Prüfen thermodynamikkens 1. lov. $\underline{\underline{Q = \Delta U + W}}$

$$Q_{ab} = \Delta U + W_{ab} \quad W_{ab} = 0$$

$$\Rightarrow Q_{ab} = \Delta U = U_b - U_a = nC_v(T_b - T_a) = nC_v(T_v - T_0)$$

(8)

$$Q_{ab} = n \frac{R}{(\gamma-1)} (T_v - T_0) = \frac{1}{(\gamma-1)} (nRT_v - nRT_0)$$

$$\underline{Q_{ab}} = \frac{1}{(\gamma-1)} [p_b V_0 - p_0 V_0] = \underline{\frac{p_0 V_0}{(\gamma-1)} [3^{(\gamma-1)} - 1]}$$

$$\underline{Q_{ab}} = \frac{400 \text{ J}}{0.4} [0.5518] = \underline{552 \text{ J}} \quad (\text{absorbiert})$$

$$\underline{Q_{bc}} = 0 \quad \text{p. def. an adiabat.}$$

$$Q_{ca} = \Delta U + W_{ca} \quad \Delta U = 0 \quad \text{gg. isotherm.}$$

$$\Rightarrow \underline{Q_{ca}} = W_{ca} = - \underline{p_0 V_0 \cdot \ln 3} \quad (< 0)$$

$$\underline{Q_{ca}} = -400 \text{ J} \cdot \ln 3 = \underline{-439 \text{ J}} \quad (=: \underline{\text{abgittvarme}})$$

e) Wirkungsgrad: $\varepsilon = \frac{W_{\text{net}}}{Q_H} = \frac{W_{\text{net}}}{Q_{ab}}$

$$\underline{\varepsilon} = \frac{p_0 V_0 \left[\frac{3^{(\gamma-1)} - 1}{(\gamma-1)} - \ln 3 \right]}{p_0 V_0 \left[\frac{3^{(\gamma-1)} - 1}{(\gamma-1)} \right]} = \underline{1 - \frac{\ln 3 (\gamma-1)}{3^{(\gamma-1)} - 1}}$$

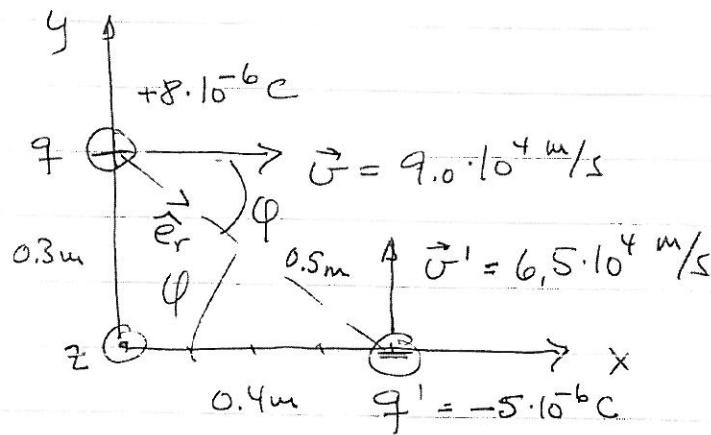
$$\underline{\varepsilon} = 1 - \frac{\ln 3 \cdot 0.4}{0.5518} = \underline{0.20}$$

Für Carnot prozess:

$$\underline{\varepsilon_c} = 1 - \frac{T_c}{T_H} = 1 - \frac{T_0}{T_0 \cdot 3^{(\gamma-1)}} = 1 - \frac{1}{3^{(\gamma-1)}} = \underline{1 - 3^{(1-\gamma)}}$$

$$\underline{\varepsilon_c} = 1 - 0.644 = \underline{0.36}$$

(9)

Oppgave 3.

$$r = \text{Avst. } qq'$$

$$\underline{\underline{r = \sqrt{0,3^2 + 0,4^2} \text{ m} = 0,5 \text{ m}}}$$

a) Magnetfelt B generert av q i pos. for q' :

$$\vec{B} = \frac{\mu_0}{4\pi} \cdot q \frac{(\vec{v} \times \hat{e}_r)}{r^2}$$

$\hat{e}_r$ enhetsvektor fra kilde (q) i retning P (der q')

$$\underline{|\vec{v} \times \hat{e}_r| = v \cdot 1 \cdot \sin\phi = \frac{3}{5}v}$$

Tallverdi: $|B| = \frac{\mu_0}{4\pi} \cdot q \frac{v \cdot \sin\phi}{r^2}$

$$\underline{|B| = \frac{4\pi \cdot 10^{-7} \text{ N/A}^2}{4\pi} \cdot \frac{8 \cdot 10^{-6} \text{ As} \cdot 9 \cdot 10^4 \text{ m/s} \cdot \frac{3}{5}}{(0,5 \text{ m})^2} = 173 \cdot 10^{-9} \text{ T}}$$

Retning: ved høyre h nds regel blir $\vec{v} \times \hat{e}_r = -v \cdot k$
der. retning inn i papirplanet

$\Rightarrow$ I posisjon for q' : $\underline{\underline{\vec{B} = -173 \cdot 10^{-9} \text{ T } k}}$

(10)

Kraftvirkning på q' i et magnetfelt B :

$$\vec{F} = q'(\vec{v}' \times \vec{B}) \quad \vec{v}' \perp \vec{B}$$

$$\vec{F} = q' v' B [\hat{j} \times (-\hat{k})] = -5 \cdot 10^{-6} \text{ C} \cdot 6,5 \cdot 10^4 \text{ m/s} \cdot (-173 \cdot 10^{-9} \text{ T}) \hat{i}$$

$$\underline{\underline{\vec{F} = 5,6 \cdot 10^3 \cdot 10^{-11} \text{ N} = 56 \text{ nN } \hat{i}}}$$

$$\vec{v}' \times \vec{B} = v' B [\hat{j} \times (-\hat{k})] = -v' B \hat{i}$$

Kraften virker i positiv x-retning. Siden $q' < 0 \Rightarrow q'(\vec{v}' \times \vec{B})$
 gir $F_x > 0$.

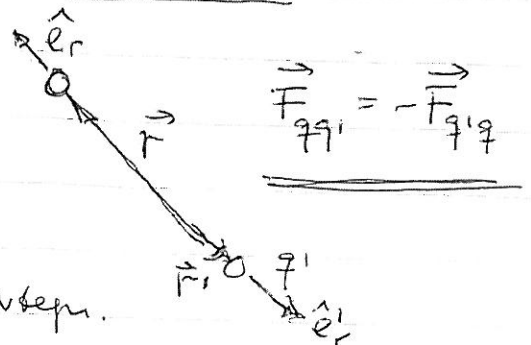
b) Elektrostatisk kraft har retning langs $\vec{r}$.

$$\text{Kraft fra } q' \text{ på } q: \vec{F}_q = \frac{q'q}{4\pi\epsilon_0} \cdot \frac{1}{r^2} \hat{e}_r$$

$$\vec{F}_q = \frac{-8 \cdot 10^{-6} \cdot 8 \cdot 10^{-6} \text{ C}^2}{4\pi \cdot 8,85 \cdot 10^{-12} \text{ C}^2/\text{Nm}^2} \cdot \frac{1}{(0,5 \text{ m})^2} \hat{e}_r$$

$$\underline{\underline{\vec{F}_q = -1,44 \cdot 10^{-12+12} \text{ N} \hat{e}_r = -1,44 \text{ N} \hat{e}_r}}$$

$\vec{r}$ har retning fra q' til q



Kraften virker tiltrekkende fordi
 q og q' har motsatt ladningstegn.

$$\text{Tilsvarende kraft på } q': \underline{\underline{\vec{F}_{q'} = -1,44 \text{ N} \hat{e}_r'}}$$

tiltrekkende

c) Verdien av elektrostatiske potensial generert av q i posisjon for q' (i avstand r)

$$V(r) = \frac{q}{4\pi\epsilon_0} \cdot \frac{1}{r}$$

$$= \frac{8 \cdot 10^{-6} \text{ C}}{4\pi \cdot 8,85 \cdot 10^{-12} \frac{\text{C}^2}{\text{Nm}^2}} \cdot \frac{1}{0,5 \text{ m}}$$

$$\underline{V(r)} = 0,144 \cdot 10^6 \text{ V} = \underline{\underline{0,144 \text{ MV}}}$$

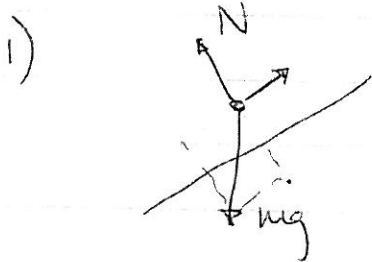
Elektrostatiske potensiell energi av systemet

$$U = \sum_{i < j} \frac{q_i q_j}{4\pi\epsilon_0 r_{ij}} = \frac{q q'}{4\pi\epsilon_0 r} = q' \cdot V$$

$$\Rightarrow \underline{\underline{U}} = 0,144 \cdot 10^6 \text{ V} \cdot (-5,0 \cdot 10^{-6} \text{ C}) = \underline{\underline{-0,72 \text{ J}}} < 0$$

Potensiell energi er negativ fordi ladningene har motsett fortegn og ikke er i ∞ avstand fra hverandre.

Oppgave 4. Flervalgssporsmål



var: C (3)

2) $W = \vec{F} \cdot \vec{s}$

$$W = (-3\hat{i} + 6\hat{j} - 9\hat{k}) \cdot (6\hat{i} - 4\hat{j} + 2\hat{k}) \hat{J}$$

$$\underline{W} = (-18 - 24 - 18) \hat{J} = \underline{-60 \hat{J}} \quad \text{var: } \underline{E}$$

3) $E = \frac{1}{2} k A^2$

$$\Rightarrow 2E = \frac{1}{2} k (A\sqrt{2})^2 \quad \text{var: } \underline{C} \quad \text{faktor } 1.4 = \sqrt{2}$$

4) $F = ma = -kx$

$$\text{maks } |a| \text{ for maks } |x| \quad \text{var: } \underline{D} \quad (4)$$

5) $\frac{1}{2} m \vec{v}^2 = \frac{3}{2} kT$

$$\langle K \rangle = \frac{3}{2} kT$$

var: B

$$m_{O_2} > m_{N_2}$$

$$\Rightarrow v_{O_2} < v_{N_2}$$

6)

$$\langle K \rangle = \frac{3}{2} kT$$

$$\frac{\langle K_1 \rangle}{\langle K_2 \rangle} = \frac{T_1}{T_2} = \frac{293}{333} = 0,88$$

Svar: D

(Se siste side).

7)

 U, S, T uav. av prosesseri Q, W avh. $\rightarrow, -$ $\Rightarrow$ Svar: A

8)

$$Q_E = \int \vec{E} \cdot d\vec{A} = \frac{Q_{\text{ind}}}{\epsilon_0} = \frac{29-9}{\epsilon_0} = \frac{9}{\epsilon_0}$$

 $\Rightarrow$ Svar: A

9)

$$V_{ab} = 12V \quad t = 2,5s \quad I = 60A$$

$$W = P \cdot t = V_{ab} \cdot I \cdot t = 12 \cdot 60 \cdot 2,5 J = 1800 J$$

Svar: A

10)

$$\Phi_m = 6t^2 + 7t + 1$$

$$\mathcal{E} = - \frac{d\Phi_m}{dt}$$

$$\mathcal{E} = - \frac{d\Phi_m}{dt} = - [12t + 7]$$

$$t = 2s \Rightarrow$$

$$\Rightarrow |\mathcal{E}| = + [12 \cdot 2 + 7] V = 31V$$

Svar: D

Flervalgsoppgave 6

14

$T = I\alpha = Tr$ - ligninger for dreiemoment på hjulet.

$T - mg = -ma$ - N2 for loddet.

$$\Rightarrow a = -\frac{T}{m} + g = -\frac{I\alpha}{mr} + g$$

Snor festet til hjul $\Rightarrow a = \alpha r$ ("ikke-sleli")

Dermed

$$a \left(1 + \frac{I}{mr^2} \right) = g$$

$$a = g \left(1 + \frac{I}{mr^2} \right)^{-1} = 9.81 \frac{\text{m}}{\text{s}^2} \left(1 + \frac{0.50 \text{ kg m}^2}{15 \text{ kg} (0.5 \text{ m})^2} \right)^{-1}$$

$$= 8.7 \text{ m/s}^2$$

Svar: B.