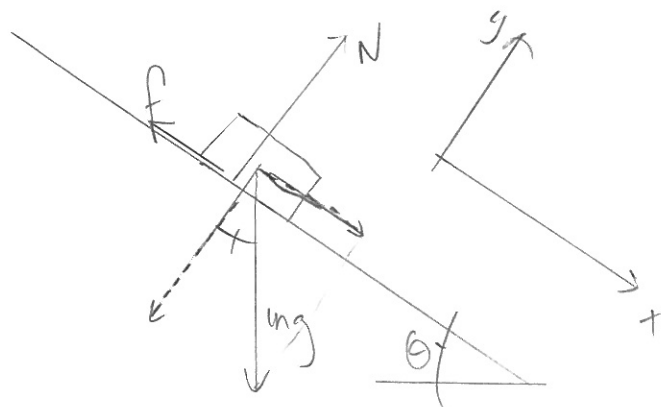


1a)



$$\sum \vec{F} = m\vec{a}.$$

y-retning: $N - mg \cos \theta = 0 \Rightarrow N = mg \cos \theta$

x-retning: $mg \sin \theta - f = ma, f = \mu_k N$

Vi eliminerer f og finner

$$\underline{a = g(\sin \theta - \mu_k \cos \theta)}$$

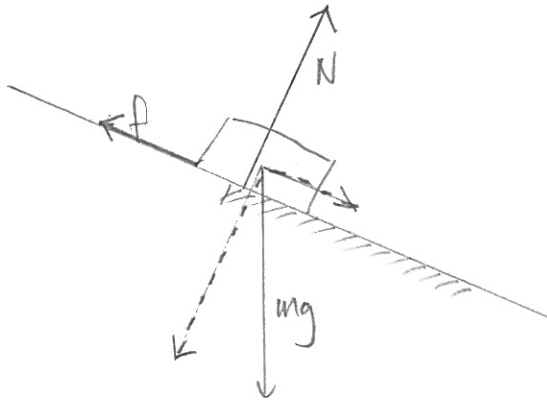
Det er rimelig å anta at friksjonen skalerer med kontaktarealet mellom blokken og skråplanet's ru del. For $x \leq 0$ er dette arealet null, mens for $x > s$ er ^{kontakt-}arealet lik blokkens areal, som er dens bredde b • lengde s .

Dermed fås: $x \leq 0$: $\mu_{\text{eff}} = 0$

$x \in (0, s)$: $\mu_{\text{eff}} = Cx, C = \frac{\mu_k}{s}$

$x > s$: $\mu_{\text{eff}} = \mu_k$

b) $a=0$, dvs $\sum F = 0$.



$$x = x_0. \quad \mu_{\text{eff}} = \frac{x_0}{s} \mu_k.$$

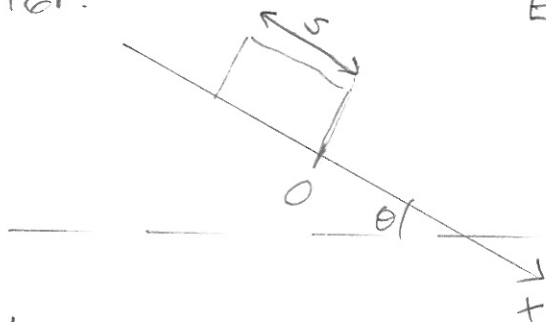
$$g(\sin \theta - \mu_{\text{eff}} \cos \theta) = 0$$

$$\mu_{\text{eff}} = \frac{x_0}{s} \mu_k = \frac{\sin \theta}{\cos \theta}$$

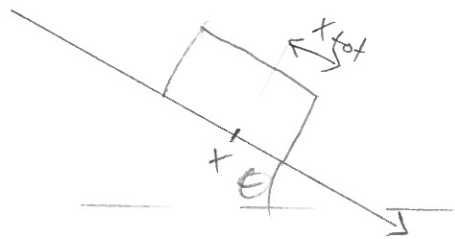
$$x_0 = \frac{1}{\mu_k} \tan \theta s.$$

c) Energibetraktninger

Før:



Efter:



Endring i potentiell energi: $\Delta U = -mgh = -mg x_{\text{tot}} \sin \theta$

Endring i kinetisk energi: $\Delta K = 0.$

Endring i termisk energi: $W_{\text{lost}} = \int_0^{x_{\text{tot}}} f_{\text{eff}} dx = \int_0^{x_{\text{tot}}} \mu_{\text{eff}} mg \cos \theta dx$

$$= \int_0^{x_{\text{tot}}} \frac{x}{s} \mu_k mg \cos \theta dx = \frac{1}{2s} \mu_k mg \cos \theta x_{\text{tot}}^2$$

1c forts.

Tapet av potensiell energi er lik økningen i termisk energi:

$$\cancel{mg} x_{\text{tot}} \sin \theta = \frac{1}{2s} \mu_k \cancel{mg} \cos \theta x_{\text{tot}}$$

$$\underline{\underline{2 \frac{1}{\mu_k} \tan \theta s = x_{\text{tot}}}} \quad \underline{\underline{\text{evt. } x_{\text{tot}} = 2x_0}}$$

$$x_{\text{tot}} \leq s:$$

$$2 \frac{1}{\mu_k} \tan \theta s \leq s$$

$$\underline{\underline{\mu_k \geq 2 \tan \theta}}$$

Oppg 2

$$a) V_D = \frac{\sigma}{2\epsilon_0} \left(\sqrt{x^2 + R^2} - x \right), \quad x > 0$$

Ladning Q på skive: $Q = \pi R^2 \sigma$

$$V_D = \frac{Q}{2\pi\epsilon_0 R^2} \left(\sqrt{x^2 + R^2} - x \right)$$

$$V_R = \frac{\lambda}{2\epsilon_0} \frac{R}{\sqrt{x^2 + R^2}}, \quad x \in (-\infty, \infty)$$

Ladning Q på ring: $Q = 2\pi R \lambda$

$$V_R = \frac{Q}{4\pi\epsilon_0} \frac{1}{\sqrt{x^2 + R^2}}$$

$$\begin{aligned} b) \quad V_R^\infty &= \frac{Q}{4\pi\epsilon_0} \frac{1}{\sqrt{x^2 + R^2}} = \frac{Q}{4\pi\epsilon_0} (x^2 + R^2)^{-\frac{1}{2}} \\ &= \frac{Q}{4\pi\epsilon_0} \left(1 + \frac{R^2}{x^2} \right)^{-\frac{1}{2}} (x^2)^{-\frac{1}{2}} \\ &\stackrel{\approx}{\underset{(\text{sm } x)}}{=} \frac{Q}{4\pi\epsilon_0} \frac{1}{x} \left(1 - \frac{R^2}{2x^2} \right) \quad // 2. \text{ orden} \\ &\stackrel{\approx}{=} \frac{Q}{4\pi\epsilon_0} \frac{1}{x} \quad // 1. \text{ orden.} \end{aligned}$$

$$\begin{aligned} V_D^\infty &= \frac{Q}{2\pi\epsilon_0 R^2} \left(\sqrt{x^2 + R^2} - x \right) \\ &= \frac{Q}{2\pi\epsilon_0 R^2} \left(x \left(1 + \frac{R^2}{x^2} \right)^{\frac{1}{2}} - x \right) \\ &\approx \frac{Q}{2\pi\epsilon_0 R^2} x \left(1 + \frac{R^2}{2x^2} - 1 \right) \\ &= \frac{Q}{4\pi\epsilon_0} \frac{1}{x} = V_R^\infty, \quad \text{som forventet fra Gauss' lov.} \end{aligned}$$

$$2c) \quad V_R = \frac{Q}{4\pi\epsilon_0} \frac{1}{\sqrt{x^2 + R^2}}$$

$$\text{Kraft } \vec{F} = q\vec{E}, \quad \vec{E} = -\nabla V = -\frac{\partial V}{\partial x} \hat{x}$$

$$\vec{E} = -\frac{\partial}{\partial x} \left(\frac{Q}{4\pi\epsilon_0} \frac{1}{\sqrt{x^2 + R^2}} \right) \hat{x} = \frac{(-Q)}{4\pi\epsilon_0} \frac{(-1)(x^2 + R^2)^{-\frac{3}{2}}}{2x} \hat{x}$$

$$= \frac{Qx}{4\pi\epsilon_0} \frac{1}{(x^2 + R^2)^{3/2}} \hat{x}$$

$$\vec{F} = q\vec{E} = \frac{eQx}{4\pi\epsilon_0} \frac{1}{(x^2 + R^2)^{3/2}} \hat{x}$$

$$= \frac{(-1,6 \cdot 10^{-19})(1,0 \cdot 10^{-9})}{4\pi \cdot 8,85 \cdot 10^{-12}} \frac{0,2}{(0,2^2 + 0,3^2)^{3/2}} \hat{x} = \underline{\underline{-6,1 \cdot 10^{-18} \text{ N} \hat{x}}}$$

(rettet mot ringen)

$$2d) \quad W = \Delta U = e\Delta V = e(V_f - V_i)$$

$$= \frac{eQ}{4\pi\epsilon_0} \left(\frac{1}{\sqrt{x_f^2 + R^2}} - \frac{1}{\sqrt{x_i^2 + R^2}} \right)$$

$$= \frac{(-1,60 \cdot 10^{-19})(1,0 \cdot 10^{-9})}{4\pi \cdot 8,85 \cdot 10^{-12}} \left(\frac{1}{\sqrt{1,0^2 + 0,3^2}} - \frac{1}{\sqrt{0,2^2 + 0,3^2}} \right) \text{ J}$$

$$\underline{\underline{= 2,6 \cdot 10^{-18} \text{ J}}}$$

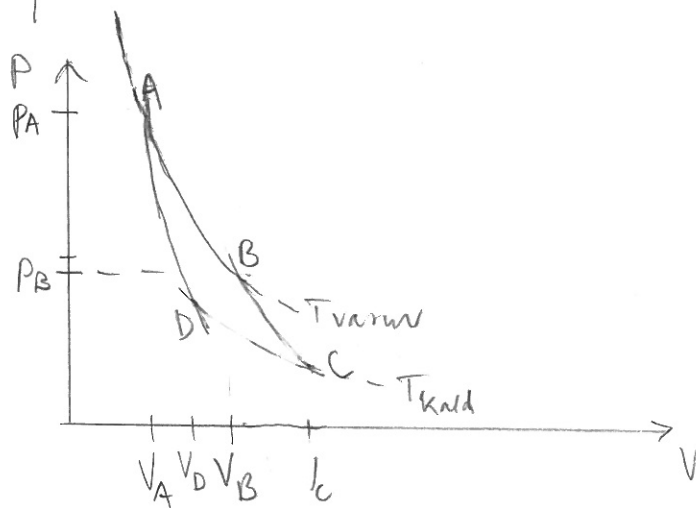
Arbeidet tilsvarer endring i kinetiske energi:

$$\frac{1}{2}mv^2 = W$$

$$v = \sqrt{\frac{2W}{m}} = \underline{\underline{2,4 \cdot 10^6 \text{ m/s}}} \quad (\ll c, \text{ ok!})$$

Oppg 3. Varmløstmeslin.

b)



a) P_B .

$$pV = nRT \Rightarrow P_A V_A = P_B V_B$$

$$P_B = P_A \frac{V_A}{V_B} = \frac{1}{2} P_A = \underline{\underline{50.5 \text{ kPa}}}$$

P_C

$$P_B V_B^\gamma = P_C V_C^\gamma$$

$$P_C = P_B \left(\frac{V_B}{V_C} \right)^\gamma = P_A \frac{1}{2} \left(\frac{2V_A}{3V_A} \right)^\gamma = P_A \frac{2^{\gamma-1}}{3^\gamma}$$

$$\underline{\underline{P_C = 25.7 \text{ kPa}}}$$

$$= P_A \frac{2^{2/3}}{3^{5/3}} \approx 0.25 P_A$$

V_A

$$P_A V_A = nRT_A, n=1, T_A=400\text{K}, P_A=1.01 \cdot 10^5 \text{ Pa}$$

$$V_A = \frac{RT_A}{P_A} = \frac{8.314 \cdot 400}{1.01 \cdot 10^5} \text{ m}^3 = \underline{\underline{32.9 \text{ dm}^3}}$$

T_C

$$P_C V_C = nRT_C$$

$$T_C = \frac{P_C V_C}{nR} = P_A \frac{2^{\gamma-1}}{3^\gamma} \frac{3 \left(\frac{RT_A}{P_A} \right)}{R}$$

$$= T_A \left(\frac{2}{3} \right)^{\frac{2}{3}}$$

$$\underline{\underline{T_C = 305 \text{ K}}}$$

3c) Ved D med begge ligningene være oppfylt:

$$P_C \frac{V_C}{V_D} = P_A \left(\frac{V_A}{V_D} \right)^\gamma$$

$$\cancel{P_A} \frac{2^{\gamma-1} \cancel{3} V_A}{3^\gamma V_D} = \cancel{P_A} \left(\frac{V_A}{V_D} \right)^\gamma$$

$$V_D^{\gamma-1} = V_A^{\gamma-1} \frac{3^{\gamma-1}}{2^{\gamma-1}}$$

$$\underline{\underline{V_D = \frac{3}{2} V_A = 49.4 \text{ L}}}$$

$$P_D V_D = n R T_D, \quad T_D = T_C = T_A \left(\frac{2}{3} \right)^{\gamma-1}$$

$$P_D = \frac{R T_D}{V_D} = R T_A \left(\frac{2}{3} \right)^{\gamma-1} \frac{1}{\frac{3}{2} V_A}$$

$$= R T_A \left(\frac{2}{3} \right)^\gamma \cdot \frac{1}{V_A}$$

$$\underline{\underline{= P_A \left(\frac{2}{3} \right)^{\frac{5}{3}} \approx 0.5}}$$

$$3d) Q_{warm} = Q_{AB}$$

$$Q = \Delta U + W, \quad \Delta U = 0 \text{ fordi } U = U(T) \text{ for ideell gas.}$$

$$Q = W = \int_A^B P(V) dV = \int_A^B \frac{nRT_A}{V} dV = nRT_A \ln \frac{V_B}{V_A}$$

$$= nRT_A \ln 2 = \underline{\underline{p_A V_A \ln 2}}$$

$$= 1,01 \cdot 10^5 \cdot 32,9 \cdot 10^{-3} \ln 2 \text{ J}$$

$$= \underline{\underline{2,3 \text{ kJ}}}$$

3e) Virkningsgrad for Carnotprosess,

$$\epsilon_c = 1 - \frac{T_{kald}}{T_{varm}} = 1 - \frac{T_C}{T_A} = 1 - \left(\frac{2}{3}\right)^{\frac{2}{3}} \approx \underline{\underline{0,24}}$$

$$\epsilon = \frac{W_{net}}{Q_{varm}}$$

$$W_{net} = \epsilon Q_{varm} = \epsilon_c Q_{varm}$$

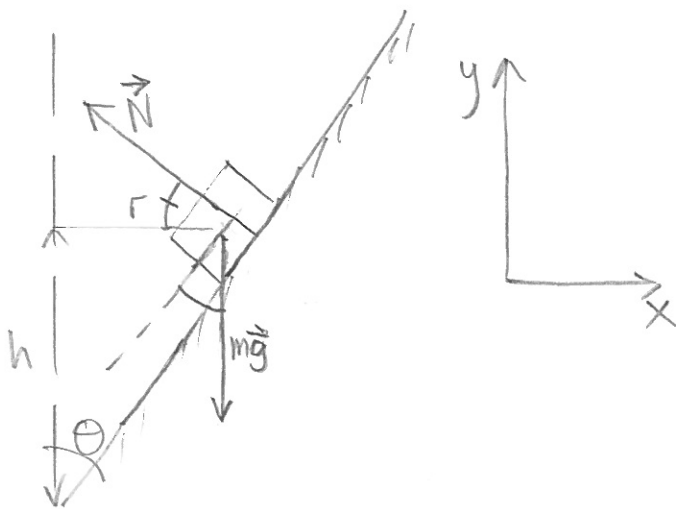
$$= \left(1 - \left(\frac{2}{3}\right)^{\frac{2}{3}}\right) p_A V_A \ln 2$$

$$\approx \underline{\underline{0,16 p_A V_A}}$$

$$= \underline{\underline{546 \text{ J}}}$$

Oppg 4

To krefter virker på klossen: tyngdekraften og
normalkraften.



Radialt er klossen akselerert, $a_x = -\frac{v^2}{r}$.

Kraftbalanse:

Ingen akselerasjon i y-retning, $a_y = 0$.

$$\Rightarrow \sum F_y = 0: \quad \underline{-mg + N \sin \theta = 0.}$$

$\therefore N = \frac{mg}{\sin \theta}$

$$\underline{\underline{\sum F_x = ma_x = -N \cos \theta = -m \frac{v^2}{r}}}$$

Eliminerer N: $\frac{mg}{\sin \theta} \cos \theta = m \frac{v^2}{r} \quad (*)$

Potensiell energi $U = mgh$

Kinetisk energi $K = \frac{1}{2}mv^2$

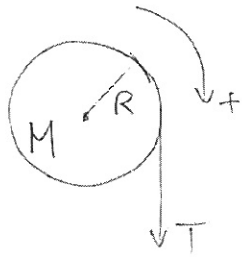
$$\frac{U}{K} = \frac{mgh}{\frac{1}{2}mv^2}$$

Vi ser at $\frac{r}{h} = \tan \theta$ (fra figur)
og $v^2 = \frac{gr}{\tan \theta}$ (fra *).

Dermed:

$$\frac{U}{K} = \frac{g \frac{r}{\tan \theta}}{\frac{1}{2} \frac{gr}{\tan \theta}} = \underline{\underline{2}}$$

Oppg 5

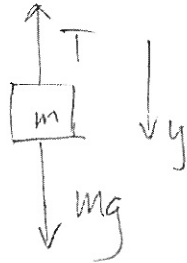


$$\sum \tau = J\alpha, \quad J = \frac{1}{2}MR^2$$

$$\tau = RT$$

$$M = \pi R^2 H \rho$$

$$\alpha = \frac{a}{R} \quad (\text{"non-slip"})$$



$$mg - T = ma$$

eliminer T: $T = m(g-a)$

Dermed, $m(g-a)R = \frac{1}{2}(\pi R^2 H \rho)R^2 \frac{a}{R}$

Løser m/lip ρ :

$$\rho = \frac{2m(g-a)}{a\pi R^2 H} = 2 \frac{m}{\pi R^2 H} \frac{g-a}{a}$$

Med $a = \frac{g}{2}$, $H = 0,30\text{m}$, $R = 0,10\text{m}$ og $m = 18,0\text{kg}$ fås

$$\rho = \frac{2 \cdot 18 \cdot \frac{9,81}{2}}{3,14(0,1)^2 \cdot 0,3 \cdot \frac{9,81}{2} \text{ m}^3}$$

$$= \underline{3,82 \cdot 10^3 \text{ kg/m}^3}$$

$$= \underline{3,82 \text{ g/cm}^3}$$