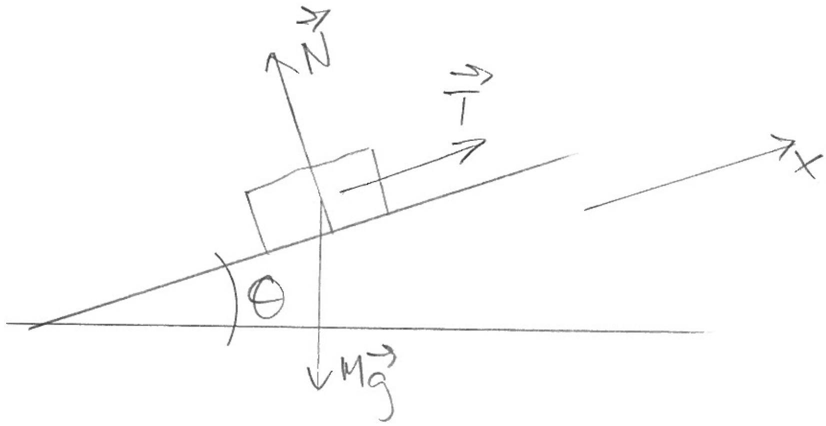
Løsningsforslag, kont. TFY 4125 Aug. 2012.

Oppg 1.



a) "$2as = v^2 - v_0^2$" Her: $v_0 = 0$.

$$a = \frac{v^2}{2s}$$

$$\sum F = ma \Rightarrow T - Mg\sin\theta = M\frac{v^2}{2s}$$

$$T = M\frac{v^2}{2s} + Mg\sin\theta$$

$$= M\left(\frac{v^2}{2s} + g\sin\theta\right)$$

b) $$W(x) = \int \vec{F}(\vec{s})\cdot d\vec{s} = T\int_0^x dx'$$

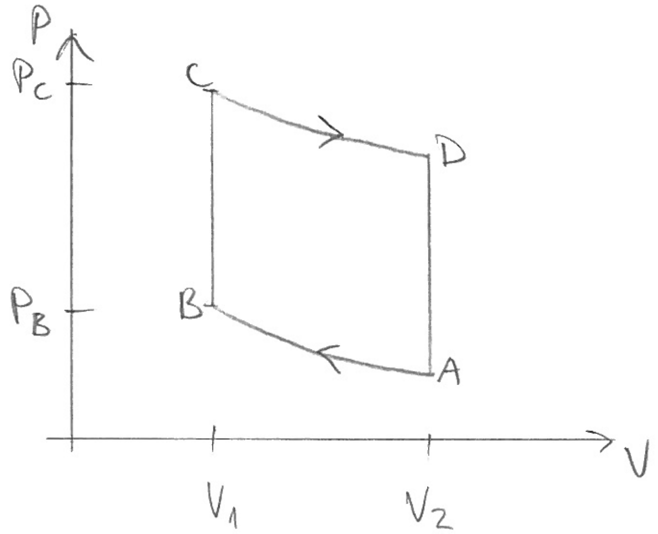
$$= \frac{1}{2}Mv^2\frac{x}{s} + Mgx\sin\theta$$

$x = s$:

$$W(s) = \frac{1}{2}Mv^2 + Mgs\sin\theta$$

$$= \Delta K + \Delta U$$ jmf arbeid - kinetisk energi - teoremet.

Oppg 2.



$$\frac{P_C}{P_B} = 3{,}00$$

$$r \equiv \frac{V_2}{V_1}$$

a) $\underline{T_B}$

$$T_A V_A^{\gamma-1} = T_B V_B^{\gamma-1}$$

$$\gamma = \frac{C_P}{C_V} = \frac{7}{5}$$

$$\left(\begin{aligned} C_V &= \tfrac{5}{2} R \\ C_P &= C_V + R = \tfrac{7}{2} R \end{aligned} \right)$$

$$T_B = T_A \frac{V_A^{\gamma-1}}{V_B^{\gamma-1}}$$

$$\underline{\underline{= T_A \, r^{\gamma-1}}}$$

$\underline{T_C}$

$$\frac{P_B \cancel{V_B}}{T_B} = \frac{P_C \cancel{V_C}}{T_C}$$

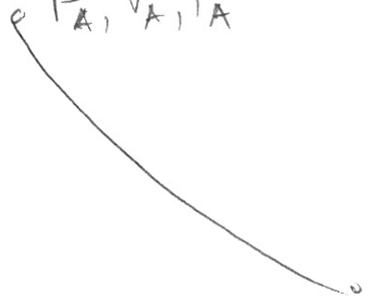
$$T_C = T_B \frac{P_C}{P_B} \underline{\underline{= 3 T_A \, r^{\gamma-1}}}$$

$\underline{T_D}$

$$T_C V_C^{\gamma-1} = T_D V_D^{\gamma-1}$$

$$T_D = T_C \frac{V_C^{\gamma-1}}{V_D^{\gamma-1}} = 3 T_A r^{\gamma-1} \frac{1}{r^{\gamma-1}} \underline{\underline{= 3 T_A}}$$

2b) Arbejd langs adiabat.



$$W_{AB} = \int_A^B P(V)\,dV, \quad PV^\gamma = \text{konst.}$$

$$PV^\gamma = P_A V_A^\gamma$$

$$P(V) = P_A V_A^\gamma V^{-\gamma}$$

$$W_{AB} = \int_{V_A}^{V_B} P_A V_A^\gamma V^{-\gamma}\,dV = P_A V_A^\gamma \frac{1}{-\gamma+1}\left(V^{-\gamma+1}\right)_{V_A}^{V_B}$$

$$= P_A V_A^\gamma \frac{1}{-\gamma+1}\left(V_B^{-\gamma+1} - V_A^{-\gamma+1}\right)$$

$$= \frac{1}{-\gamma+1}\left(P_A V_A^\gamma V_B^{-\gamma+1} - P_A V_A\right)$$

$$= \frac{1}{-\gamma+1}\left(P_B V_B^\gamma V_B^{-\gamma+1} - P_A V_A\right)$$

$$= \frac{1}{1-\gamma}\left(P_A V_A - P_B V_B\right)$$

Oppg 2c

Adiabatisk prosess: $Q = 0$, $\Delta U = -W$

Isokor prosess $\Delta V = 0 \Rightarrow W = 0$. $Q = \Delta U \cancel{+ W}$

Dermed: det gjøres arbeid av systemet, $W_1 = W_{CD}$, fra C til D

— " — på systemet, $W_2 = W_{AB}$, fra A til B.

Det tilføres varme i isokoren BC;

$$Q_2 = \Delta U_{BC} = C_V(T_C - T_B) > 0$$

Det avgis varme i isokoren DA;

$$Q_1 = \Delta U_{DA} = C_V(T_A - T_D) < 0$$

$$W_1 = W_{CD} = -\Delta U_{CD} = -C_V(T_D - T_C) > 0$$

$$W_2 = W_{AB} = -\Delta U_{AB} = -C_V(T_B - T_A) < 0$$

(Å bruke $\int P dV$ gir samme resultat, men krever mye mer regning).

Generelt:

$$\varepsilon = \frac{\text{netto arbeid}}{\text{varme tilført}} = \frac{W_1 + W_2}{Q_2} = \frac{Q_1 + Q_2}{Q_2} = 1 + \frac{Q_1}{Q_2}.$$

Her:

$$\varepsilon = 1 + \frac{\cancel{C_V}(T_A - T_D)}{\cancel{C_V}(T_C - T_B)} = 1 + \frac{T_A - 3T_A}{3T_A r^{\gamma-1} - T_A r^{\gamma-1}}$$

$$= 1 - \frac{2}{2r^{\gamma-1}} = 1 - r^{1-\gamma}$$

For $r = 10$ og $\gamma = 7/5$ fås $\varepsilon = 0.60$

Oppg 2d)

Maksimal virkningsgrad: Carnot.

$$\varepsilon_{max} = \varepsilon_c = 1 - \frac{T_{kald}}{T_{varm}}$$

Her er $T_{kald} = T_A$ og $T_{varm} = T_C$

$$\varepsilon_{max} = 1 - \frac{T_A}{T_C} = 1 - \frac{\cancel{T_A}}{3\cancel{T_A}\, r^{\gamma-1}}$$

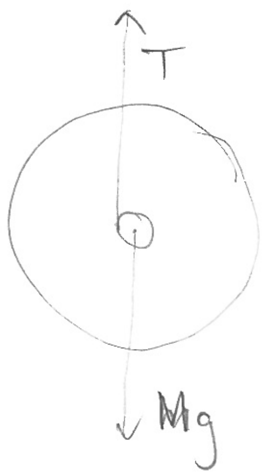
$$= 1 - \frac{1}{3} r^{1-\gamma}$$

Tallsvar:

$$\varepsilon_{max} = 1 - \frac{1}{3} 10^{1-\frac{7}{5}}$$

$$= 1 - \frac{1}{3} 10^{-\frac{2}{5}} \approx 0.87$$

Oppg 3



a) $I_0 = 2I_{skive} = \cancel{2}\cdot\frac{1}{\cancel{2}}\frac{M}{2}R^2 = \frac{1}{2}MR^2$

Tallsvar: $I_0 = \frac{1}{2}40\,kg\left(\frac{1,5}{2}\right)^2 m^2$

$= 11,25\,kgm^2$

To krefter virker på yo-yo'en, tyngdekraft og snordrag

I fall-retning: $Ma = Mg - T$. T er ukjent.

N2 for rotasjon. Vi kjenner Mg men ikke T, så vi velger å bruke Steiners sats slik at T elimineres:

$$Mgr = I\alpha,\quad I = I_0 + Mr^2$$

$$Mgr = \left(\frac{1}{2}MR^2 + Mr^2\right)\frac{a}{r}$$

$$a = \frac{\cancel{M}gr^2}{\cancel{M}\left(\frac{1}{2}R^2 + r^2\right)} = g\frac{r^2}{\frac{1}{2}R^2 + r^2}$$

$a \simeq 0,34\ m/s^2$ ("langsom!")

Oppg 3 forts

b) "$v^2 - v_0^2 = 2as$"

$$v = \sqrt{2as}$$

$$= \sqrt{2g\frac{2r^2}{R^2+2r^2}s} = 2\sqrt{\frac{gs}{2+\frac{R^2}{r^2}}}$$

Tallsvar:

$$v = \sqrt{2 \cdot 9.8\,\frac{m}{s^2}\,\frac{2 \cdot 0.1^2}{0.75^2+2\cdot 0.1^2}\,5m} = \overset{1,84}{\cancel{1,31}}\ m/s$$

$$T = Mg - Ma = M(g-a)$$

$$= M\left(g - g\frac{2r^2}{R^2+2r^2}\right)$$

$$= Mg\frac{R^2 \cancel{+2r^2} \cancel{-2r^2}}{R^2+2r^2} =$$

$$= Mg\frac{R^2}{R^2+2r^2}$$

Tallsvar: $40kg\ 9,81\ m/s^2\ \frac{0,75^2}{0,75^2+2\cdot 0,1^2}$

$$\simeq 380\ N$$

(Tilnærmet lik Mg, som forventet med stort treghetsmoment)

Oppg 3b, alternativ metode: energibevarelse.

$$Mgs = \frac{1}{2} M v^2 + \frac{1}{2} I_0 \omega^2 , \quad I_0 = \frac{1}{2} M R^2$$

$$\omega = \frac{v}{r}$$

$$\cancel{M}gs = \frac{1}{2} \cancel{M} v^2 + \frac{1}{2}\frac{1}{2} \cancel{M} R^2 \frac{v^2}{r^2}$$

$$gs = v^2 \left(\frac{1}{2} + \frac{1}{4} \frac{R^2}{r^2} \right)$$

$$\underline{\underline{v = 2 \sqrt{\frac{gs}{2 + R^2/r^2}}}}$$

Oppg 4.

a) Likevekt: $\sum \vec{F} = \vec{0}$

$\sum \vec{\tau} = \vec{0}$

Netto kraft og netto dreiemoment må være lik null.

Stabil likevekt: det virker en gjenopprettende kraft slik at systemet returnerer til likevekt etter en perturbasjon.

To krefter virker på kula: elektrostatisk frastøting og gravitasjonskraft.

Ved $y = y_0$ er $\sum \vec{F}$ lik null:

$$\frac{1}{4\pi\varepsilon_0} \frac{Qq}{y_0^2} = mg$$

$$y_0 = \sqrt{\frac{Qq}{4\pi\varepsilon_0} \frac{1}{mg}}$$

b) En liten dytt Δy ...

$$\sum F = \frac{Qq}{4\pi\varepsilon_0}\frac{1}{(y_0+\Delta y)^2} - mg = F_0 + \Delta F$$

$$\Downarrow$$

$$\Delta F = \frac{Qq}{4\pi\varepsilon_0}\frac{1}{y_0^2+2y_0\Delta y+\underbrace{\cancel{\Delta y^2}}_{\simeq 0}} - \frac{Qq}{4\pi\varepsilon_0 y_0^2}$$

gjenopprettende kraft:

$\Delta y > 0 \Rightarrow \Delta F < 0$

$\Delta y < 0 \Rightarrow \Delta F > 0$

På felles brøkstrek:

$$\Delta F = \frac{Qq}{4\pi\varepsilon_0}\left(\frac{y_0^2}{y_0^2(y_0^2+2y_0\Delta y)} - \frac{y_0^2+2y_0\Delta y}{y_0^2(y_0^2+2y_0\Delta y)}\right)$$

$$= -\frac{Qq}{4\pi\varepsilon_0}\frac{2y_0\Delta y}{y_0^4+2y_0^3\Delta y} = \frac{-Qq}{4\pi\varepsilon_0}\frac{2y_0\Delta y}{y_0^4\left(1+2\frac{\Delta y}{y_0}\right)}$$

$$\approx \frac{-Qq}{4\pi\varepsilon_0}\frac{2\Delta y}{y_0^3}$$ Gjenopprettende!

$$= -\frac{2mg}{y_0}\Delta y$$ (v. hj.a svaret i a))

N2: $\cancel{m}\dfrac{d^2(\Delta y)}{dt^2} = -2\cancel{m}g\dfrac{\Delta y}{y_0}$

∴ svingelikning med $\omega = \sqrt{\dfrac{2g}{y_0}}$