

Eksamen TFY 4165

09.08.2018

Løsningforslag

Öppgäve 1

a) $Z = Z_1^N$
 $Z_1 = e^{-\beta \epsilon_1} + e^{-\beta \epsilon_2} + e^{-\beta \epsilon_3}$

$$\epsilon_1 = 0, \quad \epsilon_2 = \epsilon, \quad \epsilon_3 = -\epsilon$$

$$Z_1 = 1 + e^{-\beta \epsilon} + e^{\beta \epsilon}$$

$$= \underline{1 + 2 \cosh(\beta \epsilon)}$$

b) $\langle \epsilon \rangle = - \frac{\partial \ln Z_1}{\partial \beta} = - \frac{\partial}{\partial \beta} \ln(1 + 2 \cosh \beta \epsilon)$

$$= - \frac{1}{1 + 2 \cosh \beta \epsilon} \cdot 2 \epsilon \sinh(\beta \epsilon)$$

$$= - 2 \epsilon \frac{\sinh \beta \epsilon}{1 + 2 \cosh \beta \epsilon}$$

$$\beta \rightarrow 0 : \quad \cosh(\beta \epsilon) \approx 1 ; \quad \sinh \beta \epsilon \approx \beta \epsilon$$

$$\underline{\langle \epsilon \rangle \approx - \frac{2}{3} \epsilon \beta^2 \rightarrow 0}$$

$$\beta \rightarrow \infty : \quad \cosh(\beta \epsilon) \gg 1 \Rightarrow$$

$$\langle \epsilon \rangle \approx - 2 \epsilon \frac{\sinh \beta \epsilon}{2 \cosh \beta \epsilon} \approx - \epsilon \tanh \beta \epsilon$$
$$\approx \underline{\underline{- \epsilon}}$$

c)

$$C_V = \left(\frac{\partial U}{\partial T} \right)_V$$

Per partikkel:

$$C_V = \left(\frac{\partial \epsilon}{\partial T} \right)_V$$

$$= -k_B \beta^2 \frac{\partial \epsilon}{\partial \beta}$$

$$= k_B \beta^2 2\epsilon \left\{ \frac{\epsilon \cosh(\beta\epsilon)}{1 + 2 \cosh \beta\epsilon} - \frac{2\epsilon \sinh(\beta\epsilon) \cosh(\beta\epsilon)}{(1 + 2 \cosh \beta\epsilon)^2} \right\}$$

$$= \underline{\underline{2 k_B (\beta\epsilon)^2 \frac{(2 + \cosh(\beta\epsilon))}{(1 + 2 \cosh \beta\epsilon)^2}}}$$

Lågtemperatur-gren: $\beta\epsilon \rightarrow \infty$

$$C_V \approx k_B (\beta\epsilon)^2 e^{-\beta\epsilon} \rightarrow 0$$

Alle partiklene er samlet i laveste energinivå. Her er det ingen energifluktuasjoner, grunntilstanden har ikke noe til å legge varme.

Svant er i samsvar med chw. partisjonnprinsipp, da system ikke har noen kvadratiske frihetsgrader.

Oppgave 2

a	:	A
b	:	D
c	:	C
d	:	A
e	:	D
f	:	D
g	:	C
h	:	C

Oppgave 3

a) 2D fartfordeling

$$F_{2D}(v) = 2\pi v dv \frac{m}{2\pi k_B T} e^{-\frac{m}{2} v^2}$$

$$v^2 = v_x^2 + v_y^2$$

$$\langle v \rangle = \int_0^\infty dv v F_{2D}(v)$$

$$= 2\alpha \underbrace{\int_0^\infty dv v^2 e^{-\alpha v^2}}_{\equiv I(\alpha)} ; \alpha \equiv \frac{m}{2k_B T}$$

$$= 2\alpha I(\alpha)$$

$$I(\alpha) = -\frac{\partial}{\partial \alpha} \int_0^\infty dv e^{-\alpha v^2} = -\frac{\partial}{\partial \alpha} \left(\frac{1}{2} \sqrt{\frac{\pi}{\alpha}} \right)$$

$$= \frac{1}{4} \frac{\sqrt{\pi}}{\alpha^{3/2}}$$

$$\langle v \rangle = 2\alpha \frac{1}{4} \sqrt{\frac{\pi}{\alpha}} \frac{1}{\alpha} = \sqrt{\frac{\pi}{4\alpha}}$$

$$= \sqrt{\frac{\pi}{2}} \sqrt{\frac{k_B T}{m}}$$

$$\langle v^2 \rangle = \langle v_x^2 \rangle + \langle v_y^2 \rangle$$

$$= \int_0^\infty dv v^2 F_{2D}(v)$$

$$= 2\alpha \int_0^\infty dv v^3 e^{-\alpha v^2} ; \alpha \equiv \frac{m}{2k_B T}$$

$$\begin{aligned}
 &= \alpha \int_0^{\infty} dx \, x \, e^{-\alpha x} \\
 &= \alpha \left(-\frac{d}{d\alpha} \right) \int_0^{\infty} dx \, e^{-\alpha x} \\
 &= \alpha \left(-\frac{d}{d\alpha} \right) \frac{1}{\alpha} = \frac{1}{\alpha}
 \end{aligned}$$

$$\underline{\underline{\langle v^2 \rangle = 2 \frac{k_B T}{m}}}$$

$$\underline{\underline{\langle v^2 \rangle - \langle v \rangle^2 = \langle (v - \langle v \rangle)^2 \rangle \geq 0}}$$

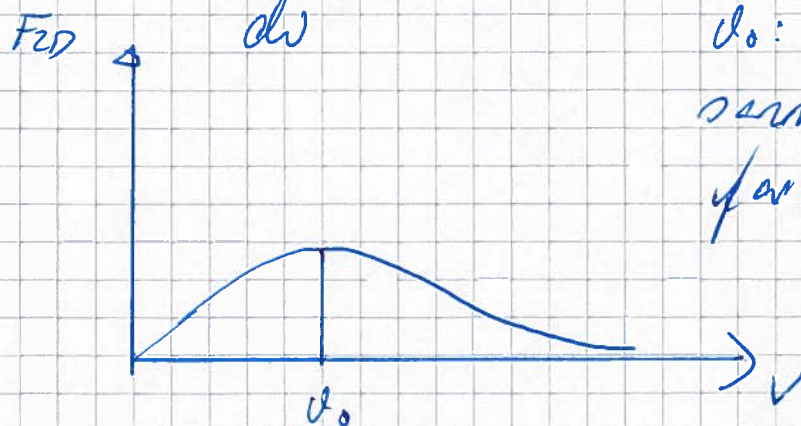
b) Fartfordelingen:

$$F_{2D}(v) dv = \frac{m}{2\pi k_B T} 2\pi v dv e^{-\frac{m}{2k_B T} v^2}$$

Den mest sannsynlige farten finnes ved å sjå for hvor v $F_{2D}(v)$ er størst. Det får vi fra

likninga

$$\frac{dF_{2D}}{dv} = 0$$



da: Most sannsynlige farten.

$$\frac{dF_{ED}}{dv} = K \left(e^{-\alpha v^2} - 2\alpha v^2 e^{-\alpha v^2} \right)$$

$$= 0 \quad \Rightarrow$$

$$1 - 2\alpha v_0^2 = 0$$

$$v_0 = \sqrt{\frac{1}{2\alpha}} = \sqrt{\frac{k_B T}{m}}$$

$$v_0 = \sqrt{\frac{1.38 \cdot 10^{-23} \cdot 300}{5.314 \cdot 10^{-26}}}$$

$$= \underline{\underline{279 \text{ m/s}}}$$

$$c) \quad \langle v_x^2 \rangle = \sqrt{\frac{m}{2\pi k_B T}} \int_{-\infty}^{\infty} dv_x v_x^2 e^{-\frac{m}{2k_B T} v_x^2}$$

$$= \sqrt{\frac{m}{2\pi k_B T}} \cdot \frac{1}{2} \sqrt{\pi} \frac{2k_B T}{m} \sqrt{\frac{2k_B T}{m}}$$

$$= \frac{k_B T}{m} \Rightarrow \underline{\underline{\langle E_{kin} \rangle = \frac{1}{2} k_B T}}$$

$$\langle v_x \rangle = 0$$

$$\underline{\underline{\langle p_x \rangle = 0}}$$