LF kont 2019

1) $\underline{\underline{E}}$ 2) $\underline{\underline{\mu_{JT} = 0}}$ 3) Sykllisk regel $\Leftrightarrow$ $\frac{\alpha_p \chi_T}{\alpha_V} = \frac{1}{p} \Leftrightarrow \underline{\underline{p = \frac{RT}{V}}}$

4) $$\chi_T = -\frac{1}{V}\left(\frac{\partial V}{\partial p}\right)_{T,N}$$

$$\frac{p}{kT} = \frac{N}{V} + \frac{N^2}{2V^2}$$

$$\frac{dp}{kT} - \frac{p\,dT}{kT^2} = \frac{dN}{V} - \frac{N\,dV}{V^2} + \frac{N\,dN}{V^2} - \frac{N^2\,dV}{V^3}$$

konstant T og N:

$$\frac{dp}{kT} = -\frac{N\,dV}{V^2}\left(1 + \frac{N}{V}\right)$$

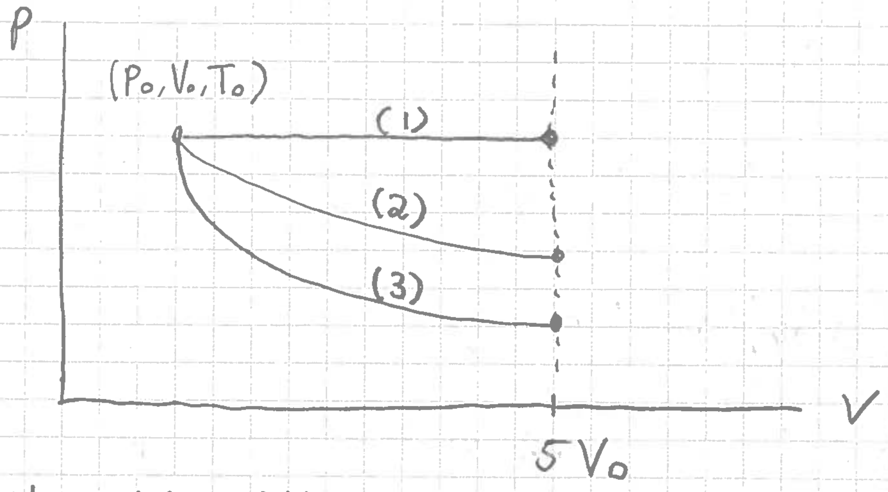
$$\Rightarrow \left(\frac{\partial V}{\partial p}\right)_{T,N} = \frac{-1}{kT} \frac{1}{\frac{N}{V^2}\left(1 + \frac{N}{V}\right)}$$

$$\Rightarrow \underline{\underline{\chi_T = \frac{1}{kT}\frac{V}{N}\left(\frac{1}{1 + N/V}\right)}}$$

5) Arbeid = all energi som krysser grenseflaten som

<u>ikke</u> skyldes ΔT.

6)



$$\underline{\underline{W_P > W_T > W_S}}$$

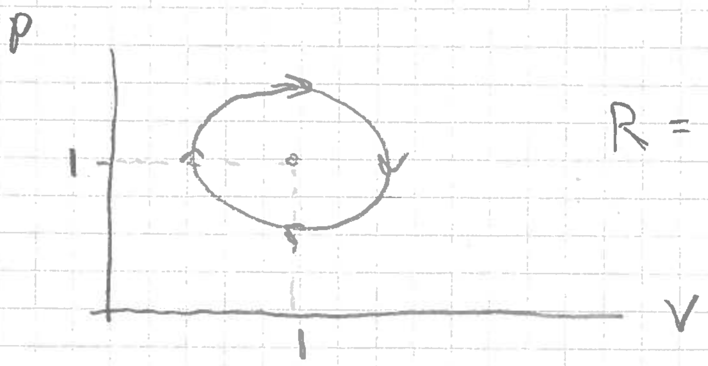
7) $C_P - C_V = Nk$

$$\underline{\underline{C_P = \frac{5}{2} Nk}}$$

8) Adiabat = konstant entropi

$\Rightarrow$ $\underline{\underline{\Delta S = 0}}$

9)



$R = 1/2$

$$W = \pi R^2 = \underline{\underline{\frac{\pi}{4}}}$$

10) $\eta_c = 1 - \frac{T_{minimum}}{T_{maksimum}}$

Ideell gass $PV = nRT$

$nRT = (1 + \frac{1}{2}\sin\varphi)(1 + \frac{1}{2}\cos\varphi)$

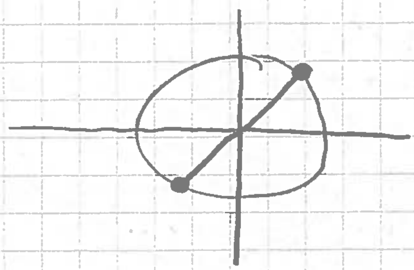
For å finne $T_{max/min}$ må vi kreve $dT/d\varphi = 0$

$$nR\frac{dT}{d\varphi} = (\tfrac{1}{2}\cos\varphi)(1+\tfrac{1}{2}\cos\varphi) + (1+\tfrac{1}{2}\sin\varphi)(-\tfrac{1}{2}\sin\varphi)$$

$$= \frac{1}{2}(\cos\varphi - \sin\varphi) + \frac{1}{4}(\cos^2\varphi - \sin^2\varphi) = 0$$

T_{max} : $\varphi = \pi/4$

T_{min} : $\varphi = 5\pi/4$



$$nRT_{min} = \left(1 - \frac{1}{2\sqrt{2}}\right)^2$$

$$nRT_{max} = \left(1 + \frac{1}{2\sqrt{2}}\right)^2$$

$$\eta_c = 1 - \frac{T_{min}}{T_{max}} \simeq \underline{\underline{77\,\%}}$$

11) ΔS = tilst. funksjon $= 0$

12) $dS > 0$ i irrev. prosess.

13) En isotrop funksjon er retningsuavhengig

$$F(x,y,z) = F(\vec{r}) = F(|\vec{r}|)$$

$$\boxed{F(v_x, v_y, v_z) = A(v_x^2 + v_y^2 + v_z^2)}$$

14) $$v_{rms} = \sqrt{\langle v^2 \rangle} = \left(\int_0^\infty v^2 f(v)\, dv \right)^{1/2}$$

$$= \dots = \underline{\underline{\sqrt{\frac{3kT}{m}}}}$$

15) $\underline{\underline{C}}$

16) 2D : $f_{translasjon} = \cancel{3}\ 2$

$f_{rot} = 1$

$\left.\begin{array}{l} f_{vib} = 1 \\ f_{pot.\ energi} = 1 \end{array}\right\}$ bidrar ikke ved romtemperatur

$$C_v = \frac{1}{2} k (2+1) = \underline{\underline{\frac{3}{2} k}}$$

17) $F = NkT \ln\left(\frac{N\lambda^3}{V}\right) - NkT$

$= -NkT \ln V$ + irrelevante ledd (uten V)

$p = -\left(\frac{\partial F}{\partial V}\right)_T = \underline{\underline{\frac{NkT}{V}}}$ = ideell gass

18) $p\sqrt{V} = AT$

$H = U + pV = pV \Rightarrow U = 0 \Rightarrow \underline{C_V = 0}$

$C_p - C_V = T\left(\frac{\partial p}{\partial T}\right)_V \left(\frac{\partial V}{\partial T}\right)_p$

$= T \frac{\partial}{\partial T}\left\{\frac{AT}{\sqrt{V}}\right\} \frac{\partial}{\partial T}\left\{\frac{A^2T^2}{p^2}\right\}$

$= T\left\{\frac{A}{\sqrt{V}}\right\}\left\{2\frac{A^2T}{p^2}\right\} = \left\{\frac{A}{\sqrt{V}}\right\} 2 \{V\}$

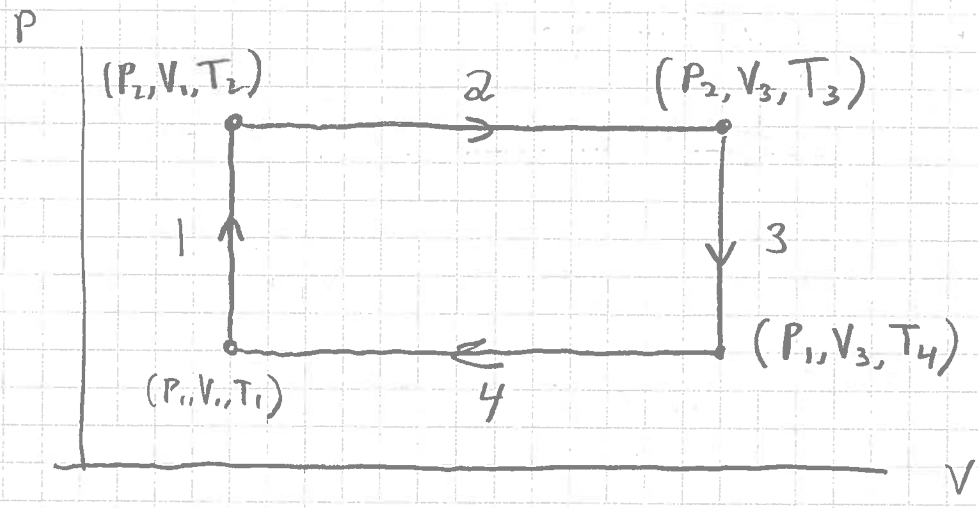
$= \underline{\underline{2A\sqrt{V}}}$

integrerer over alternativ rev. prosess.

19) $\Delta S = \int dS = \int \underbrace{C_V}_{0} \frac{dT}{T} + \underbrace{\left(\frac{\partial p}{\partial T}\right)_V}_{\frac{A}{\sqrt{V}}} dV$

$= 2A \left[\sqrt{V}\right]_{V_0}^{V_1} = \underline{\underline{2A\left(\sqrt{V_1} - \sqrt{V_0}\right)}}$

20)



$$dQ = dU + p\,dV$$

Isochor: $dQ = dU \Rightarrow \Delta Q = C_V \Delta T$

Isobar: $dQ = dU + \underbrace{p}_{\text{konstant}}\,dV \Rightarrow \Delta Q = C_V \Delta T + p\,\Delta V$

$$\Delta Q = C_V \Delta T + nR\,\Delta T = \Delta T \underbrace{(C_V + nR)}_{C_P}$$

$$= C_P\,\Delta T$$

$Q_1 = C_V (T_2 - T_1) > 0$

$Q_2 = C_P (T_3 - T_2) > 0$

$Q_3 < 0$

$Q_4 < 0$

$W = \text{Areal} = (P_2 - P_1)(V_3 - V_1)$

$$\eta = \frac{W}{Q_{inn}} = \frac{(P_2 - P_1)(V_3 - V_1)}{C_V(T_2 - T_1) + C_P(T_3 - T_2)}$$

21) $dS > 0 \Leftrightarrow$ minimer F for å oppnå LV

$\Rightarrow$ Tilst. ligning følger fra $dF = 0$

$$\boxed{A \Leftrightarrow B \Rightarrow E}$$

22) $$dS = \underbrace{C_V \frac{dT}{T}}_{\text{Isoterm} = 0} + \underbrace{\left(\frac{\partial P}{\partial T}\right)_V}_{\frac{nR}{V}} dV$$

$$\Delta S = \int dS = nR \int_{V_1}^{V_2} \frac{dV}{V} = nR \ln \frac{V_2}{V_1} > 0$$

30. juni 2021

23) Siden vannmengdene har samme temperatur T_0 er det ingen temperatur-utjevning i prosessen.

Da prosessen er irreversibel må vi integrere langs en alternativ rev. vei (Isobar)

$$\Delta S_1 = \int_{T_0}^{T_0} \frac{C_p m \, dT}{T} = 0$$

$$\Delta S_2 = \int_{T_0}^{T_0} \frac{C_p m \, dT}{T} = 0$$

Adiabatisk:

$$\Delta S_{univers} = \Delta S_1 + \Delta S_2 = \underline{\underline{0}}$$

Det blir kun ~~ekstra~~ ΔS hvis temperaturene er forskjellige.

24) $\underline{\underline{\text{Ingen}}}$

25) $dF = -S\,dT - p\,dV$

$$S = -\left(\frac{\partial F}{\partial T}\right)_V \quad \& \quad p = -\left(\frac{\partial F}{\partial V}\right)_T$$

$$-\frac{\partial^2 F}{\partial V \partial T} = \left(\frac{\partial S}{\partial V}\right)_T = \left(\frac{\partial p}{\partial T}\right)_V$$

26) $E = \frac{1}{2m}\left(p_x^2 + p_y^2\right) + \frac{1}{2}m\omega^2\left(x^2+y^2\right)$

kvadratiske

4 frihetsgrader

$$C_V = \frac{4}{2}k = 2k$$

27) $\Delta S = 0$ Like mange tilstander med $E = -J/2$ som med $E = +J/2$

entalpi

28) $C_{\mathcal{H}} = \left(\frac{\partial H}{\partial T}\right)_{\mathcal{H}}$

$$H = U - \mu_0 \mathcal{H} M = -\mu_0 \mathcal{H} M$$

$$= -\mu_0 A \frac{\mathcal{H}^2}{T}$$

$$C_{\mathcal{H}} = \mu_0 A \left(\frac{\mathcal{H}}{T}\right)^2$$

29) C Dette er magnetisk kjøling.

30) Når $T=\infty$ er den termiske energien ∞.
Da er det like sannsynlig å være i tilstandene ~~$E = E_1 \, 2 \, E_2$~~ E_1, E_2, og E_3.

$$P_3 = P_2 = P_1 = \underline{\underline{1/3}}$$

31) $\underline{dW = PdV}$

$$H = U + PV$$

$$dH = \underbrace{dU + PdV}_{TdS} + VdP$$

Naturlige variable er S og P.

$\underline{\text{Med } dW = y\,dx:}$

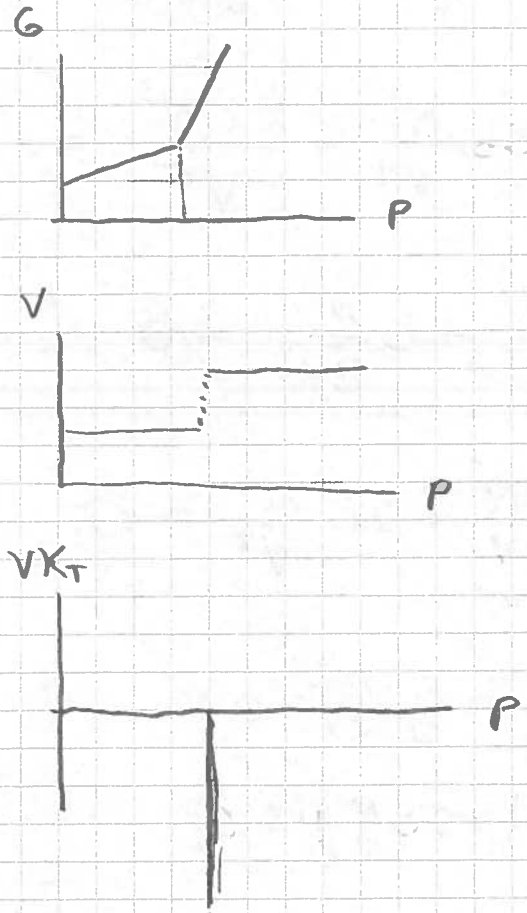
——— " ——— $\underline{\underline{S \text{ og } y}}$

32) $f = c + 2 - q$

$$= 3 + 2 - 3 = \underline{\underline{2}}$$

33) $\kappa_T V = -\left(\frac{\partial V}{\partial P}\right)_T$

$$V = \left(\frac{\partial G}{\partial P}\right)_T$$



$\underline{\underline{A}}$

34) $p = \frac{kTN}{V} - \frac{b}{2}\frac{N^2}{V^2} + \frac{cN^3}{6V^3}$

$$\frac{\partial p}{\partial V} = -\frac{kTN}{V^2} + b\frac{N^2}{V^3} - \frac{cN^3}{2V^4} = 0 \quad (1)$$

$$\frac{\partial^2 p}{\partial V^2} = 2\frac{kTN}{V^3} - 3b\frac{N^2}{V^4} + \frac{2cN^3}{V^5} = 0 \quad (2)$$

(1): $-kT + b\frac{N}{V} - c\frac{N^2}{2V^2} = 0$

(2): $2kT - 3b\frac{N}{V} + 2c\frac{N^2}{V^2} = 0$

(1): $-kT + \frac{N}{V}\left(b - \frac{c}{2}\frac{N}{V}\right) = 0$

(2): $2kT + \frac{N}{V}\left(-3b + 2c\frac{N}{V}\right) = 0$

34) $p = kTn - \frac{b}{2}n^2 + \frac{c}{6}n^3$

$$\frac{\partial p}{\partial n} = kT - bn + \frac{c}{2}n^2 = 0 \qquad (a)$$

$$\frac{\partial^2 p}{\partial n^2} = -b + cn = 0 \Rightarrow b = cn \qquad (b)$$

$$(a): \quad kT - cn^2 + \frac{c}{2}n^2 = 0$$

$$kT = \frac{c}{2}n^2$$

34) $p = \frac{NkT}{V-Nb} - a\frac{N^2}{V^2}$

b: Volumet okkupert av et gassmolekyl

a: et mål på den attraktive vekselvirkningen mellom gassmolekylene.

$\underline{\underline{B}}$

35) $p = \frac{NkT}{V} - \frac{aN^2}{V^2}$

$$\frac{\partial p}{\partial V} = -\frac{NkT}{V^2} + 2\frac{aN^2}{V^3} = 0 \Rightarrow -kT + 2\frac{aN}{V} = 0 \qquad (a)$$

$$\frac{\partial^2 p}{\partial V^2} = 2\frac{NkT}{V^3} - 6\frac{aN^2}{V^4} = 0 \Rightarrow \cancel{2}kT - \overset{3}{\cancel{6}}\frac{aN}{V} = 0 \qquad (b)$$

$$\left.\begin{array}{l}(a):\quad a = \frac{kT_cV_c}{2N} \\ (b):\quad a = \frac{kT_cV_c}{3N}\end{array}\right\}$$

eneste mulighet å oppfylle begge lignlinger er at $T_c \cdot V_c = 0$. Så en av disse må være null.

$\Rightarrow$ Det finnes ingen ikke-trivielle kritiske punkt

$\Rightarrow$ $\underline{\underline{\text{Nei}}}$

36) $$\frac{dP}{dT} = \frac{L_f}{T\,\Delta V}$$

$$\Delta V = V_g - V_v \simeq V_g = \frac{NkT}{P}$$

$$\frac{dP}{dT} = \frac{L_f\,P}{NkT^2}$$

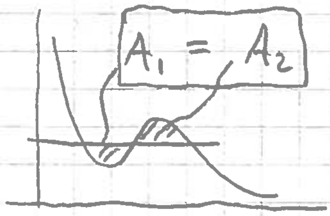
$$\frac{Nk}{L_f}\int_{P_1}^{P}\frac{dP}{P} = \int_{T_1}^{T}\frac{dT}{T^2}$$

$$\frac{Nk}{L_f}\ln\frac{P}{P_1} = -\left[\frac{1}{T}\right]_{T_1}^{T} = -\left[\frac{1}{T} - \frac{1}{T_1}\right]$$

$$\underline{\underline{\ln\frac{P}{P_1} = -\frac{L_f}{Nk}\left[\frac{1}{T} - \frac{1}{T_1}\right]}}$$

37) $\underline{\underline{C}}$

38) Likevektstrykket er bestemt av Maxwell-konstruksjonen.



$$\underline{\underline{P/P_c = 0.8}}$$

39) LV $\Leftrightarrow$ $\mu_{glass} = \mu_{krystall}$

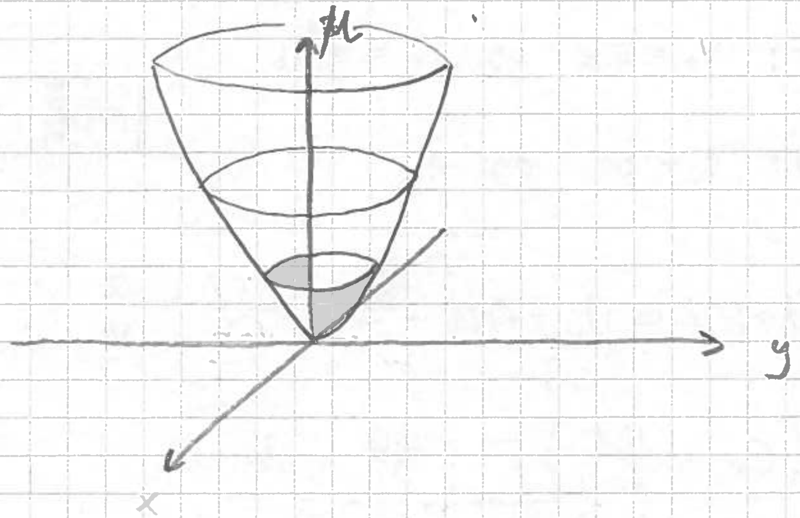
$$\mu_g = \frac{\beta}{2}T^2 - \beta T^2 = -\frac{\beta}{2}T^2$$

$$\mu_k = \frac{\alpha}{4}T^4 - \frac{\alpha}{3}T^4 = -\frac{\alpha}{12}T^4$$

$$\Rightarrow \frac{\beta}{2}T_m^2 = \frac{\alpha}{12}T_m^4$$

$$T_m^2 = \frac{6\beta}{\alpha} \Rightarrow \underline{\underline{T_m = \sqrt{\frac{6\beta}{\alpha}}}}$$

40) Partikler strømmer fra høy til lav μ.



Partiklene strømmer mot origo.

41) $G = U + pV - TS$

$$\left(\frac{\partial S}{\partial V}\right)_T = \left(\frac{\partial p}{\partial T}\right)_V = \frac{\partial}{\partial T}\left\{\frac{a}{3}T^4\right\} = \frac{4aT^3}{3}$$

$$S(V,T) = \frac{4aT^3}{3}V + \underbrace{C(T)}_{=0}$$

$$G = aVT^4 + \frac{a}{3}VT^4 - \frac{4a}{3}VT^4 = 0\ !$$

siden $\mu = G/N \Rightarrow \underline{\underline{\mu = 0}}$

42) Før: $V_0 = 5b$ og $T_0 = \frac{a}{2Rb}$

Slutt: $V_s = \infty$ og T_s

$$\underline{H} = U + PV = U_0 + C_vT - \frac{a}{V} + \frac{RTV}{V-b} - \frac{a}{V}$$

$$= \underline{\left(C_v + \frac{RV}{V-b}\right)T - \frac{2a}{V} + U_0}$$

Joule - Thomson $\Leftrightarrow$ $H_{før} = H_{etter}$:

$$\left(C_v + R\frac{V_0}{V_0-b}\right)T_0 - \frac{2a}{V_0} = \left(C_v + R\right)T_s$$

$$\Rightarrow T_s = \frac{39}{50}T_0 \simeq \underline{\underline{0.78\,T_0}}$$

43) $\vec{J} = -\chi \nabla T$

$\nabla \cdot \vec{J} = 0$ (stasjonær)

$J = -\chi \dfrac{dT}{dx}$

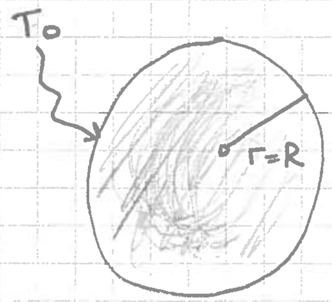
$\dfrac{dJ}{dx} = 0$

$$0 = \frac{d}{dx}\left(-\chi(x)\frac{dT}{dx}\right) = -\left\{\underbrace{\frac{d\chi}{dx}}_{-e^{-x}}\frac{dT}{dx} + \chi\frac{d^2T}{dx^2}\right\}$$

$$0 = e^{-x}\frac{dT}{dx} - e^{-x}\frac{d^2T}{dx^2}$$

$$\Rightarrow \boxed{\frac{d^2T}{dx^2} - \frac{dT}{dx} = 0}$$

44)



Stasjonær varmeligning med kulesymmetry:

$$\frac{d}{dr}\left(r^2 \frac{dT}{dr}\right) = 0$$

$$r^2 \frac{dT}{dr} = C_1$$

$$\frac{dT}{dr} = \frac{C_1}{r^2}$$

$$T(r) = C_2 - \frac{C_1}{r}$$

$$T(r \to \infty) = 0 = C_2$$

$$T(R) = T_0 = -\frac{C_1}{R} \Rightarrow C_1 = -RT_0$$

$$\Rightarrow T(r) = R\left(\frac{T_0}{r}\right)$$

45) $P(x,t) = \frac{1}{\sqrt{4\pi D t}} e^{-\frac{(x - V_d t)}{4Dt}}$

$$\langle x \rangle = \int_{-\infty}^{\infty} \frac{1}{\sqrt{4\pi D t}} e^{-\frac{(x-V_d t)^2}{4Dt}} x \, dx$$

La $y = x - V_d t \Rightarrow dy = dx$ og $a_1 = 4Dt$

$a_2 = 4\pi D t$

$$\langle x \rangle = \frac{1}{\sqrt{a_2}} \int_{-\infty}^{\infty} (y + V_d t) e^{-\frac{y^2}{a_1}}$$

$$= \underbrace{\frac{1}{\sqrt{a_2}} \int_{-\infty}^{\infty} y e^{-\frac{y^2}{a_1}}}_{0} + \underbrace{\left(\frac{1}{\sqrt{a_2}} \int_{-\infty}^{\infty} e^{-\frac{y^2}{a_1}} \right)}_{1} V_d t$$

$$= \underline{\underline{V_d t}}$$ $\Rightarrow$ Partiklene strømmer mot høyre

$$\langle x^2 \rangle = \int_{-\infty}^{\infty} \frac{1}{\sqrt{4\pi D t}} e^{-\frac{(x-V_d t)^2}{4Dt}} x^2 \, dx$$

$$y = x - V_d t,$$

$$\langle x^2 \rangle = \frac{1}{\sqrt{4\pi D t}} \int_{-\infty}^{\infty} \underbrace{(y+V_d t)^2}_{y^2 + 2yV_d t + (V_d t)^2} e^{-\frac{y^2}{4Dt}} \, dy$$

$$= \frac{1}{\sqrt{4\pi D t}} \left\{ \int_{-\infty}^{\infty} y^2 e^{-\frac{y^2}{4Dt}} + 2V_d t \underbrace{y e^{-\frac{y^2}{4Dt}}}_{0} + (V_d t)^2 \underbrace{e^{-\frac{y^2}{4Dt}}}_{1} \right\} dy$$

$$= (V_d t)^2 + \underbrace{\frac{1}{\sqrt{4\pi D t}} \int_{-\infty}^{\infty} y^2 e^{-\frac{y^2}{4Dt}} \, dy}_{I}$$

$$\lambda = \frac{1}{4Dt}$$

$$I = \int_{-\infty}^{\infty} y^2 e^{-\lambda y^2} dy = -\frac{d}{d\lambda} \underbrace{\int_{-\infty}^{\infty} e^{-\lambda y^2} dy}_{\sqrt{\frac{\pi}{\lambda}}}$$

$$= \frac{1}{2} \sqrt{\pi} \frac{1}{\lambda^{3/2}}$$

$$\langle x^2 \rangle = (V_d t)^2 + \cancel{\frac{1}{\sqrt{4\pi D t}}} \left(\frac{1}{2} \cancel{\sqrt{\pi}} \cancel{\sqrt{4Dt}} \; 4Dt \right)$$

$$= \underbrace{(V_d t)^2}_{\text{Verdrin beveger seg}} + \underbrace{2Dt}_{\text{diffusion}}$$