1) $\frac{\alpha_p \varkappa_T}{\alpha_V} = \frac{1}{p} \Rightarrow \frac{RT}{p^2 V} = \frac{1}{p} \Rightarrow \underline{\underline{p = \frac{RT}{V}}}$

2) $\underline{\underline{C}}$ 3) $\underline{\underline{B}}$

4) $\alpha_L = \frac{1}{L}\left(\frac{\partial L}{\partial T}\right)_p \Rightarrow \alpha_L = \frac{\Delta L / L}{\Delta T}$ original lengde

$\Rightarrow \Delta L = L \cdot \alpha_L \Delta T$

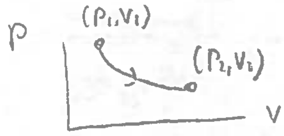
$\Rightarrow t_2 = 2\pi\sqrt{\frac{L + L\alpha_L \Delta T}{g}} = 2\pi\sqrt{\frac{L(1+\alpha_L\Delta T)}{g}} = t_1\sqrt{1+\alpha_L \Delta T}$

$= 1\,s\sqrt{1+20\cdot 10^{-6}\cdot 10} \simeq \underline{\underline{1.0001\ s}}$

5) B, C, E 6) B, C 7) A

8) $L_f = P \cdot t = 500\,\frac{J}{s} \cdot 15\,s = 500 \cdot 15\ J = 7500\ J = \underline{\underline{7.5\ kJ}}$

9) Adiabat $pV^\gamma = konst$



$$W = \int_{V_1}^{V_2} p\,dV = \int_{V_1}^{V_2} \frac{konst}{V^\gamma}\,dV = konst\left[\frac{V^{1-\gamma}}{1-\gamma}\right]_{V_1}^{V_2} = \frac{konst}{1-\gamma}\left[\frac{V_2}{V_2^\gamma} - \frac{V_1}{V_1^\gamma}\right]$$

$$= \underline{\underline{\frac{1}{1-\gamma}\left[P_2 V_2 - P_1 V_1\right]}}$$

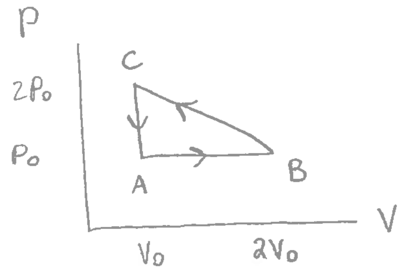
10) H_2 : Diatomisk $f = f_{trans} + f_{rot} + f_{vib} = 3 + 2 + 2$

Ved 40 K bidrar kun translasjon $\Rightarrow \underline{\underline{\tilde{f} = 3}}$

$\Rightarrow C_V = \underline{\underline{\frac{3}{2}k}}$

11) $\underline{\underline{\epsilon_R = \frac{T_1}{T_2 - T_1}}}$ 12) Carnotmaskin $\Rightarrow \eta = 1 - \frac{T_1}{T_2} = \underline{\underline{\frac{T_2 - T_1}{T_2}}}$

13)

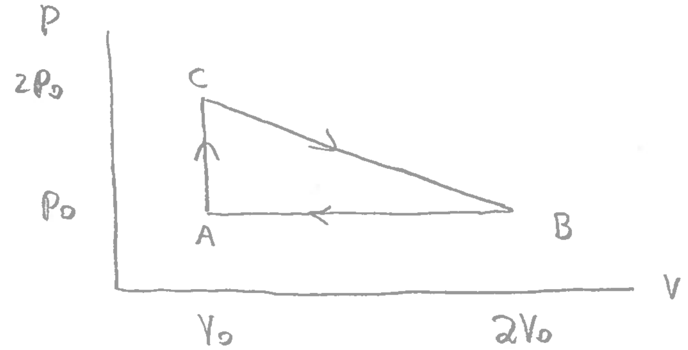


mot klokka

$$Q_t = \Delta U_t + W_t = W_t = -V_0 P_0 \frac{1}{2} = \underline{\underline{-\frac{1}{2} P_0 V_0}}$$

$\hookrightarrow 0$ per syklus

14)



Carnots teorem: $\eta < \eta_c = 1 - \frac{T_{Lavest}}{T_{Høyest}}$

Vi må bestemme laveste og høyeste temperatur. $P = \frac{nRT}{V}$

Laveste temperatur: $T_A \Rightarrow nRT_A = P_0 V_0$

Høyeste temp: et eller annet sted på linja fra C til B.

Parametrisering: $P_{CB} = -\frac{P_0}{V_0} V_{BC} + 3P_0$ (rett linje gitt 2 punkt)

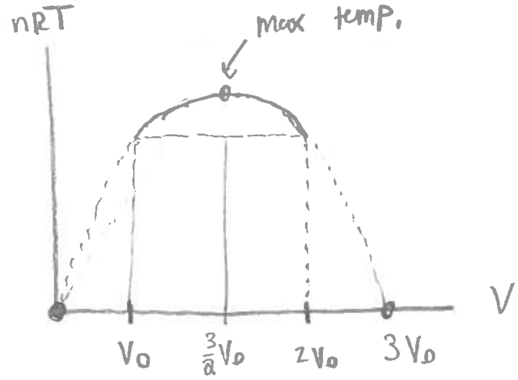
Temperatur på linja CB:

$$nRT = PV = -\frac{P_0}{V_0} V^2 + 3P_0 V = V\left(3P_0 - \frac{P_0}{V_0} V\right)$$

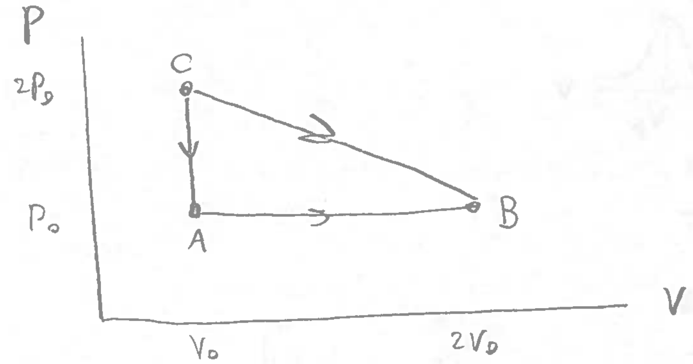
$3P_0 = \frac{P_0}{V_0} V \Rightarrow V = 3V_0$

$$nRT_{max} = \frac{3}{2} V_0 \left(3P_0 - \frac{3}{2} P_0\right) = \frac{3}{2} V_0 \left(\frac{3}{2} P_0\right) = \frac{9}{4} P_0 V_0$$

$$\eta_c = 1 - \frac{P_0 V_0}{\frac{9}{4} P_0 V_0} = 1 - \frac{4}{9} = \frac{5}{9} \approx \underline{\underline{0.56}}$$



15)



$pV = nRT$

$$\frac{p}{nR} = \frac{T}{V}$$

$$dS = \frac{C_v}{T}\,dT + \underbrace{\left(\frac{\partial P}{\partial T}\right)_V}_{\frac{nR}{V}} dV$$

$$\Rightarrow \Delta S = C_v \ln\frac{T_{slutt}}{T_{start}} + nR\ln\frac{V_{slutt}}{V_{start}}$$

CA: Isokor, $\Delta T < 0 \Rightarrow \underline{\underline{\Delta S(C\to A) < 0}}$

AB: Isobar: $\Delta S = C_v \ln\frac{T_B}{T_A} + nR\ln 2$; $\frac{T_A}{V_0} = \frac{T_B}{2V_0} \Rightarrow \frac{T_B}{T_A} = 2$

$$= \underline{\underline{(C_v + nR)\ln 2}} = C_p \ln 2$$

CB: Samme temperatur i C & B (symmetri, evt ved utregning som i forrige oppgave)

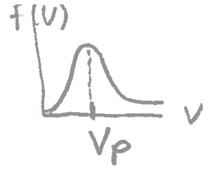
$\underline{\underline{\Delta S = nR\ln 2}}$

$$\underbrace{(C_v + nR)\ln 2}_{\Delta S(A\to B)} > \underbrace{nR\ln 2}_{\Delta S(C\to B)} > \Delta S(C\to A)$$

$$\Rightarrow \underline{\underline{\Delta S(A\to B) > \Delta S(C\to B) > \Delta S(C\to A)}}$$

16) $\underline{\underline{C}}$

17) Maxwells fartsfordeling



$$f(v) = 4\pi \left(\frac{m}{2\pi kT}\right)^{3/2} v^2 e^{-\frac{mv^2}{2kT}}$$

$$\frac{df}{dv} = 0 = 4\pi \left(\frac{m}{2\pi kT}\right)^{3/2} \left\{ 2v e^{-\frac{mv^2}{2kT}} + v^2 \cdot e^{-\frac{mv^2}{2kT}} \cdot -\frac{mv}{kT} \right\}$$

$$= 4\pi \left(\frac{m}{2\pi kT}\right)^{3/2} e^{-\frac{mv^2}{2kT}} \underbrace{\left[2 - \frac{mv^2}{kT} \right]}_{0\ (v = v_p)} v$$

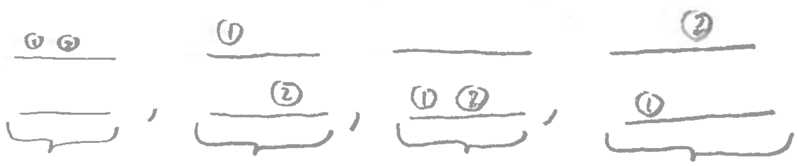
$$\Rightarrow \quad \underline{\underline{v_p = \sqrt{\frac{2kT}{m}}}}$$

18) $E_2 = \frac{\beta\Delta}{2}$

to partikler ① ②

$E_1 = -\frac{\beta\Delta}{2}$

Mikrotilstander:



$E = \beta\Delta$, $E = 0$, $E = -\beta\Delta$, $E = 0$

$$\Delta S = S(E = \beta\Delta) - S(E = 0) = \underbrace{k \ln 1}_{0} - k \ln 2 = \underline{\underline{-k \ln 2}}$$

19) $\underbrace{\uparrow\uparrow\uparrow}_{E=J/2}$, $\underbrace{\uparrow\uparrow\downarrow}_{0}$, $\underbrace{\uparrow\downarrow\uparrow}_{-J/2}$, $\underbrace{\uparrow\downarrow\downarrow}_{0}$

$\underbrace{\downarrow\uparrow\uparrow}_{0}$, $\underbrace{\downarrow\uparrow\downarrow}_{-J/2}$, $\underbrace{\downarrow\downarrow\uparrow}_{0}$, $\underbrace{\downarrow\downarrow\downarrow}_{J/2}$

$$Z = \sum_j e^{-\beta E_j} = 2e^{-\beta\frac{J}{2}} + 2e^{\beta\frac{J}{2}} + 4e^0$$

$$= \underline{\underline{4\left(1 + \cosh\left(\beta\frac{J}{2}\right)\right)}}$$

20) $F = F_i + kT B_2(T)\frac{N^2}{V}$

$P = -\left(\frac{\partial F}{\partial V}\right)_T$ Frm TDI

$$\left(\frac{\partial F}{\partial V}\right)_T = \left(\frac{\partial F_i}{\partial V}\right)_T - kT B_2(T)\frac{N^2}{V^2}$$

$$P = \frac{NkT}{V} + \frac{NkT}{V}\cdot\frac{N}{V}B_2(T) = \underline{\underline{\frac{NkT}{V}\left(1 + \frac{N}{V}B_2(T)\right)}}$$

21) $S = -\left(\frac{\partial F}{\partial T}\right)_V$

$$\left(\frac{\partial F}{\partial T}\right)_V = \left(\frac{\partial F_i}{\partial T}\right)_V + k\frac{N^2}{V}\left[B_2(T) + T\frac{dB_2}{dT}\right]$$

$$\Rightarrow \underline{\underline{S = S_i - \frac{kN^2}{V}\left\{B_2(T) + T\frac{dB_2}{dT}\right\}}}$$

22) $B_2(T) = -\frac{a}{kT}$

$$\frac{dB_2}{dT} = \frac{a}{kT^2} \quad ; \quad \frac{d^2B_2}{dT^2} = -\frac{2a}{kT^3}$$

$$C_V = C_{V,i} - k\frac{N^2}{V}T\left(\frac{2a}{kT^2} + -\frac{2a}{kT^2}\right) = C_{V,i} = \frac{3}{2}k$$

EPP: kun bidrag fra translasjon ved alle temperaturer

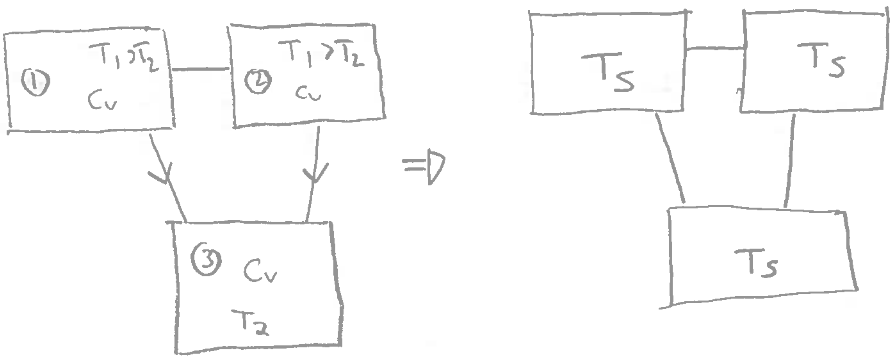
$\Rightarrow$ gassen må være monoatomisk

23) A, B, C, D, E Alt er ekvivalent

24) $dS = C_V\frac{dT}{T} + \frac{nR}{V}\underbrace{dV}_{0} \Rightarrow \Delta S = C_V \ln\frac{T_{slutt}}{T_{start}}$

$$\Rightarrow \Delta S = C_V\ln\frac{T_0/2}{T_0} = C_V\ln\frac{1}{2} = -C_V\ln 2 = -\frac{3}{2}Nk\ln 2$$

25)



$$\underbrace{Q}_{} = \Delta U + \underbrace{W}_{\to 0} \Rightarrow \Delta U = \Delta U_{①} + \Delta U_{②} + \Delta U_{③} = 0$$

$$\Rightarrow T_S = \frac{T_1 + T_1 + T_2}{3} = \frac{2T_1 + T_2}{3}$$

$$\Delta S = \Delta S_{①} + \Delta S_{②} + \Delta S_{③} = 2C_V\ln\frac{T_S}{T_1} + C_V\ln\frac{T_S}{T_2} = C_V\ln\frac{T_S^3}{T_1^2T_2}$$

$$= C_V\ln\frac{(2T_1+T_2)^3}{9T_1^2T_2}$$

26) $dW = -f\,dL$

$P \to -f,\ V \to L$

$$S(L,T) = g(T) - c\left(\frac{L^2}{2L_0} + \frac{L_0^2}{L}\right)$$

$$dF = -S\,dT + -p\,dV$$

$$\Rightarrow \left(\frac{\partial S}{\partial V}\right)_T = \left(\frac{\partial P}{\partial T}\right)_V$$

$$\Rightarrow \left(\frac{\partial S}{\partial L}\right)_T = -\left(\frac{\partial f}{\partial T}\right)_L$$

$$+\left(\frac{\partial f}{\partial T}\right)_L = +c\left(\frac{L}{L_0} - \frac{L_0^2}{L^2}\right)$$

$$\Rightarrow f(T,L) = cT\left(\frac{L}{L_0} - \frac{L_0^2}{L^2}\right) + h(L)$$

$$f(T, L=L_0) = 0 = cT(1-1) + h(L) \Rightarrow h(L) = 0$$

$$\Rightarrow f(T,L) = cT\left(\frac{L}{L_0} - \frac{L_0^2}{L^2}\right)$$

27) $U = aVT^4$

$$T\,dS = dU + P\,dV = \left(\frac{\partial U}{\partial V}\right)_T dV + \left(\frac{\partial U}{\partial T}\right)_V dT + P\,dV$$

$$= (aT^4 + P)\,dV + 4aVT^3\,dT$$

$$\Rightarrow dS = \underline{\left(\frac{P}{T} + aT^3\right)} dV + \underline{\underline{4aVT^2\,dT}}$$

$$S = V \cdot s(T)$$

$$dS = \left(\frac{\partial S}{\partial V}\right)_T dV + \left(\frac{\partial S}{\partial T}\right)_V dT = \underline{s(T)\,dV} + \underline{\underline{\left(\frac{\partial S}{\partial T}\right)_V dT}}$$

$$\Rightarrow s(T) = aT^3 + \frac{P}{T} \quad \text{og} \quad \left(\frac{\partial S}{\partial T}\right)_V = 4aVT^2$$

$$\Rightarrow S(T,V) = \frac{4}{3}aVT^3 + f(V)$$

$\hookrightarrow 0$ (initial betingelse)

$$= \underline{\underline{\frac{4}{3}aVT^3}}$$

28) $P = \frac{a}{3}T^4$

$$H = U + PV = aVT^4 + \frac{a}{3}VT^4 = \frac{4}{3}aVT^4 = 4PV$$

$$C_P = \left(\frac{\partial H}{\partial T}\right)_P = 4P\left(\frac{\partial V}{\partial T}\right)_P = 4P\left(\frac{\partial T}{\partial V}\right)_P^{-1}$$

$$\left(\frac{\partial T}{\partial V}\right)_P \Rightarrow T(V,P) = \left(\frac{3P}{a}\right)^{1/4} \Rightarrow \left(\frac{\partial T}{\partial V}\right)_P = 0 \Rightarrow \left(\frac{\partial V}{\partial T}\right)_P = \underline{\underline{\infty}}$$

$$\Rightarrow \underline{\underline{C_P = \infty}}$$

29) $dG = Vdp - SdT + \mu dN \quad ; G = \mu N$

$= N d\mu + \mu dN$

$pV = NkT \Rightarrow \frac{N}{V} = \frac{p}{kT}$

$\int \frac{dy}{dx} dx$

$\Rightarrow N d\mu = V dp - S dT$

$\Rightarrow \frac{N}{V} = \left(\frac{dp}{d\mu}\right)_T = \frac{p}{kT}$

$\Rightarrow \left(\frac{\partial \mu}{\partial p}\right)_T = \frac{kT}{p}$

$\Rightarrow \mu - \mu_0 = kT \ln \frac{p}{p_0}$

$\Rightarrow \mu = \mu_0 + kT \ln \frac{p}{p_0}$

30) Siden de støkiometriske tallene alle er 1 er LV-betingelsen ~~at alle kjemiske potensial skal være like. DVS alternativ C~~

$\mu(A) + \mu(B) = \mu(C) + \mu(D)$

$\Rightarrow$ eneste mulighet er $\underline{\underline{C}}$

31) Ideell blanding:

$$\mu_j = \mu_j^\circ + RT \ln x_j$$

molbrøk Salt : x

$\Rightarrow$ molbrøk Vann : $1-x$

$$LV: \quad \mu^\circ_{H_2O}(P,T) = \underbrace{\mu_{H_2O}(P+\Delta P, T)}_{\underbrace{\mu^\circ_{H_2O}(P+\Delta P,T)}_{\mu^\circ_{H_2O}(P,T) + \Delta P \left(\frac{\partial \mu_0}{\partial P}\right)_T} + \underbrace{RT\ln(1-x)}_{-RTx}}$$ (ideell blanding)

$$\Rightarrow \frac{RTx}{\Delta P} = \underbrace{\left(\frac{\partial \mu^\circ_{H_2O}}{\partial P}\right)_T}_{\frac{V_{H_2O}}{N_{H_2O}} \simeq \frac{V}{N}}$$

$$\Rightarrow \frac{RT}{\Delta P}\frac{N_{salt}}{N} = \frac{V}{N} \Rightarrow \underline{\underline{\Delta P = \frac{N_{salt} RT}{V}}}$$

32) $\underline{\underline{A, B}}$

C er feil fordi at kvasistatiske prosesser hvor vi ikke ser bort i fra dissipative effekter (f.eks friksjon) ikke er reversible.

33) $T\,dS = C_V\,dT + T\left(\frac{\partial P}{\partial T}\right)_V dV$

$\mathcal{M} = C\frac{\mathcal{H}}{T} \Rightarrow \mathcal{H} = \frac{\mathcal{M}T}{C}$

$P \to -\mu_0\mathcal{H}, \; V \to \mathcal{M}$

$$\Rightarrow T\,dS = \underbrace{C_{\mathcal{M}}}_{\left(\frac{\partial U}{\partial T}\right)_{\mathcal{M}}=0} dT - T\mu_0 \underbrace{\left(\frac{\partial \mathcal{H}}{\partial T}\right)_{\mathcal{M}}}_{\frac{\mathcal{M}}{C}} d\mathcal{M} = -T\frac{\mu_0}{C}\mathcal{M}\,d\mathcal{M}$$

$$\Rightarrow \int_{S_0}^{S} dS = -\frac{\mu_0}{C}\int_0^{\mathcal{M}_1} \mathcal{M}\,d\mathcal{M} = -\frac{\mu_0 \mathcal{M}_1^2}{2C}$$

$$\Rightarrow \Delta S = -\frac{\mu_0\mathcal{M}_1^2}{2C}$$

34) <u>B</u>

35) $$dT = \frac{\mu_0 T}{C_{\mathcal{H}}}\left|\left(\frac{\partial \mathcal{M}}{\partial T}\right)_{\mathcal{H}}\right| d\mathcal{H}$$

$$C_{\mathcal{H}} = \left(\frac{\partial H}{\partial T}\right)_{\mathcal{H}} = \frac{\partial}{\partial T}\left\{-\mu_0 \mathcal{H}\cdot C\left(\frac{\mathcal{H}}{T}\right)^{\alpha}\right\}$$

$$= -\mu_0 C\mathcal{H}^{\alpha+1}\cdot -\alpha\frac{1}{T^{\alpha+1}} = \alpha\mu_0 C\left(\frac{\mathcal{H}}{T}\right)^{\alpha+1}$$

$$\left(\frac{\partial \mathcal{M}}{\partial T}\right)_{\mathcal{H}} = \frac{\partial}{\partial T}\left\{C\frac{\mathcal{H}^{\alpha}}{T^{\alpha}}\right\} = C\mathcal{H}^{\alpha}\frac{\partial}{\partial T}\left\{T^{-\alpha}\right\} = C\mathcal{H}^{\alpha}\cdot -\alpha\frac{1}{T^{\alpha+1}}$$

$$dT = \mu_0 T\cdot\frac{\cancel{T^{\alpha+1}}}{\mathcal{H}^{\alpha+1}\,\cancel{\alpha\mu_0 C}}\cdot\frac{\cancel{\alpha C}\,\mathcal{H}^{\alpha}}{\cancel{T^{\alpha+1}}}\,d\mathcal{H} = \frac{T}{\mathcal{H}}\,d\mathcal{H}$$

$$\Rightarrow \frac{dT}{T} = \frac{d\mathcal{H}}{\mathcal{H}} \Rightarrow \ln T = \ln\mathcal{H} + C$$

$$T = A\mathcal{H}$$

36)

$$V = \frac{RT}{P} + \left(b - \frac{a}{RT}\right)$$

$$\mu_{JT} = \frac{1}{C_P}\left[T\left(\frac{\partial V}{\partial T}\right)_P - V\right]$$

$$= \frac{1}{C_P}\left[T\left(\frac{R}{P} + \frac{a}{RT^2}\right) - V\right] = \frac{1}{C_P}\left[\frac{RT}{P} + \frac{a}{RT} - V\right]$$

$$= \frac{1}{C_P}\left[\cancel{V} - b + \frac{a}{RT} + \frac{a}{RT} - \cancel{V}\right] = \frac{1}{C_P}\left[\frac{2a}{RT} - b\right]$$

inversionstemperatur : $\mu_{JT}(T_*) = 0$

$$\Rightarrow \frac{2a}{RT_*} - b = 0 \Rightarrow \underline{\underline{T_* = \frac{2a}{Rb}}}$$

37) $P = \frac{NkT}{V-Nb} - a\frac{N^2}{V^2}$

$$\frac{\partial P}{\partial V} = -\frac{NkT}{(V-Nb)^2} + 2a\frac{N^2}{V^3} \Rightarrow \frac{NkT_c}{(V-Nb)^2} = 2a\frac{N^2}{V^3} \qquad (1)$$

$$\frac{\partial^2 P}{\partial V^2} = 2\frac{NkT}{(V-Nb)^3} - 6a\frac{N^2}{V^4} \Rightarrow \frac{NkT_c}{(V-Nb)^3} = 3a\frac{N^2}{V^4} \qquad (2)$$

$$\Rightarrow \frac{(1)}{(2)} \Rightarrow \frac{\cancel{NkT_c}}{(V_c-Nb)^2} \cdot \frac{(V_c-Nb)^3}{\cancel{NkT_c}} = (V_c - Nb) = \frac{2\cancel{aN^2}}{V_c^3} \cdot \frac{V_c^4}{3\cancel{aN^2}} = \frac{2}{3}V_c$$

$$\Rightarrow \frac{V_c}{3} = Nb \Rightarrow V_c = 3\underline{\underline{Nb}}$$

$$\Rightarrow NkT_c = 2a\frac{\cancel{N^2}}{3^3 N^{\cancel{3}} b^3} \cdot (2Nb)^2 = \frac{2a}{27Nb^3} \cdot 4N^2b^2$$

$$= \frac{8aN}{27b} \Rightarrow \underline{\underline{T_c = \frac{8a}{27kb}}}$$

38) $P = \frac{RT}{V-b} - \frac{a}{V^2}$

$$dU = \underbrace{\left(\frac{\partial U}{\partial T}\right)}_{C_V} dT + \underbrace{\left(\frac{\partial U}{\partial V}\right)}_{T\left(\frac{\partial P}{\partial T}\right)_V - P \,=\, \frac{RT}{V-b} - P \,=\, \frac{a}{V^2}} dV$$

$$dU = C_V \, dT + a \frac{dV}{V^2}$$

$$\Rightarrow U - U_0 = C_V T - \frac{a}{V} \Rightarrow \underline{\underline{U = C_V T - \frac{a}{V} + U_0}}$$

39) $\frac{dP}{dT} = \frac{l}{T\Delta V} = \frac{L_F}{T\Delta V}$ $\qquad \Delta V = V_g - V_v \simeq V_g = \left(\frac{AT}{P}\right)^2$

$$\frac{dP}{dT} = \frac{L_f P^2}{A^2 T^3} \Rightarrow \int_{P_1}^{P} \frac{dP}{P^2} = \frac{L_f}{A^2} \int_{T_1}^{T} \frac{dT}{T^3}$$

$$= -\left[\frac{1}{P} - \frac{1}{P_1}\right] = \frac{L_f}{A^2}\left[-\frac{1}{2} T^{-2}\right]_{T_1}^{T} = \frac{-L_f}{2A^2}\left[\frac{1}{T^2} - \frac{1}{T_1^2}\right]$$

$$\Rightarrow \underline{\underline{\frac{1}{P} - \frac{1}{P_1} = \frac{L_f}{2A^2}\left(\frac{1}{T^2} - \frac{1}{T_1^2}\right)}}$$

40) $P \to -\mu_0 \mathcal{H}$ $\quad V \to \mathcal{M}$

$$-\mu_0 \frac{d\mathcal{H}}{dT} = \frac{\Delta S}{\Delta \mathcal{M}} \Rightarrow \Delta S = -\mu_0 \underbrace{\Delta \mathcal{M}}_{\text{positiv}} \cdot \frac{d\mathcal{H}}{dT}$$

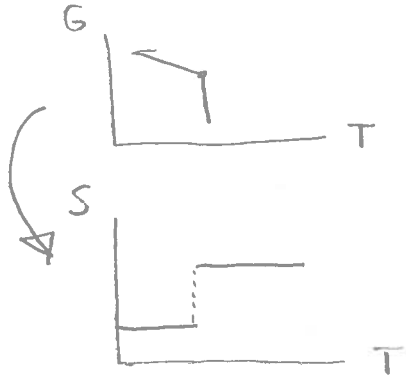
A: $\Delta S > 0$ $\quad$ B: $\Delta S = 0$ $\quad$ C: $\Delta S < 0$ $\quad$ D: $\Delta S = 0$ $\quad$ E: $\Delta S = +\infty$

$$\underline{\underline{\Delta S_E > \Delta S_A > \Delta S_B = \Delta S_D > \Delta S_C}}$$

41) $G = U + PV - TS$ $\qquad\qquad$ $TdS = dU + pdV$

$$dG = \cancel{dU} + \cancel{pdV} + Vdp - \cancel{TdS} - SdT = Vdp - SdT$$

$$\Rightarrow S = -\left(\frac{\partial G}{\partial T}\right)$$



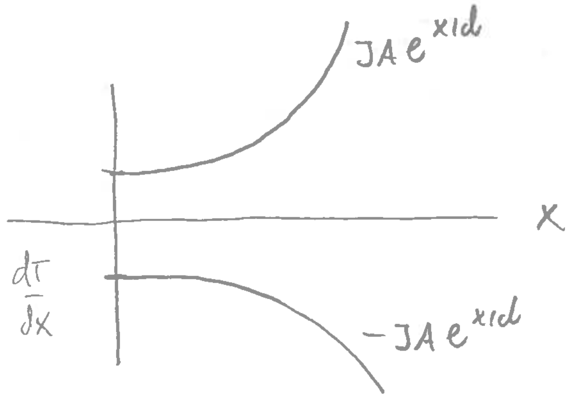
$\underline{\underline{A}}$

42) $$\Delta T = J \sum_{i=1}^{N} \frac{a_i}{\kappa_i} = J\left(\frac{b_1}{k_1} + \frac{b_2}{k_2}\right)$$

$$J = \frac{20}{\frac{1}{0.14} + \frac{2}{0.047}} = \underline{\underline{0.40}} \ \frac{W}{m^2}$$

43) $J = -\kappa(x) \dfrac{dT}{dx}$ $\qquad$ Siden energi er bevart er $J = \text{konst.}$ i $\underline{\underline{1D}}$

$$\Rightarrow \frac{dT}{dx} = -\frac{J}{\kappa(x)} = -JA e^{x/d}$$



$\frac{dT}{dx}$ blir mindre og mindre med økende x

$\Rightarrow$ Eneste mulighet er $\underline{\underline{B}}$

44) $\nabla \cdot J = 0 \qquad J = -\mathcal{K}\nabla T$

$$0 = \frac{dJ}{dx} = \frac{d}{dx}\left(-\mathcal{K}\frac{dT}{dx}\right)$$

$$\Rightarrow \quad 0 = \frac{d\mathcal{K}}{dx}\frac{dT}{dx} + \mathcal{K}\frac{d^2T}{dx^2}$$

$$\mathcal{K}(x) = Ae^{-x/d}$$

$$\frac{d\mathcal{K}}{dx} = Ae^{-x/d} \cdot -\frac{1}{d} = -\frac{A}{d}e^{-x/d}$$

$$\Rightarrow \quad 0 = -\frac{\cancel{A}}{d}\cancel{e^{-x/d}}\frac{dT}{dx} + \cancel{Ae^{-x/d}}\frac{d^2T}{dx^2}$$

$$\Rightarrow \quad \boxed{0 = \frac{d^2T}{dx^2} - \frac{1}{d}\frac{dT}{dx}}$$

45) Symmetrisk fordeling $\Rightarrow \langle \vec{r} \rangle = \langle \vec{r} \rangle^2 = 0$

$$\langle \vec{r}^{\,2} \rangle = \langle x^2+y^2 \rangle = \int_{-\infty}^{\infty}\int_{-\infty}^{\infty} (x^2+y^2)\, \tilde{p}(x,y,t)\, dx\,dy$$

$$= \int_0^{2\pi}\int_0^{\infty} r^2 \cdot \frac{1}{4\pi Dt} e^{-\frac{r^2}{4Dt}}\, r\, dr\, d\theta$$

$$= \frac{2\pi}{4\pi Dt} \underbrace{\int_0^{\infty} r^3 e^{-\frac{r^2}{4Dt}}\, dr}_{\frac{1}{2\left(\frac{1}{4Dt}\right)^2}}$$

$$= \frac{1}{2Dt} \frac{16 D^2 t^2}{2} = 4Dt$$

$$\Rightarrow \mathrm{Var}(\vec{r}) = \langle \vec{r}^{\,2} \rangle - \langle \vec{r} \rangle^2 = \underline{\underline{4Dt}}$$