

Institutt for fysikk

**Eksamensoppgave i
TFY4170 - Fysikk II**

Faglig kontakt under eksamen: Justin Wells

Tlf.: 45 16 36 97

Eksamensdato: 29.05.2019

Eksamenstid (fra-til): 0900-1300

Hjelpemiddelkode: C

Tillatte hjelpemidler: *Rottmann Matematisk formelsamling (or equivalent)*

Annen informasjon:

The exam gives 100% of the final grade.

The exam is only given in English - However, You may answer the exam in English, Bokmål or Ny Norsk.

Målform/språk: English.

The expectation value of a physical quantity $Q(x,t)$ is:

$$\langle Q \rangle = \int_{-\infty}^{\infty} \psi^*(x,t) Q(x,t) \psi(x,t) dx$$

Probability within a finite range of x :

$$P(a < x < b) = \int_a^b \psi(x)^* \psi(x) dx$$

Energy and momentum operators:

$$p_{x,op} = -i\hbar \frac{\partial}{\partial x} \quad E_{op} = i\hbar \frac{\partial}{\partial t}$$

de Broglie relations

$$\lambda = \frac{h}{p} \Rightarrow p = \hbar k$$

$$f = \frac{E}{h} \Rightarrow E = \hbar \omega$$

Schrödinger equation (1D, time independent)

$$\frac{\hbar^2}{2m} \frac{\partial^2}{\partial x^2} \psi(x) + (E - U(x)) \psi(x) = 0$$

Standard error in variable p :

$$\Delta p = \sqrt{\langle p^2 \rangle - \langle p \rangle^2}$$

$$\langle p^2 \rangle = \int_{-\infty}^{\infty} \psi^*(x,t) p_{x,op}^2 \psi(x,t) dx$$

Heisenberg's uncertainty principle:

$$= \int_{-\infty}^{\infty} \psi^*(x,t) \left(-i\hbar \frac{\partial}{\partial x} \right) \psi(x,t) dx$$

$$= (\hbar k)^2 \int_{-\infty}^{\infty} |A|^2 dx$$

$$= (\hbar k)^2$$

$$\Delta p = \sqrt{\langle p^2 \rangle - \langle p \rangle^2} = 0$$

$$\Delta p = 0, \quad \Delta x = \infty$$

Question 1: This question is about the confinement of waves and particles in a box.

Consider a tennis ball trapped in a box of side length 1m. The ball's initial position is $x=0$, and the initial velocity is v m/s in the $+x$ direction.

Assume that all collisions are perfectly elastic (i.e. no energy lost).

- a) What is the average velocity of the ball (averaged over many reflections)?
- b) What is the average momentum of the ball?
- c) Write an expression for the average kinetic energy of the ball?

Consider instead a sinusoidal sound wave entering the same box.

- d) Write an equation to describe the sound wave as a function of x and t
- e) Assume perfect reflection from the walls of the box: write an expression for the reflected wave.
- f) Under certain conditions, the wave may form a "standing wave" in the box.

What is this condition?

- g) Write an equation for the resulting standing wave.

We will now repeat the question for an electron trapped in a quantum dot (length = 10 nm).

- h) Write an expression for the wavefunction ψ of the electron in the box
- i) What is the kinetic energy of the electron?
- j) What is the expectation value of the electron's position (i.e. $\langle x \rangle$)?
- k) What is the expectation value of the electron's momentum (i.e. $\langle p_x \rangle$)?
- l) Do your answers to parts i, j and k agree with the answers for a larger object (such as the tennis ball in a, b and c)? Which properties are conceptually similar, and which are conceptually different?
- m) What is the uncertainty in position?
- n) What is the uncertainty in momentum?
- o) Comment on whether your answers to parts m and n are consistent with Heisenberg's uncertainty principle.



Question 2: In this question, we will investigate a LED flatscreen television.

In a traditional LED, an electron is excited into an unoccupied energy level. When it “relaxes” back into the original lower energy level a photon is produced. One can generally assume that the photon energy will be the same as the energy of the *bandgap*. For example, an LED made of diamond would emit ultraviolet light.

- a) What is a “bandgap”?
- b) Why do some materials have a bandgap and others do not?
- c) If I want to make a traditional LED with a different colour, which parameter(s) could I change?

A new generation of “Quantum dot LED” (or QLED) screens are now available. In these screens, the colours are generated by making use of transitions between energy levels in a quantum dot.

- d) How is it possible to create specific colours (like red, green and blue) from quantum dots? (i.e. which physical parameter(s) do you need to control in order to achieve this)?

When white light hits an LED screen, an interesting array of colours is produced (see picture).

- e) Can you explain what is happening here?
- f) The most commonly sold screen in Elkjøp is “24.5 inch” and has 1920 x 1080 pixels. Is this consistent with your answer to part e) and the picture? i.e. use this knowledge about pixel size to try to make your previous answer into a quantitative estimate.

[Some information which may be helpful:

24.5 inch means that the diagonal distance across the screen is about 600 mm

the wavelength of red light is ~ 680 nm, blue is 470 nm and green ~ 540 nm]

Question 3: This question involves explaining some historical scientific observations

When light hits a solid material, it may result in electrons be emitted.

- a) What would happen if you increase/decrease the intensity of the light?
- b) increase/decrease the wavelength?
- c) increase/decrease the amount of time the sample is exposed to the light?

When x-rays hit a crystalline sample they may be transmitted, reflected or scattered by the material.

- d) A detector placed at 30 degrees from the incoming beam collects x-rays which have the same wavelength as the incoming x-rays. What mechanism(s) can cause this?
- e) The same detector (at the same angle) also collects x-rays which have a different wavelength than the incoming c-rays. What mechanism(s) can cause this?

Fundamental constants

Quantity	Symbol	Approximate value
Acceleration of free fall (Earth's surface)	g	9.81 m s^{-2}
Gravitational constant	G	$6.67 \times 10^{-11} \text{ N m}^2 \text{ kg}^{-2}$
Avogadro's constant	N_A	$6.02 \times 10^{23} \text{ mol}^{-1}$
Gas constant	R	$8.31 \text{ J K}^{-1} \text{ mol}^{-1}$
Boltzmann's constant	k	$1.38 \times 10^{-23} \text{ J K}^{-1}$
Stefan–Boltzmann constant	σ	$5.67 \times 10^{-8} \text{ W m}^{-2} \text{ K}^{-4}$
Coulomb constant	k	$8.99 \times 10^9 \text{ N m}^2 \text{ C}^{-2}$
Permittivity of free space	ϵ_0	$8.85 \times 10^{-12} \text{ C}^2 \text{ N}^{-1} \text{ m}^{-2}$
Permeability of free space	μ_0	$4\pi \times 10^{-7} \text{ T m A}^{-1}$
Speed of light in vacuum	c	$3.00 \times 10^8 \text{ m s}^{-1}$
Planck's constant	h	$6.63 \times 10^{-34} \text{ J s}$
Elementary charge	e	$1.60 \times 10^{-19} \text{ C}$
Electron rest mass	m_e	$9.110 \times 10^{-31} \text{ kg} = 0.000549 \text{ u} = 0.511 \text{ MeV c}^{-2}$
Proton rest mass	m_p	$1.673 \times 10^{-27} \text{ kg} = 1.007276 \text{ u} = 938 \text{ MeV c}^{-2}$
Neutron rest mass	m_n	$1.675 \times 10^{-27} \text{ kg} = 1.008665 \text{ u} = 940 \text{ MeV c}^{-2}$
Unified atomic mass unit	u	$1.661 \times 10^{-27} \text{ kg} = 931.5 \text{ MeV c}^{-2}$