

Department of Physics

Examination paper for TFY4195 Optics

Academic contact during examination:

Dag Werner Breiby

Phone:

9845 4213

Examination date:

___ August 2018

Examination time (from-to):

09:00 – 13:00

Permitted examination support material:

Code C:

Approved calculator with empty memory.

K. Rottmann: Matematisk Formelsamling

S. Barnett & T.M. Cronin: Mathematical Formulae

O. Øgrim & B.E. Lian: Størrelser og enheter i fysikk og teknikk

Other information:

Each subtask a) b) etc. in task 1-4 will be given equal weight, with in total 100 % for the 11 subtasks.

Number of pages (front page excluded):

4

Number of pages enclosed:

2

Informasjon om trykking av eksamensoppgave

Originalen er:

1-sidig ☐ **2-sidig** ☐

sort/hvit ☐ **farger** ☐

skal ha flervalgskjema ☐

Checked by:

Date

Signature

Task 1 Mixed challenges

- a) Give shortly Fermat's principle. Derive Snell's law from Fermat's principle. You may choose the coordinate system and variables yourself, but you should write down the assumptions you make.
- b) A ray of light in air falls onto a planar glass surface with index of refraction n_2 . We assume no absorption. Sketch the reflectance R_s and R_p as function of incidence angle θ_1 . What is the interpretation of the Brewster angle θ_B ?

We now assume that the light is unpolarized and that the incidence angle equals the Brewster angle, $\theta_i = \theta_B$. The net transmittance is known to be $T = 0.86$. What is the reflectance (reflected intensity ratio) of the s-polarized light?

- c) A thin plano-convex lens made of glass with index of refraction n has diameter $D = 40$ mm and radius of curvature $R = 100$ mm. When the focal length f is 180 mm in air for $\lambda = 632.8$ nm (He-Ne laser), what is the value of n for this wave length? Would you normally expect that the focal length is shorter or longer for $\lambda = 454.6$ nm (Argon laser)? Explain.

Task 2 Geometrical optics

A thin lens L of focal length $f > 0$ stands in air (with index of refraction $n = 1$). The lens is mounted in a frame with inner diameter D_L .

- a) Show that the matrix $\mathbf{M}$ given by

$$\mathbf{M} = \begin{pmatrix} A & B \\ C & D \end{pmatrix} = \begin{pmatrix} A & \frac{-1}{f} \\ s_1 + s_2 - \frac{s_1 s_2}{f} & 1 - \frac{s_2}{f} \end{pmatrix} \quad (1)$$

describes the path of rays from the object plane in a distance $s_1 > f$ in front of the lens to a plane at a distance s_2 after the lens. Determine also the matrix element A .

What conditions must generally be fulfilled for this kind of matrix formalism to be valid?

- b) We now put an aperture with adjustable circular opening of radius r_b in the plane X a distance x after the lens, with $0 < x \leq f$.

Assume that a ray starts on the optical axis in the object plane and is marginal at L (that is, the ray just barely makes it within the frame of the lens).

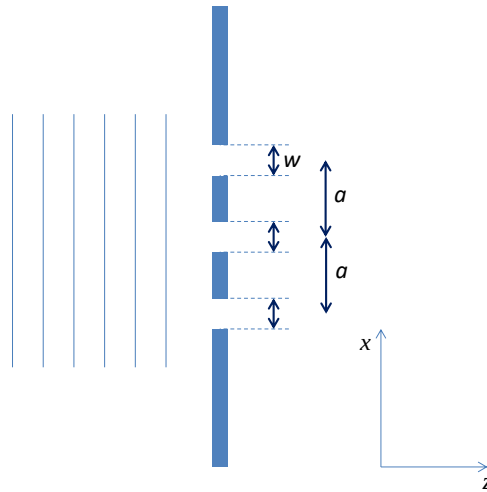
- Show by using the matrix from a) that the y -position of this ray in the plane X is given by

$$y_x = \frac{1}{2} D_L x \left(\frac{1}{s_1} + \frac{1}{x} - \frac{1}{f} \right).$$

- Use this to express a condition for r_b which makes the aperture the *aperture stop* of the system.
 - Find expressions for the location of the entrance and exit pupils of the system (given that the aperture is the aperture stop).
- c) Explain shortly the connection between the stop and a *telecentric* system for the special case that the matrix element $D = 0$ in equation (1). What is the practical understanding of a telecentric system?

Note: We use the Hecht-convention where rays are represented with vectors $\begin{bmatrix} n\alpha \\ y \end{bmatrix}$, where n is the index of refraction, α is the angle between the ray and the optical axis, and y is the distance from the optical axis.

Task 3 Interference from grating



We have a diffraction grating consisting of three slits of width $w = 10 \mu\text{m}$, separated by a distance $a = 100 \mu\text{m}$, as shown in the sketch. A monochromatic light source ($\lambda = 633 \text{ nm}$) sends light (approximately plane wave) through the lattice. The three slits all have the same rectangular cross section that can be described by $f_n(x) = f_0 \text{rect}(x/w)$, where w is the slit width and the function $\text{rect}(u)$ is defined by

$$\text{rect}(u) = \begin{cases} 1, & |u| \leq \frac{1}{2} \\ 0, & |u| > \frac{1}{2} \end{cases}$$

- Find an expression for the intensity distribution in the Fraunhofer-regime (far field) for *one* single of the three slits. At what angle θ_1 is the first minimum seen? What happens to θ_1 if w increases?
- Explain how the intensity from the three slits together at a long distance from the slits can be described by

$$I \propto \left| F \{ f_{\text{tot}}(x) \} \right|^2 = \left| F \left\{ \sum_{n=-1}^1 f_n(x) \otimes \delta(x - na) \right\} \right|^2, \quad (1)$$

where F denotes Fourier transformation and $\otimes$ is convolution. Have we made any assumptions about the coherence of the light source to arrive at this expression?

Find a more specific expression for the intensity, starting out from eq. (1). Comment on use of the convolution theorem.

- A positive lens is used to image the lattice. The lens has a focal length $f = 15.0 \text{ cm}$ and diameter $D = 2.00 \text{ cm}$. If the lens is placed at a distance $s = 18.0 \text{ cm}$ behind the lattice, how many diffraction orders will pass through the lens, and what consequences will this have for the imaging?

Task 4 You as a consultant

- a) Explain shortly why laser light is
- (approximately) monochromatic
 - (approximately) parallel
- b) Show that the relative phase change $\Delta\varphi$ between the ordinary and extraordinary ray when going through a uniaxial crystal with its optical axis parallel to the entrance plane is given by

$$\Delta\varphi = \frac{2\pi}{\lambda} |n_o - n_e| d.$$

Use this to calculate the thickness d for a piece of quartz to act as a quarter wave plate, given the wavelength $\lambda = 632 \text{ nm}$.

Finally, argue why the Jones matrix

$$M_{1/4} = \exp(i\pi/4) \begin{pmatrix} 1 & 0 \\ 0 & -i \end{pmatrix}$$

mathematically can serve as model for a quarter wave plate.

Given: For quartz, $n_o = 1.5442$ and $n_e = 1.5533$.

Formelliste for emnet TFY4195 Optikk**(VEDLEGG)**Vektorstørrelser er i **uthevet** skrift.**Fysiske konstanter:**

Ett mol: $M(^{12}\text{C}) = 12,000 \text{ g}$
 $k_B = 1,3807 \cdot 10^{-23} \text{ J/K}$
 $\epsilon_0 = 8,8542 \cdot 10^{-12} \text{ C}^2/\text{Nm}^2$
 $e = 1,6022 \cdot 10^{-19} \text{ C}$
 $c = 2,998 \cdot 10^8 \text{ m/s}$

$1\text{u} = 1,6605 \cdot 10^{-27} \text{ kg}$
 $R = N_A k_B = 8,3145 \text{ J mol}^{-1} \text{ K}^{-1}$
 $\mu_0 = 4\pi \cdot 10^{-7} \text{ N/A}^2$
 $m_e = 9,1094 \cdot 10^{-31} \text{ kg}$
 $h = 6,6261 \cdot 10^{-34} \text{ Js}$

$N_A = 6,0221 \cdot 10^{23} \text{ mol}^{-1}$
 $0^\circ\text{C} = 273,15 \text{ K}$
 $g = 9,81 \text{ m/s}^2$

Elektrisitet og magnetisme:

$$\oint_S \mathbf{E} \cdot d\mathbf{A} = \oint_S E_n dA = \frac{Q_{\text{inni}}}{\epsilon_0}$$

$$\nabla \cdot \mathbf{D} = \rho$$

$$\oint_S \mathbf{B} \cdot d\mathbf{A} = \oint_S B_n dA = 0$$

$$\nabla \cdot \mathbf{B} = 0$$

$$\oint_C \mathbf{E} \cdot d\mathbf{s} = \epsilon = -\frac{d\Phi_m}{dt} = -\frac{d}{dt} \int_S B_n dA = -\int_S \frac{\partial B_n}{\partial t} dA$$

$$\nabla \times \mathbf{E} = -\frac{\partial \mathbf{B}}{\partial t}$$

$$\oint_C \mathbf{B} \cdot d\mathbf{s} = \mu_0 (I_{\text{inni}} + I_d), \quad I_d = \epsilon_0 \frac{d\Phi_E}{dt} = \epsilon_0 \int_S \frac{\partial E_n}{\partial t} dA$$

$$\nabla \times \mathbf{B} = \mu_0 \mathbf{J} + \mu_0 \epsilon_0 \frac{\partial \mathbf{E}}{\partial t}$$

$$\mathbf{S} = c^2 \epsilon_0 \mathbf{E} \times \mathbf{B} \quad I \equiv \langle S \rangle_T = \frac{\epsilon_0 c}{2} E_0^2 = \epsilon_0 c \langle E^2 \rangle_T$$

$$\mathbf{D} = \epsilon \mathbf{E} \quad \mathbf{B} = \mu \mathbf{H}$$

Bølger, refraksjon og refleksjon:

$$\nabla^2 \psi = \frac{1}{v^2} \frac{\partial^2 \psi}{\partial t^2} \quad (\nabla^2 + k^2)U = 0 \quad I(r) = |U(r)|^2 \quad I = I_1 + I_2 + 2\sqrt{I_1 I_2} \cos \delta$$

$$U(\mathbf{r}) = \exp(i\mathbf{k} \cdot \mathbf{r}) \quad U(r) = \frac{A}{r} \exp(ikr) \approx \frac{A}{z} \exp(ikz) \exp\left[ik \frac{x^2 + y^2}{2z}\right]$$

$$n_1 \sin \theta_1 = n_2 \sin \theta_2$$

$$r_s = r_\perp = \left(\frac{E_{0r}}{E_{0i}} \right)_\perp = \frac{n_i \cos \theta_i - n_t \cos \theta_t}{n_i \cos \theta_i + n_t \cos \theta_t}$$

$$r_p = r_\parallel = \left(\frac{E_{0r}}{E_{0i}} \right)_\parallel = \frac{n_t \cos \theta_i - n_i \cos \theta_t}{n_i \cos \theta_t + n_t \cos \theta_i}$$

$$t_s = t_\perp = \left(\frac{E_{0t}}{E_{0i}} \right)_\perp = \frac{2n_i \cos \theta_i}{n_i \cos \theta_i + n_t \cos \theta_t}$$

$$t_p = t_\parallel = \left(\frac{E_{0t}}{E_{0i}} \right)_\parallel = \frac{2n_i \cos \theta_i}{n_i \cos \theta_t + n_t \cos \theta_i}$$

Jones og Stokes vektorer:

$$\mathbf{E} = \begin{bmatrix} E_x(t) \\ E_y(t) \end{bmatrix} \quad \mathbf{S} = \begin{bmatrix} 2I_0 \\ 2I_1 - 2I_0 \\ 2I_2 - 2I_0 \\ 2I_3 - 2I_0 \end{bmatrix}$$

Geometrisk optikk:

$$\frac{1}{s} + \frac{1}{s'} = \frac{1}{f} \quad \frac{1}{f} = \frac{n_{lens} - n_{medium}}{n_{medium}} \left(\frac{1}{R_1} - \frac{1}{R_2} \right)$$

$$\begin{bmatrix} n\alpha \\ y \end{bmatrix} \quad R_1 = \begin{pmatrix} 1 & -P_1 \\ 0 & 1 \end{pmatrix}, \quad P_1 = \frac{n_{t1} - n_{i1}}{R} \quad T_{21} = \begin{pmatrix} 1 & 0 \\ d_{21}/n_{t1} & 1 \end{pmatrix}$$

Diffraksjon:

$$U(X, Y, z) = \frac{1}{i\lambda z} e^{ikz} e^{ik \frac{(X^2 + Y^2)}{2z}} \iint U(x, y, 0) \exp \left[\frac{ik}{2z} (x^2 + y^2) - i(k_x x + k_y y) \right] dx dy$$

$$k_x = \frac{kX}{z}, k_y = \frac{kY}{z}, k = \frac{2\pi}{\lambda}$$

Fouriertransformasjon:

$$F(k_x, k_y) = \iint f(x, y) \exp(i(k_x x + k_y y)) dx dy$$

$$\mathcal{F}\{f(x - x_0)\} = \exp(ik_x x_0) \mathcal{F}\{f(x)\}$$

$$h(x) = f(x) \otimes g(x) \Rightarrow \mathcal{F}\{h(x)\} = \mathcal{F}\{f(x)\} \mathcal{F}\{g(x)\}$$

$$\mathcal{F}\left\{\text{circ}\left(\frac{\rho}{a}\right)\right\} = \pi a^2 2 \frac{J_1(\kappa a)}{\kappa a}, \quad \kappa = \sqrt{k_x^2 + k_y^2}, \quad \rho = \sqrt{x^2 + y^2}, \quad J_1(3,83) = 0. \quad \rho_1 = 1,22 \frac{R\lambda}{2a}$$

$$\mathcal{F}\left\{\text{rect}\left(\frac{x}{w}\right)\right\} = w \frac{\sin(k_x w / 2)}{k_x w / 2}$$

$$\mathcal{F}\left\{\sum_{n=1}^N \delta(x - na)\right\} = e^{ik_x a(N+1)/2} \frac{\sin(k_x a N / 2)}{\sin(k_x a / 2)}$$