

(d)

Problem 1

$$1) \begin{pmatrix} 1 & d/n \\ 0 & 1 \end{pmatrix}$$

$$2) \begin{pmatrix} 1 & 0 \\ -P & 1 \end{pmatrix},$$

$$P = +\frac{(n'-n)}{R}$$

$$P = \frac{1}{f}, \text{ for lens}$$

$$3) \begin{pmatrix} \beta & C \\ -P & 1/\beta \end{pmatrix}$$

d) must derive

$$\begin{pmatrix} 1 & d/n' \\ 0 & 1 \end{pmatrix} \begin{pmatrix} A & B \\ C & D \end{pmatrix} \begin{pmatrix} 1 & d/n \\ 0 & 1 \end{pmatrix} = \begin{pmatrix} A' & B' \\ C' & D' \end{pmatrix}$$

$$= \begin{pmatrix} A + C \frac{d}{n'} & B + A \frac{d}{n} + D \frac{d}{n'} + C \frac{dd'}{nn'} \\ C & D + C \frac{d}{n} \end{pmatrix}$$

$$\beta = 1 \text{ (Principal planes)} \text{ hence } A' = D' = 1$$

$$\therefore A + C \frac{d}{n'} = 1 \Rightarrow \boxed{h' = \frac{(1-A)}{C} n'}$$

$$D + C \frac{d}{n} = 1 \Rightarrow \boxed{h = \frac{(1-D)}{C} n}$$

also accepting when $n = n' = 1$

(e)

$$\begin{pmatrix} 1 & 0 \\ f_2 & 1 \end{pmatrix} \begin{pmatrix} 1 & d \\ 0 & 1 \end{pmatrix} \begin{pmatrix} 1 & 0 \\ +\frac{1}{f_1} & 1 \end{pmatrix} \begin{pmatrix} 1 & d \\ 0 & 1 \end{pmatrix} \begin{pmatrix} 1 & 0 \\ -\frac{1}{f_1} & 1 \end{pmatrix} =$$

$$\begin{pmatrix} 1 & 0 \\ -\frac{1}{f_2} & 1 \end{pmatrix} \begin{pmatrix} 1 & d \\ 0 & 1 \end{pmatrix} \begin{pmatrix} 1 & 0 \\ \frac{1}{f_1} & 1 \end{pmatrix} \begin{pmatrix} 1 - \frac{d}{f_1} & d \\ -\frac{1}{f_1} & 1 \end{pmatrix}$$

$$\begin{pmatrix} 1 & 0 \\ -\frac{1}{f_2} & 1 \end{pmatrix} \begin{pmatrix} 1 & d \\ 0 & 1 \end{pmatrix} \begin{pmatrix} 1 - \frac{d}{f_1} & d \\ \frac{1}{f_1} - \frac{d}{f_1^2} & 1 - \frac{d}{f_1} \end{pmatrix}$$

$$\begin{pmatrix} 1 - \frac{d}{f_1} - \frac{d^2}{f_1^2} & d + \frac{1}{f_1} d^2 + d \\ -\frac{1}{f_2} (1 - \frac{d}{f_1}) - \frac{d}{f_1^2} (\frac{d}{f_1} + 1) & -\frac{d}{f_2} + (-\frac{d}{f_2} + 1) (\frac{d}{f_1} + 1) \end{pmatrix}$$

$$\begin{pmatrix} 1 - \frac{d}{f_1} - \frac{d^2}{f_1^2} & 2d + \frac{d^2}{f_1} \\ -\frac{1}{f_2} + \frac{d}{f_1 f_2} + \frac{d^2}{f_1^2 f_2} - \frac{d}{f_1^2} & 1 - \frac{2d}{f_2} - \frac{d^2}{f_1 f_2} + \frac{d}{f_1^2} \end{pmatrix}$$

(f)

f) power is $P = -C$

$$\therefore P = - \left[-\frac{1}{f_2} + \frac{d}{f_1 f_2} + \frac{d^2}{f_1^2 f_2} - \frac{d}{f_1^2} \right]$$

Problem 2

$$i) \quad U(x, y, z) = e^{i\vec{k}_1 \cdot \vec{r}} + e^{i\vec{k}_2 \cdot \vec{r}} + e^{i\vec{k}_3 \cdot \vec{r}}$$

also accept

$$U(x, y, z) = e^{i(\vec{k}_1 \cdot \vec{r} + \phi_1)} + e^{i(\vec{k}_2 \cdot \vec{r} + \phi_2)} + e^{i(\vec{k}_3 \cdot \vec{r} + \phi_3)}$$

where

$$\vec{k}_i \cdot \vec{r} = k_{ix}x + k_{iy}y + k_{iz}z$$

for this case $k_{iy} = 0$

$$\text{and} \quad k_{x1} = k \sin \theta_1$$

$$k_{x2} = k \sin \theta_2$$

$$k_{x3} = k \sin \theta_3$$

and

$$k_{z1} = k \cos \theta_1$$

$$k_{z2} = k \cos \theta_2$$

$$k_{z3} = k \cos \theta_3$$

steps regarding θ_i

(c)

$$I = U U^*$$

$$\propto [e^{ik_{x1}x} + e^{ik_{x2}x} + e^{ik_{x3}x}]^*$$

$$\propto [e^{ik_{x1}x} + e^{ik_{x2}x} + e^{ik_{x3}x}] [e^{-ik_{x1}x} + e^{-ik_{x2}x} + e^{-ik_{x3}x}]$$

$$\propto 1 + e^{i(k_{x1}-k_{x2})x} + e^{i(k_{x1}-k_{x3})x} + e^{i(k_{x2}-k_{x1})x} + 1 + e^{i(k_{x2}-k_{x3})x} + e^{i(k_{x3}-k_{x1})x} + e^{i(k_{x3}-k_{x2})x} + 1$$

$$= 3 + 2 \cos[(k_{x1}-k_{x2})x] + 2 \cos[(k_{x1}-k_{x3})x] + 2 \cos[(k_{x2}-k_{x3})x]$$

(This for case where arbitrary phases $\phi_1 = \phi_2 = \phi_3 = 0$)

$$k_{x1} = k \sin \theta_1$$

$$k_{x2} = k \sin \theta_2$$

$$k_{x3} = k \sin \theta_3$$

(b)

$$U(x, y, 0) = e^{ik_{x1}x} + e^{ik_{x2}x} + e^{ik_{x3}x}$$

$$k_{x1} = k \sin \theta_1$$

$$k_{x2} = k \sin \theta_2$$

$$k_{x3} = k \sin \theta_3$$

Problem 3

(a)

Fresnel's diffraction integral

$$U(x, y, z) = \frac{1}{i\lambda z} e^{ik[z + \frac{1}{2z}(x^2 + y^2)]} \iint_{-\infty}^{\infty} U(x', y', 0) e^{-i\frac{k}{2z}(x'^2 + y'^2)} e^{-i\frac{k}{2z}(x^2 + y^2)} dx' dy'$$

here $U(x', y', 0) = A t(x', y')$

also, for Fraunhofer diffraction we have $2z \gg k(x^2 + y^2)$, hence last term in integral U and Fresnel integral reduces to

$$U(x, y, z) = \frac{1}{i\lambda z} e^{ik[z + \frac{1}{2z}(x^2 + y^2)]} \iint A t(x', y') e^{-i\frac{k}{2z}(x'^2 + y'^2)} dx' dy'$$

$$= \frac{1}{i\lambda z} e^{ik[z + \frac{1}{2z}(x^2 + y^2)]} A \iint t(x', y') e^{-i\frac{k}{2z}(x'^2 + y'^2)} dx' dy'$$

$$= \frac{1}{i\lambda z} e^{ik[z + \frac{1}{2z}(x^2 + y^2)]} A T\left(\frac{kx}{z}, \frac{ky}{z}\right)$$

$$\text{where } T\left(\frac{kx}{z}, \frac{ky}{z}\right) = \iint t(x', y') e^{-i\frac{k}{2z}(x'^2 + y'^2)} dx' dy'$$

hence

$$U(x, y, z) = \frac{1}{i\lambda z} e^{ik[z + \frac{1}{2k}(x^2 + y^2)]} A T\left(\frac{kx}{z}, \frac{ky}{z}\right)$$

also

$$I(x, y, z) = U U^* \\ = \left(\frac{A}{\lambda z}\right)^2 \left| T\left(\frac{kx}{z}, \frac{ky}{z}\right) \right|^2$$

(b)

Fraunhofer diffraction given by Fourier transform

$$FT\{t(x)\} = \int_{-d-L}^{d+L} e^{-ik_x x} dx + \int_{d-L}^{d+L} e^{-ik_x x} dx$$

$$\text{where } k_x = \frac{kx}{z}$$

$$\begin{aligned} &= \left[\frac{e^{-ik_x x}}{-ik_x} \right]_{-d-L}^{d+L} + \left[\frac{e^{-ik_x x}}{-ik_x} \right]_{d-L}^{d+L} \\ &= \left[\frac{e^{-ik_x(d+L)} - e^{-ik_x(-d-L)}}{-ik_x} \right] + \left[\frac{e^{-ik_x(d+L)} - e^{-ik_x(d-L)}}{-ik_x} \right] \\ &= e^{+ik_x d} \left[\frac{e^{-ik_x L} - e^{ik_x L}}{-ik_x} \right] + e^{-ik_x d} \left[\frac{e^{-ik_x L} - e^{ik_x L}}{-ik_x} \right] \\ &= L e^{ik_x d} \left[\frac{2 \sin(k_x L)}{k_x L} \right] + L e^{-ik_x d} \left[\frac{2 \sin(k_x L)}{k_x L} \right] \\ &= 2L e^{ik_x d} \text{sinc}(k_x L) + 2L e^{-ik_x d} \text{sinc}(k_x L) \\ &= 2L \text{sinc}(k_x L) [e^{ik_x d} + e^{-ik_x d}] \\ &= 2L \text{sinc}(k_x L) 2 \cos(k_x d) \\ &= 4L \text{sinc}(k_x L) \cos(k_x d) \end{aligned}$$

$$\Rightarrow \text{Fraunhofer pattern} \\ \therefore I(x, y, z) = \left(\frac{4L}{\lambda z}\right)^2 \text{sinc}^2(k_x L) \cos^2(k_x d), \quad k_x = \frac{kx}{z}$$

(b) also take this:
(and variations)

Fraunhofer diffraction given by Fourier transform

$$T\{t(x)\} = \int_{-d-L}^{d+L} e^{-ik_x x} dx + \int_{d-L}^{d+L} e^{-ik_x x} dx$$

$$\text{where } k_x = \frac{kx}{z}$$

$$\begin{aligned} &= \left[\frac{e^{-ik_x x}}{-ik_x} \right]_{-d-L}^{d+L} + \left[\frac{e^{-ik_x x}}{-ik_x} \right]_{d-L}^{d+L} \\ &= \left[\frac{e^{-ik_x(d+L)} - e^{-ik_x(-d-L)}}{-ik_x} \right] + \left[\frac{e^{-ik_x(d+L)} - e^{-ik_x(d-L)}}{-ik_x} \right] \\ &= \frac{1}{k_x} \left[e^{ik_x(d+L)} - e^{ik_x(d-L)} + e^{-ik_x(d-L)} - e^{-ik_x(d+L)} \right] \end{aligned}$$

$$= \frac{1}{ik_x} \{ 2 \sin[k_x(d+L)] - 2 \sin[k_x(d-L)] \}$$

$$= \frac{2}{k_x} \{ \sin[k_x(d+L)] - \sin[k_x(d-L)] \}, \quad k_x = \frac{kx}{z}$$

$$= 2(d+L) \frac{\sin[k_x(d+L)]}{k_x(d+L)} - 2(d-L) \frac{\sin[k_x(d-L)]}{k_x(d-L)}$$

$$= 2(d+L) \text{sinc}[k_x(d+L)] - 2(d-L) \text{sinc}[k_x(d-L)] \quad k_x = \frac{kx}{z}$$

$$\text{Intensity } I = \left(\frac{1}{\lambda z}\right)^2 \left| \{ FT \} \right|^2$$

(c)

approximation made

$$2z \gg k(x'^2 + y'^2)$$

for this case we can make

$$2z \gg kx'^2$$

$$\text{and here } x' \leq 2(d+L)$$

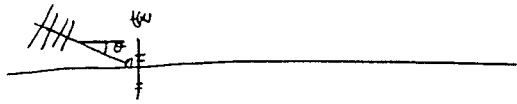
$$\therefore 2z \gg k[2(d+L)]^2$$

$$z \gg \frac{k}{2} 4(d+L)^2$$

$$z \gg \frac{2\pi}{\lambda} 2(d+L)^2$$

$$z \gg \frac{4\pi}{\lambda} (d+L)^2$$

(d)



$$u(x, y, z) = \frac{1}{i\lambda z} e^{ik[z + \frac{1}{2z}(x'^2 + y'^2)]} \iint u(x', y', 0) e^{-i\frac{k}{2}(xx' + yy')} dx' dy'$$

for thin case $u(x', y', 0) = e^{ik \sin \theta x} t(x)$

$$u(x, y, z) = \frac{1}{i\lambda z} e^{ik[z + \frac{1}{2z}(x'^2 + y'^2)]} \iint e^{ikx' \sin \theta} t(x') e^{-i\frac{k}{2}(xx' + yy')} dx' dy'$$

$$= \frac{1}{i\lambda z} e^{ik[z + \frac{1}{2z}(x'^2 + y'^2)]} \int t(x') e^{-i\frac{k}{2}(x + z \sin \theta)x'} dx'$$

$$= \frac{1}{i\lambda z} e^{ik[z + \frac{1}{2z}(x'^2 + y'^2)]} T\left[\frac{k}{2}(x + z \sin \theta)\right]$$

where $T(\)$ is Fourier transform

$\Rightarrow$ diffraction pattern is displaced by $z \sin \theta$

(e)

the Fourier plane in image space is at the plane conjugate to the light source, hence, the Fourier plane is at distance d' such that

$$\frac{1}{d} + \frac{1}{d'} = \frac{1}{f}$$

$$\frac{1}{50\text{cm}} + \frac{1}{d'} = \frac{1}{10\text{cm}}$$

$$\Rightarrow \boxed{d' = 12.5\text{cm}}$$

$\Rightarrow$ Fourier plane is at 12.5 cm to the right of the H' principal plane