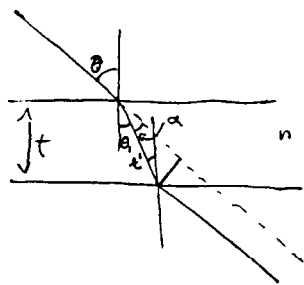


1(a)

PROBLEM 1



$$\alpha = \theta - \theta_1$$

also $\sin \theta = n \sin \theta_1$
 $\theta_1 = \sin^{-1} \left[\frac{\sin \theta}{n} \right]$

now,

$$\cos \theta_1 = \frac{t}{t'}$$

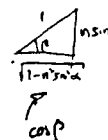
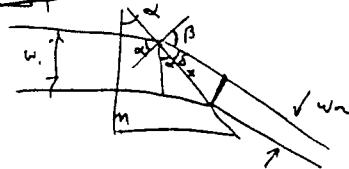
$$\Rightarrow t' = \frac{t}{\cos \theta_1}$$

$$d = t' \sin \alpha$$

$$= \frac{t}{\cos \theta_1} \sin \left[\theta - \sin^{-1} \left\{ \frac{\sin \theta}{n} \right\} \right]$$

$$d = \frac{t}{\cos \theta_1} \sin [\theta - \theta_1] \quad \text{where } \theta_1 = \sin^{-1} \left[\frac{\sin \theta}{n} \right]$$

1(b)



have, $\cos \alpha = \frac{w_r}{x}$

$$\sin \beta = n \sin \alpha \Rightarrow \beta = \sin^{-1} [n \sin \alpha]$$

$$\delta = 90 - \beta$$

also, $\sin \delta = \frac{w_r}{x}$

$$\therefore w_r = x \sin \delta$$

$$= \left[\frac{w_r}{\cos \alpha} \right] \sin \delta$$

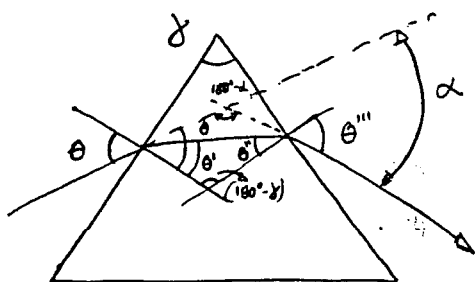
$$= \left[\frac{w_r}{\cos \alpha} \right] \sin \left(\frac{\pi}{2} - \beta \right)$$

$$= \left[\frac{w_r}{\cos \alpha} \right] \sqrt{1 - n^2 \sin^2 \alpha}$$

$$\sin \left(\frac{\pi}{2} - \beta \right) = \sin \left(\frac{\pi}{2} \right) \cos \beta - \cos \left(\frac{\pi}{2} \right) \sin \beta$$

$$\text{since } \sin^2 \beta + \cos^2 \beta = 1$$

1(c)



from quadrilateral in figure

$$180^\circ - \alpha + \theta + \theta'' + 180^\circ - \gamma = 360^\circ$$

$$\therefore \alpha = \theta + \theta'' - \gamma$$

now, from Snell's law,

$$n \sin \theta' = \sin \theta$$

$$\sin \theta' = \frac{1}{n} \sin \theta$$

now $\theta' + (180^\circ - \gamma) + \theta'' = 180^\circ$
 $\theta'' = \gamma - \theta'$

$$n \sin \theta'' = \sin \theta'''$$

$$n \sin [\gamma - \theta'] = \sin \theta'''$$

$$\sin \theta''' = n \sin [\gamma - \theta']$$

$$= n \sin(\gamma) \cos(\theta') - n \cos(\gamma) \sin(\theta')$$

$$= n \sin(\gamma) \cos(\theta') - n \cos(\gamma) \sin(\theta')$$

$$\sin \theta' = \frac{1}{n} \sin \theta$$

$$\cos \theta' = \sqrt{1 - \frac{1}{n^2} \sin^2 \theta}$$

$$\sin \theta''' = \left[n \sin(\gamma) \sqrt{1 - \frac{1}{n^2} \sin^2 \theta} \right] - n \cos(\gamma) \frac{1}{n} \sin(\theta)$$

$$\therefore \cos \theta''' = \sqrt{1 - \sin^2 \theta'''}$$

$$\therefore \theta''' = \sin^{-1} \left\{ \left[n \sin(\gamma) \sqrt{1 - \frac{1}{n^2} \sin^2 \theta} \right] - n \cos(\gamma) \sin(\theta) \right\}$$

∴

$$\alpha = \theta + \theta''' - \gamma$$

where θ''' given in last page.

to find minimum d then must take derivative and equate to zero.

ie must solve $\frac{\partial \alpha}{\partial \theta} = 0$

If they get this far then they get full credit!

(it gets very messy from here!)

Problem 2

2a)

using $\frac{1}{o} + \frac{1}{i} = \frac{1}{f}$

for first lens have

$$\frac{1}{20} + \frac{1}{i} = +\frac{1}{10}$$

$$\therefore \boxed{i = 20}$$

magnification is $\beta_1 = -\frac{20}{20} = -1$

for second lens have

$$o = -5$$

(since $i = 20$ above and separation between lenses is 15 cm)

$$\therefore \frac{1}{-5} + \frac{1}{i} = \frac{1}{-10}$$

$$\therefore \frac{1}{i} = -\frac{1}{10} + \frac{1}{5}$$

$$\therefore \boxed{i = 10}$$

magnification is $\beta_2 = \frac{-10}{-5} = +2$

MATRIX APPROACH

$$\begin{pmatrix} 1 & 0 \\ -1/f_2 & 1 \end{pmatrix} \begin{pmatrix} 1 & d_2 \\ 0 & 1 \end{pmatrix} \begin{pmatrix} 1 & 0 \\ -1/f_1 & 1 \end{pmatrix} \begin{pmatrix} 1 & d_1 \\ 0 & 1 \end{pmatrix} \begin{pmatrix} 1 & 0 \\ -1/f_1 & 1 \end{pmatrix} =$$

$$= \begin{pmatrix} 1 & 0 \\ -1/15 & 1 \end{pmatrix} \begin{pmatrix} 1 & 20 \\ 0 & 1 \end{pmatrix} \begin{pmatrix} 1 & 0 \\ 1/10 & 1 \end{pmatrix} \begin{pmatrix} 1 & 15 \\ 0 & 1 \end{pmatrix} \begin{pmatrix} 1 & 0 \\ -1/10 & 1 \end{pmatrix}$$

$$= \begin{pmatrix} 1 & 20 \\ -1/15 & -20/15 + 1 \end{pmatrix} \begin{pmatrix} 1 & 15 \\ 1/10 & 15/10 + 1 \end{pmatrix} \begin{pmatrix} 1 & 0 \\ -1/10 & 1 \end{pmatrix}$$

$$= \begin{pmatrix} 1 & 20 \\ -1/15 & -1/3 \end{pmatrix} \begin{pmatrix} 1 & 15 \\ 1/10 & 5/2 \end{pmatrix} \begin{pmatrix} 1 & 0 \\ -1/10 & 1 \end{pmatrix}$$

$$= \begin{pmatrix} 1 & 20 \\ -1/15 & -1/3 \end{pmatrix} \begin{pmatrix} 1 - 15/10 & 15 \\ 1/10 - 1/4 & 5/2 \end{pmatrix}$$

$$= \begin{pmatrix} 1 & 20 \\ -1/15 & -1/3 \end{pmatrix} \begin{pmatrix} -1/2 & 15 \\ -3/20 & 5/2 \end{pmatrix}$$

$$= \begin{pmatrix} -1/2 - 3 & 15 + 50 \\ -1/3 + 1/15 & -1 - 5/2 \end{pmatrix}$$

$$M_{vv'} = \begin{pmatrix} -7/2 & 65 \\ 1/12 & -11/6 \end{pmatrix}$$

if object is 20 cm away then

$$\begin{pmatrix} 1 & d \\ 0 & 1 \end{pmatrix} \begin{pmatrix} -7/2 & 65 \\ 1/12 & -11/6 \end{pmatrix} \begin{pmatrix} 1 & 20 \\ 0 & 1 \end{pmatrix}$$

$$= \begin{pmatrix} 1 & d \\ 0 & 1 \end{pmatrix} \begin{pmatrix} -7/2 & -70 + 65 \\ 1/12 & 70/12 - 11/6 \end{pmatrix}$$

$$= \begin{pmatrix} 1 & d \\ 0 & 1 \end{pmatrix} \begin{pmatrix} -7/2 & -5 \\ 1/12 & -1/6 \end{pmatrix}$$

$$= \begin{pmatrix} -7/2 + 1/12 d & -5 - 1/6 d \\ 1/12 & -1/6 \end{pmatrix}$$

hence, system matrix is

$$M = \begin{pmatrix} A' & B' \\ C' & D' \end{pmatrix} = \begin{pmatrix} -\frac{7}{2} + \frac{1}{12}d & -5 - \frac{1}{6}d \\ \frac{1}{12} & -\frac{1}{6} \end{pmatrix}$$

for imaging have $B' = 0$

$$\therefore -5 - \frac{1}{6}d = 0$$

$$-5 = \frac{1}{6}d$$

$$\therefore \boxed{d = -30 \text{ cm}}$$

MAGNIFICATION is given by $+A'$

$$\therefore -\frac{7}{2} + \frac{1}{12}d = M$$

$$\therefore \boxed{M = -6}$$

2(b)

Vertical polarization $\begin{pmatrix} 0 \\ 1 \end{pmatrix}$

polarizer at 45° $P = \begin{pmatrix} \frac{1}{2} & \frac{1}{2} \\ \frac{1}{2} & \frac{1}{2} \end{pmatrix}$

$\lambda/4$ plate

$$M = \begin{pmatrix} 1 & 0 \\ 0 & e^{i\pi/2} \end{pmatrix} = \begin{pmatrix} 1 & 0 \\ 0 & i \end{pmatrix}$$

$\therefore$ output polarization is

$$= \begin{pmatrix} 1 & 0 \\ 0 & i \end{pmatrix} \begin{pmatrix} \frac{1}{2} & \frac{1}{2} \\ \frac{1}{2} & \frac{1}{2} \end{pmatrix} \begin{pmatrix} 0 \\ 1 \end{pmatrix}$$

$$= \begin{pmatrix} 1 & 0 \\ 0 & i \end{pmatrix} \begin{pmatrix} \frac{1}{2} \\ \frac{1}{2} \end{pmatrix}$$

$$= \frac{1}{2} \begin{pmatrix} 1 \\ i \end{pmatrix} \quad \text{ie circular polarization}$$

for third lens have $\phi = 10$
(from last result and since second and third lenses separated by 20 cm)

$$\therefore \frac{1}{\phi} + \frac{1}{i} = \frac{1}{15}$$

$$\frac{1}{i} = \frac{1}{15} - \frac{1}{10}$$

$$\therefore \boxed{i = -30} \quad \text{magnification is } \beta_3 = -\frac{20}{10} = +3$$

and this is distance from last lens to find image

$$\Rightarrow \boxed{i = -30 \text{ cm}}$$

total magnification is

$$\beta_{\text{tot}} = \beta_1 \beta_2 \beta_3 \\ = (-1)(+2)(+3)$$

$$\text{MAGNIFICATION} = -6$$

Problem 3

3(a)

wave in one arm of interferometer

$$U_1 = e^{ikr}$$

wave propagating in other arm of interferometer

$$U_2 = e^{ik(r+2d)}$$

$\Rightarrow$ total E-field is

$$\boxed{\begin{aligned} E_{\text{TOT}} &= U_1 + U_2 \\ E_{\text{TOT}} &= e^{ikr} + e^{ik(r+2d)} \end{aligned}}$$

3(b)

Intensity given by $E_{\text{tot}} E_{\text{tot}}^*$

$$I = [e^{ikr} + e^{ik(r+2d)}][e^{-ikr} + e^{-ik(r+2d)}]$$

$$= 1 + 1 + e^{i[kr - kr - 2kd]} + e^{-i[kr - kr - 2kd]}$$

$$= 2 + 2 \cos(2kd)$$

$I \propto 1 + \cos(2kd)$

it is ok to ignore the 2 here

note, the optical path length is

$$\Delta \text{OPL} = 2kd$$

$$= 2 \frac{2\pi}{\lambda} d$$

if originally have constructive interference, then will get destructive interference when the optical path length changes by $\frac{\pi}{2}$.

The OPL changed by changing λ , so then

$$2 \frac{2\pi}{\lambda} d - 2 \frac{2\pi}{(\lambda + \Delta\lambda)} d = \frac{\pi}{2}$$

$$\frac{\lambda + \Delta\lambda - \lambda}{\lambda(\lambda + \Delta\lambda)} = \frac{1}{8d}$$

$$\Delta\lambda = \frac{1}{8d} [\lambda^2 + \lambda \Delta\lambda]$$

$$\Delta\lambda - \frac{1}{8d} \lambda \Delta\lambda = \frac{\lambda^2}{8d}$$

$$\Delta\lambda [1 - \frac{\lambda}{8d}] = \frac{\lambda^2}{8d}$$

$$\Delta\lambda = \frac{\lambda^2}{8d [1 - \frac{\lambda}{8d}]}$$

and if d is very large, then

$$\Delta\lambda \approx \frac{\lambda^2}{8d}$$

Problem 4

4-a) $A(k_x, k_y; 0) = \iint U(x, y, 0) e^{-i(k_x x + k_y y)} dx dy$

4-b) $A(k_x, k_y; z) = \iint U(x, y, z) e^{-i(k_x x + k_y y)} dx dy$

4-c)

from (4-b) have

$$U(x, y, z) = \left(\frac{1}{2\pi}\right)^2 \iint A(k_x, k_y; z) e^{i(k_x x + k_y y)} dk_x dk_y$$

however $U(x, y, z)$ must satisfy Helmholtz eq

$$\nabla^2 U + k^2 U = 0$$

hence, by plugging into eq. get

$$(\text{recall } \nabla^2 = \frac{\partial^2}{\partial x^2} + \frac{\partial^2}{\partial y^2} + \frac{\partial^2}{\partial z^2})$$

$$\left(\frac{\partial^2}{\partial x^2} + \frac{\partial^2}{\partial y^2} + \frac{\partial^2}{\partial z^2}\right) U + k^2 U = 0$$

$$\left(\frac{\partial^2}{\partial x^2} + \frac{\partial^2}{\partial y^2} + \frac{\partial^2}{\partial z^2}\right) \left[\left(\frac{1}{2\pi}\right)^2 \iint A(k_x, k_y; z) e^{i(k_x x + k_y y)} dk_x dk_y\right]$$

$$+ k^2 \left[\left(\frac{1}{2\pi}\right)^2 \iint A(k_x, k_y; z) e^{i(k_x x + k_y y)} dk_x dk_y\right] = 0$$

$$\frac{\partial^2 A}{\partial z^2} - k^2 A(k_x, k_y; z) = k_y^2 A(k_x, k_y; z) + k^2 A(k_x, k_y; z) = 0$$

$$\frac{\partial^2 A}{\partial z^2} + [k^2 - k_x^2 - k_y^2] A(k_x, k_y; z) = 0$$

$\Rightarrow$ set

$$\frac{d^2}{dz^2} A(k_x, k_y; z) + [k^2 - k_x^2 - k_y^2] A(k_x, k_y; z) = 0$$

and solution to this eq is

$$A(k_x, k_y; z) = A(k_x, k_y; 0) e^{i\sqrt{k^2 - k_x^2 - k_y^2} z}$$

(4-d)

Convergent wave $U(x, y, z) = A_0 \frac{d}{dz} e^{ik[z - \frac{1}{2(d-z)}(x^2 + y^2)]}$

and at $z=0$ (object plane)

$$U_0(x, y, 0) = U_0(x, y, 0) t(x, y) \\ = A_0 e^{i\frac{k}{2d}(x^2 + y^2)} t(x, y)$$

hence, from Fresnel's integral

$$U(x, y, z) = \frac{1}{i\lambda z} e^{ik[z + \frac{1}{2d}(x^2 + y^2)]} \iint A_0 e^{-i\frac{k}{2d}(x'^2 + y'^2)} t(x', y') \\ \cdot e^{-i\frac{k}{2}(x' + x)^2} e^{i\frac{k}{2}(y' + y)^2} dx' dy'$$

$$= \frac{1}{i\lambda z} e^{ik[z + \frac{1}{2d}(x^2 + y^2)]} \iint t(x', y') e^{-i\frac{k}{2d}(x'^2 + y'^2)} e^{i\frac{k}{2}[\frac{1}{d} - 1](x'^2 + y'^2)} dx'$$

and for $z=d$ this reduces to

$$U(x, y, d) = \frac{A_0}{i\lambda d} e^{ik[d + \frac{1}{2d}(x^2 + y^2)]} \iint t(x', y') e^{-i\frac{k}{2d}(x'^2 + y'^2)} dx' dy'$$

$$= \frac{A_0}{i\lambda d} e^{ik[d + \frac{1}{2d}(x^2 + y^2)]} \text{FT}\{t(x, y)\}$$