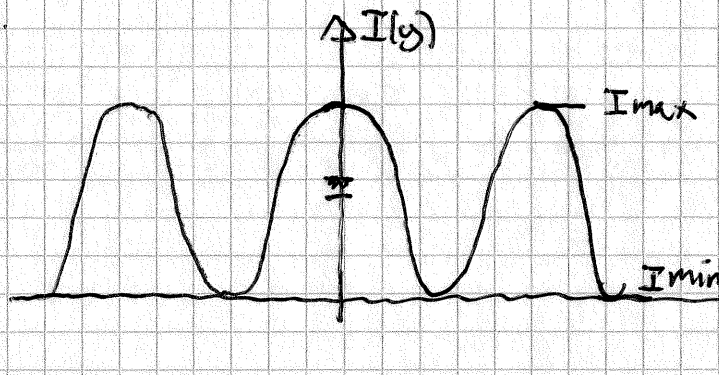


Oppgave 1 - Young's experiment

- a) With a point source, we have high spatial coherence. Assume that diffraction effects can be neglected (i.e., that slit widths are $\gg \lambda$)



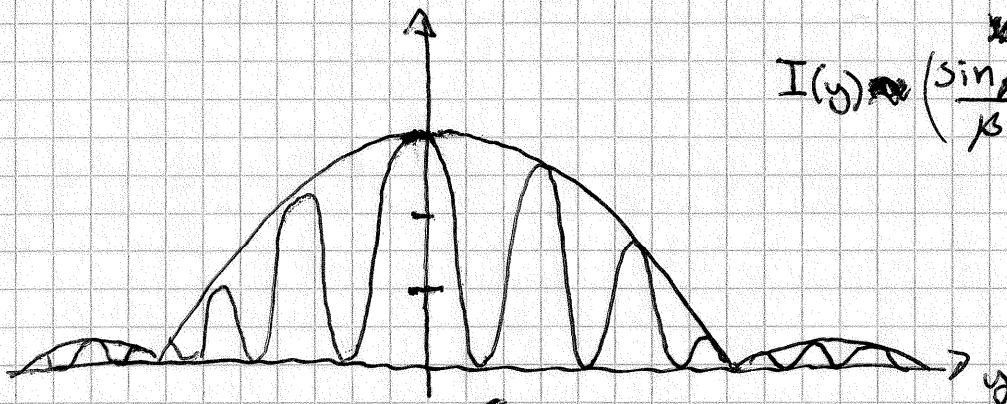
Identical slits, $V=1$

$$I(y) = 2 I_0 (1 + \cos ks)$$

path length difference

$$V = \frac{I_{\max} - I_{\min}}{I_{\max} + I_{\min}}$$

Note, one may include diffraction effects, which become important for $\lambda \rightarrow$ slit width. Difficult to realize without the use of a lens.



$$I(y) \approx \left(\frac{\sin \beta}{\beta} \right)^2 2 I_0 (1 + \cos ks)$$

$\beta \propto \frac{\text{slit width}}{\lambda}$

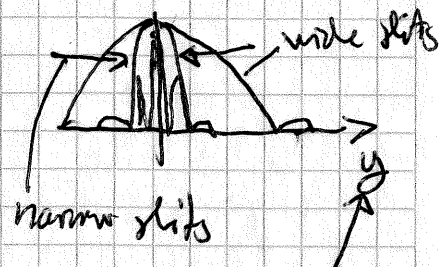
Note: Both answers are acceptable, since it was not clearly stated that diffraction effects could be neglected

- b) Upon increasing the source width (i.e. upon going from a point source to an extended source), the spatial coherence is reduced $\Rightarrow V \rightarrow 0$, effectively ~~adding~~ ~~adding~~ adding intensities in obs. plane from each source point. The contrast of the interference pattern is reduced and the pattern gets blurred ($V < 1$, and $V \rightarrow 0$ with increasing width)

Note: A number of students may have misread the question, and thought that either the slit widths increased or the distance between the slits increased.

b) note contd.

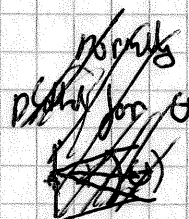
increasing slit width $\Rightarrow$ ~~more~~ less diffraction



increasing slit spacing (more likely misunderstanding)

- changes period of oscillations

see also note below.

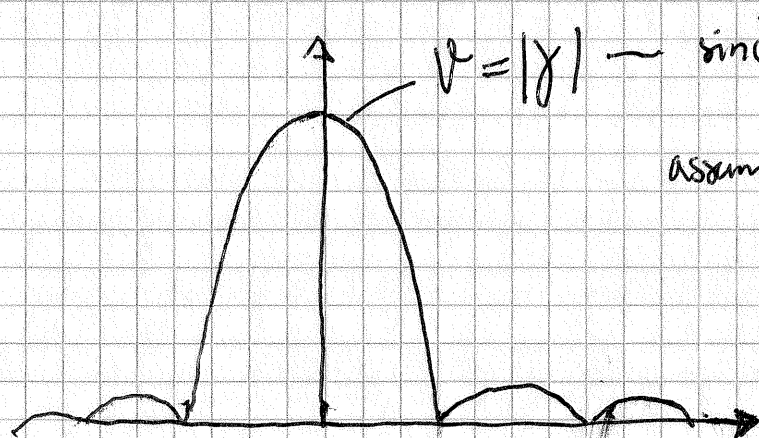


c) The visibility $V = \frac{I_{\max} - I_{\min}}{I_{\max} + I_{\min}} = | \gamma(0, a) |$

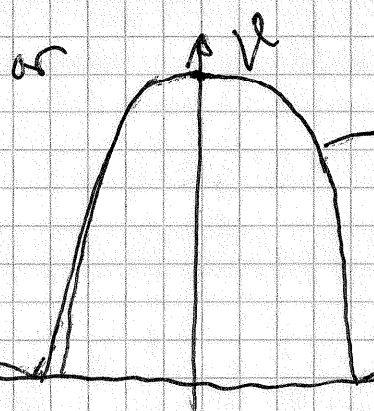
distance between sources

degree of coherence

$| \gamma(0, a) |$ is given by the Fourier Transform of the source. For a rectangular source it becomes a sinc function and for a ~~rectangular~~ circular source it becomes a Bessel function.



assumed rectangular source



source width (rectangular)

assumed circular source

source diameter (circular source)

2. Use $\frac{1}{s_o} + \frac{1}{s_i} = \frac{1}{f}$ in two steps

1st $\frac{1}{f} + \frac{1}{s_i'} = -\frac{1}{4} \Rightarrow \frac{1}{s_i'} = -\frac{1}{4} - \frac{1}{f} = \frac{-4-f}{f \cdot 4} \Rightarrow s_i' = -\frac{28}{11}$

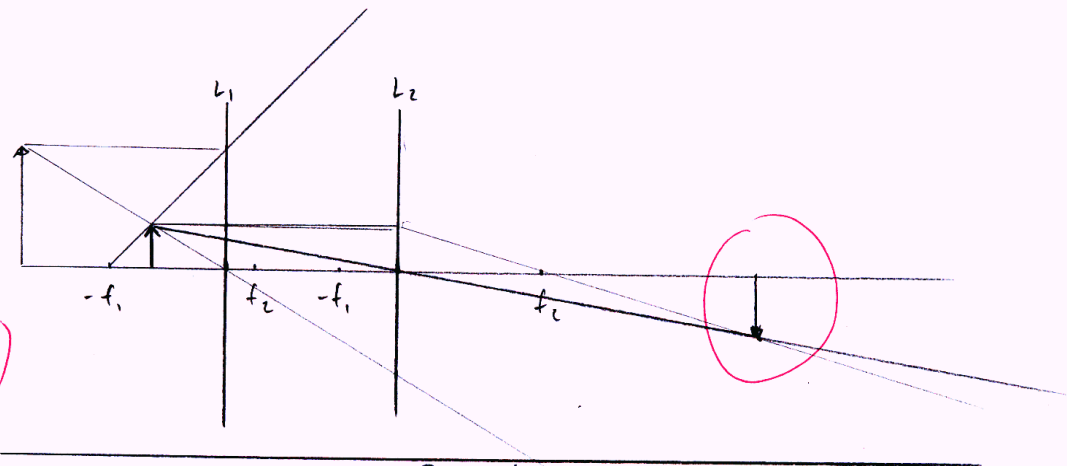
2nd $s_o' = 6 + \frac{28}{11} = \frac{94}{11}$

$\frac{11}{94} + \frac{1}{s_i'} = \frac{1}{5} \Rightarrow \frac{1}{s_i'} = \frac{1.94}{5.44} - \frac{11.5}{94.5} = \frac{94-55}{470} \Rightarrow s_i' = \frac{470}{39} \approx 12.1 \text{ cm after lens 2.}$

A)

$M = \left(-\frac{s_i}{s_o}\right) \left(-\frac{s_i'}{s_o'}\right) = \left(-\frac{-\frac{28}{11}}{\frac{94}{11}}\right) \left(-\frac{\frac{470}{39}}{\frac{94}{11}}\right) = \left(\frac{4}{11}\right) \left(-\frac{11.5}{39}\right) = -\frac{20}{39}$

B)



3.5

A)

slit 0.5 mm
 $\lambda = 642.8 \text{ nm}$
 $z = 20 \text{ m}$

Y



$\left(\frac{\sin \pi x}{\pi x}\right)^2$ function

$Y_{\min}$

distance between 1st minimum on each side

$2Y_{\min} = \frac{2z \cdot \lambda}{b} = 5.06 \text{ cm}$

B)

B) See eq. problem 7:

$\int e^{\frac{ik \cdot Y \cdot y}{2}} dy$ gives FT

$k = \frac{2\pi}{\lambda}$

$\int_{-\frac{b}{2}}^{\frac{b}{2}} e^{\frac{i2\pi Y y}{2\lambda}} dy = \left[\frac{e^{i\alpha y}}{i\alpha} \right]_{-\frac{b}{2}}^{\frac{b}{2}} = \left(\frac{e^{i\frac{b\alpha}{2}} - e^{-i\frac{b\alpha}{2}}}{2i\alpha \frac{1}{2}} \right) = \frac{\sin\left(\frac{b\alpha}{2}\right)}{\left(\frac{b\alpha}{2}\right)} = \frac{\sin\left(\frac{b\pi Y}{2\lambda}\right)}{\left(\frac{b\pi Y}{2\lambda}\right)}$

intensity $\Rightarrow$ square $\Rightarrow I_Y \propto$

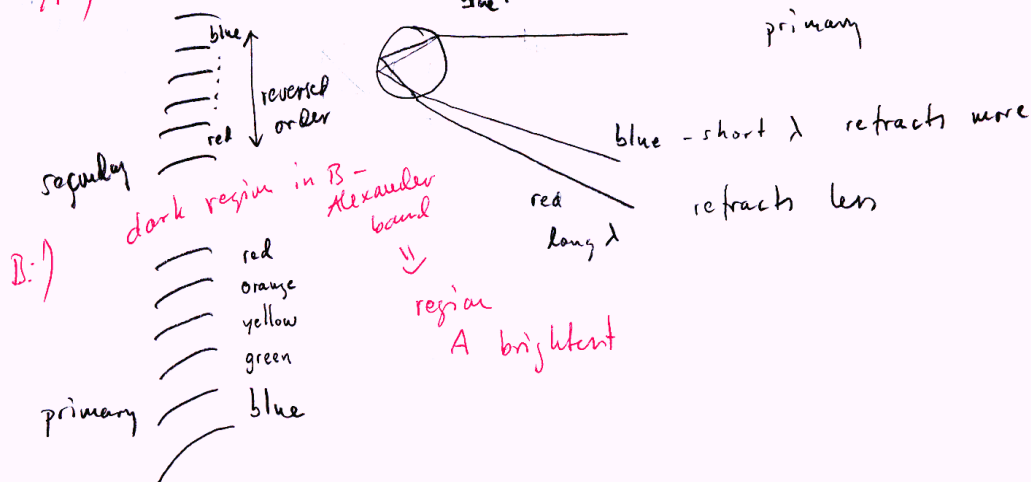
$\frac{\sin^2\left(\frac{b\pi Y}{2\lambda}\right)}{\left(\frac{b\pi Y}{2\lambda}\right)^2}$

min at

$Y \frac{b\pi}{2\lambda} = \pi \Rightarrow Y_{\min} = \frac{2 \cdot \lambda}{b}$

4.

A.)



B.)



5.

Ole Christian Rømer (1644-1710): Proved speed of light finite. Good estimation.

Johannes Kepler (1571-1630): Total internal reflection

Thomas Young (1773-1829): Interference, transverse waves.

James Maxwell (1831-1879): EM theory: unifying static & dynamic EM.

↑
A

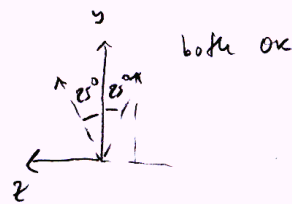
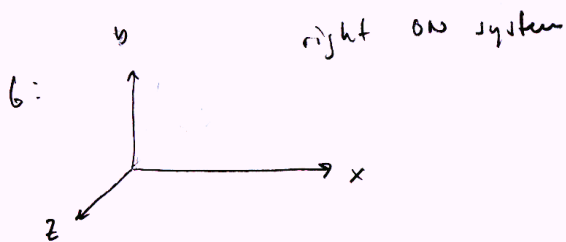
↑
B

7. A) Quadratic phase-exponential in integral should vanish i.e. $\frac{1}{z_1} + \frac{1}{z_2} = \frac{1}{f}$. The FT is to be found where the point source is imaged !!

B) $F(f_x) = \int_{-\infty}^{\infty} f(x) e^{i2\pi x \cdot f_x} dx$ identity: $f_x = \frac{1}{\lambda} \left(\frac{x}{z_1} + \frac{y}{z_2} \right)$
 $f_y = \frac{1}{\lambda} \left(\frac{y}{z_1} + \frac{v}{z_2} \right)$

↑
spatial frequency

other conventions are possible



$$\Rightarrow E_{0y} = E_0 \cdot \cos 25^\circ$$

$$E_{0z} = \pm E_0 \sin 25^\circ$$

several
Amplitude

Generally: $E(x,t) = A \cdot \cos(kx - \omega t + \phi)$

$$k = \frac{2\pi}{\lambda} \quad \frac{\omega}{k} = c \Rightarrow \omega = \frac{2\pi}{\lambda} \cdot c$$

$$\Rightarrow E(x,t) = A \cdot \cos\left(\frac{2\pi}{\lambda}(x - ct) + \phi\right)$$

Amplitude zero at $x=0, t=0 \Rightarrow$

$$0 = A \cos(0 + \phi) \Rightarrow \phi = 90^\circ \text{ (or } 270^\circ \text{)}$$

A: $\Rightarrow \vec{E}(x,t) = (E_0 \cos 25^\circ \hat{y} \pm E_0 \sin 25^\circ \hat{z}) \cos\left[\frac{2\pi}{\lambda}(x - ct) + 90^\circ\right]$

B: $\vec{\nabla} \times \vec{E} = -\frac{\partial \vec{B}}{\partial t}$

$$\hat{x} \left(\frac{\partial E_z}{\partial y} - \frac{\partial E_y}{\partial z} \right) + \hat{y} \cdot \left(0 - \frac{\partial}{\partial x} E_z(x,t) \right) + \hat{z} \left(\frac{\partial}{\partial x} E_y(x,t) + 0 \right)$$

$$= 0 \quad = 0 \quad \underbrace{-\frac{\partial}{\partial t} B_y} \quad \underbrace{-\frac{\partial}{\partial t} B_z}$$

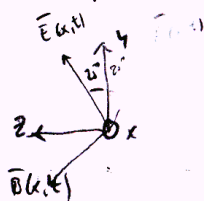
$\hat{x}$	$\hat{y}$	$\hat{z}$
$\frac{\partial}{\partial x}$	$\frac{\partial}{\partial y}$	$\frac{\partial}{\partial z}$
0	$E_y(x,t)$	$E_z(x,t)$

$\vec{E}$ & $\vec{B}$ must have the same phase (time & x-dependence)

$$\Rightarrow \hat{y}: \left(-\frac{2\pi}{\lambda} E_0 \sin 25^\circ \right) = -\left(-\frac{2\pi}{\lambda} c \right) B_y \Rightarrow B_y = -\frac{E_0}{c} \sin 25^\circ$$

$$\hat{z}: \left(\frac{2\pi}{\lambda} E_0 \cos 25^\circ \right) = \left(-\frac{2\pi}{\lambda} c \right) B_z \Rightarrow B_z = \frac{E_0}{c} \cos 25^\circ$$

$$\therefore \vec{B}(x,t) = \frac{1}{c} \cdot \left(-E_0 \sin 25^\circ \hat{y} + E_0 \cos 25^\circ \hat{z} \right) \cdot \cos\left[\frac{2\pi}{\lambda}(x - ct) + 90^\circ\right]$$



⑧ winning $L = \frac{3.125 + 2.083}{2} = 2.604$

Taken together the system matrix will be:

$$\begin{bmatrix} y_{out} \\ x_{out} \end{bmatrix} = \begin{bmatrix} 1 & 0 \\ -\frac{1}{f_2} & 1 \end{bmatrix} \begin{bmatrix} 1 & L \\ 0 & 1 \end{bmatrix} \begin{bmatrix} 1 & 0 \\ -\frac{1}{f_1} & 1 \end{bmatrix} \begin{bmatrix} y_{in} \\ x_{in} \end{bmatrix}$$

lens 2 free prop. lens 1

include numbers

$$M_{tot} = \begin{bmatrix} 1 - \frac{L}{f_1} & L \\ \frac{1}{f_2} \left(\frac{L}{f_1} - 1 \right) - \frac{1}{f_1} & 1 - \frac{L}{f_2} \end{bmatrix} = \begin{bmatrix} 0.16672 & 2.604 \\ -0.4000 & -0.25 \end{bmatrix}$$

A) from formula page

$$p = \frac{D}{C} = 0.685 \rightarrow F_1$$

$$q = -\frac{A}{C} = 0.4168 \rightarrow F_2$$

$$r = \frac{D-1}{C} = 3.125 \rightarrow H_1$$

$$s = \frac{1-A}{C} = -2.0832 \rightarrow H_2$$

$$v = \frac{D-1}{C} = 3.125 \rightarrow N_1$$

$$w = \frac{1-A}{C} = -2.0832 \rightarrow N_2$$

B) use thin lens formula

$$1) \frac{1}{10} + \frac{1}{s_i} = \frac{1}{3.125} \Rightarrow s_i = 4.5454$$

$$2) \text{ now } s_o' = L - 4.5454 = -1.9414$$

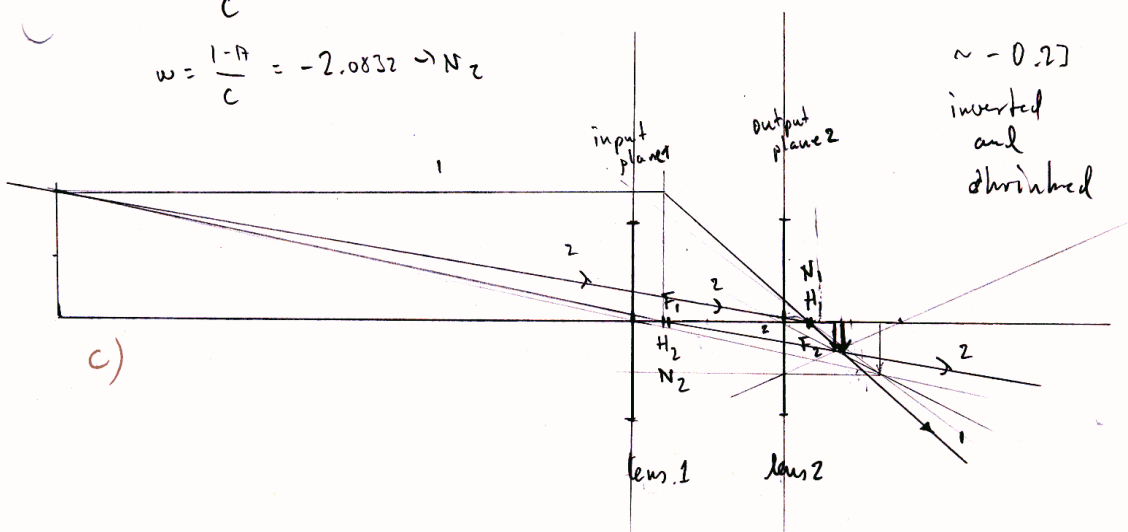
$$\frac{-1}{1.9414} + \frac{1}{s_i'} = \frac{1}{2.083} \Rightarrow s_i' = 1 \text{ cm}$$

$$M_{tt} = M_1 \cdot M_2 = \begin{pmatrix} -\frac{s_i}{s_o} & -\frac{s_i'}{s_o'} \end{pmatrix} = \begin{pmatrix} -\frac{4.54}{10} & -\frac{1}{-1.94} \end{pmatrix}$$

$$= -0.227$$

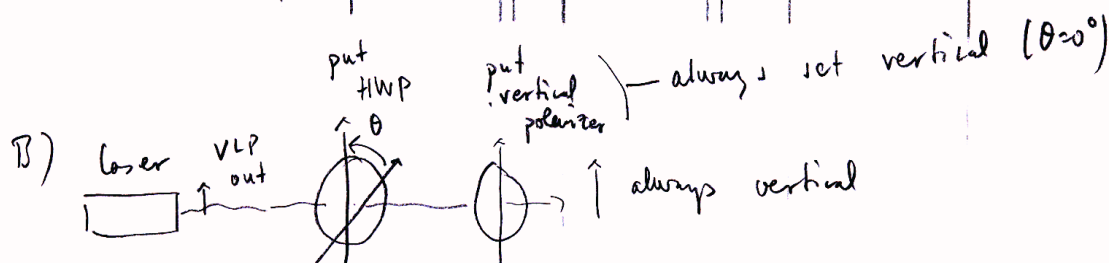
$$\sim -0.23$$

inverted and shrunk



9)

		0° out	0°	45° out	45°	90° out	90°	in
H:)	QWP	$\begin{pmatrix} 0 \\ \frac{1}{\sqrt{2}} + \frac{i}{\sqrt{2}} \end{pmatrix}$ VLP	$\begin{pmatrix} \frac{1}{\sqrt{2}} + \frac{i}{\sqrt{2}} & 0 \\ 0 & \frac{1}{\sqrt{2}} - \frac{i}{\sqrt{2}} \end{pmatrix}$	$\begin{pmatrix} \frac{1}{\sqrt{2}} \\ \frac{1}{\sqrt{2}} \end{pmatrix}$ RCP	$\begin{pmatrix} \frac{1}{\sqrt{2}} & \frac{i}{\sqrt{2}} \\ \frac{i}{\sqrt{2}} & \frac{1}{\sqrt{2}} \end{pmatrix}$	$\begin{pmatrix} 0 \\ \frac{1}{\sqrt{2}} + \frac{i}{\sqrt{2}} \end{pmatrix}$ VLP	$\begin{pmatrix} \frac{1}{\sqrt{2}} - \frac{i}{\sqrt{2}} & 0 \\ 0 & \frac{1}{\sqrt{2}} + \frac{i}{\sqrt{2}} \end{pmatrix}$	$\begin{pmatrix} 0 \\ 1 \end{pmatrix}$
	HWP	$\begin{pmatrix} 0 \\ 1 \end{pmatrix}$ VLP	$\begin{pmatrix} 1 & 0 \\ 0 & -1 \end{pmatrix}$	$\begin{pmatrix} 1 \\ 0 \end{pmatrix}$ HLP	$\begin{pmatrix} 0 & 1 \\ 1 & 0 \end{pmatrix}$	$\begin{pmatrix} 0 \\ 1 \end{pmatrix}$ VLP	$\begin{pmatrix} -1 & 0 \\ 0 & 1 \end{pmatrix}$	$\begin{pmatrix} 0 \\ 1 \end{pmatrix}$



HWP rotates polarization depending on θ

$$\Rightarrow \text{out} = \begin{pmatrix} \sin 2\theta \\ -\cos 2\theta \end{pmatrix} = \begin{pmatrix} \cos 2\theta \sin 2\theta \\ \sin 2\theta \cos 2\theta \end{pmatrix} \begin{pmatrix} 0 \\ 1 \end{pmatrix}$$

Then vertical polarizer takes out vertical component

$$\text{final} = \begin{pmatrix} 0 \\ -\cos 2\theta \end{pmatrix} = \begin{pmatrix} 0 & 0 \\ 0 & 1 \end{pmatrix} \begin{pmatrix} \sin 2\theta \\ -\cos 2\theta \end{pmatrix}$$

vertical polarizer ↑ from HWP

$$\text{intensity} \propto (-\cos 2\theta)^2$$

C) 0 out $\Rightarrow$ set $\theta = 45^\circ$ $(\cos 90^\circ)^2 = 0$

50% out $\Rightarrow$ set $\cos^2 2\theta = \frac{1}{2} \Rightarrow \theta = 22.5^\circ$

100% out $\Rightarrow$ set $\theta = 0^\circ$ $(\cos 0^\circ)^2 = 1$

HWP

LP always vertical

Problem 10:

(A) $\hat{r}_{012}^{s,p} = \hat{r}_{01}^{s,p} + t_{01}^{s,p} r_{12}^{s,p} t_{10}^{s,p} e^{i\delta}$

(optional, according to text in ~~exam~~ problem, use that $t_{01}^{s,p} t_{10}^{s,p} = 1 - (r_{01}^{s,p})^2$, bonus?)

(B) $d_1 \rightarrow 0 \Rightarrow \delta \rightarrow 0$

$\hat{r}_{02}^{s,p} = \hat{r}_{01}^{s,p} + t_{01}^{s,p} r_{12}^{s,p} t_{10}^{s,p}$ (also accept this answer)

This happens for $\delta = 0, 2\pi, 4\pi, 6\pi, \dots$

(C) $R^{s,p} = |\hat{r}_{012}^{s,p}|^2 = |\hat{r}_{01}^{s,p}|^2 + |t_{01}^{s,p}|^2 |t_{10}^{s,p}|^2 |r_{12}^{s,p}|^2 + 2 \operatorname{Re} \left\{ \hat{r}_{01}^{s,p} (t_{01}^{s,p})^* (t_{10}^{s,p})^* r_{12}^{s,p} \right\}$
 $= \underbrace{|\hat{r}_{01}^{s,p}|^2 + |t_{01}^{s,p}|^2 |t_{10}^{s,p}|^2 |r_{12}^{s,p}|^2}_a + \underbrace{2 |\hat{r}_{01}^{s,p}| |t_{01}^{s,p}| |t_{10}^{s,p}| |r_{12}^{s,p}| \cos(\delta')}_b$

May analyze ~~the~~ the details of δ' to include phase shifts upon reflection:

$\hat{r}_{01}^{s,p} = |\hat{r}_{01}^{s,p}| e^{i\delta_{01}^{s,p}}$, $r_{12}^{s,p} = |r_{12}^{s,p}| e^{i\delta_{12}^{s,p}}$, $t_{01}^{s,p} = |t_{01}^{s,p}| e^{i\delta_{01}^{s,p}}$

$\delta' = -\delta + \delta_{01}^{s,p} - \delta_{01}^{s,p} - \delta_{10}^{s,p} - \delta_{12}^{s,p}$

$\delta_{01}^{s,p} = \delta_{10}^{s,p} = 0$ from consideration of transmission coefficient (recall that $\hat{r}_{12}$ is transparent) $\{ (1 - r_{01}^2) > 0 \}$

$\delta_{01}^s = \pi$ and $\delta_{12}^s = \pi \Rightarrow \delta' = -\delta$ for s -pol. light transparent materials

$\delta_{01}^p = 0$ and $\delta_{12}^p = 0$, for $\theta_i < \theta_{\text{critical}} \Rightarrow \delta' = -\delta$

Hence $a = |\hat{r}_{01}^{s,p}|^2 + |t_{01}^{s,p}|^2 |t_{10}^{s,p}|^2 |r_{12}^{s,p}|^2$ $\left\{ \begin{array}{l} \text{for } \theta_i < \theta_{\text{critical}}, \\ \text{and transparent materials} \end{array} \right.$
 $b = 2 |\hat{r}_{01}^{s,p}| |t_{01}^{s,p}| |t_{10}^{s,p}| |r_{12}^{s,p}|$
 $|\delta'| = |\delta|$

optional,
bonus

(C)

Also accept answers of form:

$$R^{s1p} = (r_{01}^{s1p})^2 + \underbrace{(1 - r_{01}^{s1p})^2}_{\text{or } t_{01}^{s1p} t_{10}^{s1p}} r_{12}^{s1p} + 2 \underbrace{r_{01}^{s1p} (1 - r_{01}^{s1p})}_{\text{or } t_{01}^{s1p} t_{10}^{s1p}} r_{12}^{s1p} \cos(\delta)$$

where it is clearly stated that all reflection/transmission coefficients are assumed real, and that phase shifts upon reflection have been neglected.