A1

a) principle of least time, or equivalently minimal optical path

$$\mathcal{L}[AB] = \int n \, d\ell$$

$$\mathcal{L}[AB] = \min$$

$$\frac{c}{n} = v \qquad d\ell = v\,dt$$

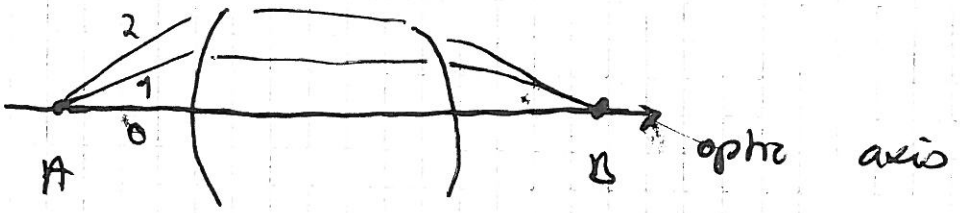
$$n = \frac{c}{v} = c\,dt \qquad \rightarrow \quad \frac{1}{v} = \frac{dt}{d\ell}$$

$$dt\, v = \frac{d\ell}{dt}$$

$$\mathcal{L}[AB] = \int n v \, dt = c \int dt$$

$\Rightarrow$ equivalent to minimal time.

b)



Consider image from A to B.

perfect image in geom. optics approx
(negligible aberrations)

⇒ all rays should be isochronous

consider rays $\angle^{0}[AB] = \angle^{1}[AB] = \angle^{2}[AB]$

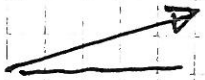
should have identical optical path length or
one time of transit.

The main approximation in Gaussian 1st order optics are:

Assume → 1) Spherical surfaces give perfect images 1p

2) Paraxial rays 2p

– rays making small angle with optic axis

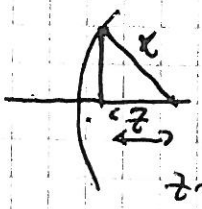


consider $\vec{s} = (\sin\alpha, \sin\beta, \cos\gamma)$

paraxial rays $(\alpha, \beta, 1)$

The from each refract surface

$$\vec{n}\,(n_2\vec{s}_2 - n_1\vec{s}_1) = 0$$



→ simplifies to

$$\begin{bmatrix} y \\ \alpha \end{bmatrix}' = \begin{bmatrix} 1 & 0 \\ \frac{n'}{n} & 1 \end{bmatrix} \begin{bmatrix} y \\ \alpha \end{bmatrix}$$

3) No absorption or reflection losses 1p

4) Rotationally symmetric surfaces. 1p

5p

Answer (for A1.d)

$$\nabla n(x) = \frac{d}{d\sigma}(n\vec{s})$$

$$\vec{s} = \frac{d\vec{r}}{d\sigma} = \frac{dx}{d\sigma}\hat{x} + \frac{dy}{d\sigma}\hat{y} + \frac{dz}{d\sigma}\hat{z}$$

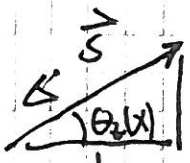
The eikonal in component form:

15p

$$\frac{dn}{dx} = \frac{d}{d\sigma}\left(n(x)\frac{dx}{d\sigma}\right)$$

$$\frac{dn}{dy} = \frac{d}{d\sigma}\left(n(x)\frac{dy}{d\sigma}\right) \qquad 0 = 0$$

$$\frac{dn}{dz} = \frac{d}{d\sigma}\left(n\frac{dz}{d\sigma}\right) = 0$$



$$\frac{dx}{d\sigma} = \sin\theta_z(x)$$

$$\frac{dz}{d\sigma} = \cos\theta_z(x)$$

$$\frac{d}{d\sigma}\left(n(x)\cos\theta_z(x)\right) = 0$$

$$n(x)\cos\theta_z(x) = \text{const}$$

$$n(0)\cos\theta_z(0) = C$$

15p

Here $n(x)\cos\theta_z(x) = n(0)\cos\theta_z(0)$, Generalized Snell's law.

My k

$$\cos\theta_z = \sin\theta_x$$

Mag x-comp is less of $\theta_x(x)$

$$\Rightarrow \quad \underline{n(x)\sin\theta_x(x) = n(0)\sin\theta_x(0)}$$

i.e. a generalized of Snell's Law.

$$\sin\theta_x(x) = \frac{n(0)\sin\theta_x(0)}{1.3 - |x|}$$

if $\dfrac{n(0)\sin\theta_x(0)}{1.3-|x|} \geq 1$, then ray must turn by total internal reflection

1p RAY TURNS BY TOTAL INTERNAL REFLECTION.

~~$n(x)\sin\theta_x(x)$~~

At turning point $\theta_z(x) = 0$

$$n(x_{tp}) = n(0)\cos\theta_z(0)$$

$$1.3 - |x| = 1.3\cos\theta_z(0)$$

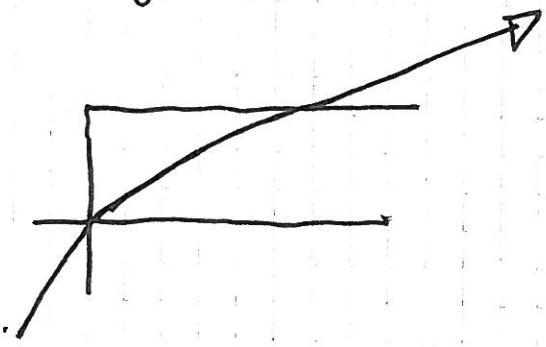
$$|x_{tp}| = 1.3\left(1 - \cos\theta_z(0)\right)$$

$$= 1.3\left(1 - \sqrt{1 - \sin^2\theta_z(0)}\right)$$

$$= 1.3\left(1 - \sqrt{1 - \left(\tfrac{1}{1.3}\right)^2 \sin^2\theta_0}\right)$$

[2p] $|x_{tip}| = 1.3 - \sqrt{1.3^2 - \sin^2\theta_0} = 1.3 - \sqrt{1.3^2 - \sin^2 80} = 0.1$

The ray will exit



if $\frac{n(0)\cos\theta_2(0)}{1} < 1$ $\quad 1.3 - 1.3 = 1$

$$\cos\theta_2(0) = \frac{1}{n(0)}\sqrt{n(0)^2 - \sin^2\theta_0}$$

$$\sqrt{n(0)^2 - \sin^2\theta_0} < 1$$

$$n(0)^2 - 1 < \sin^2\theta_0$$

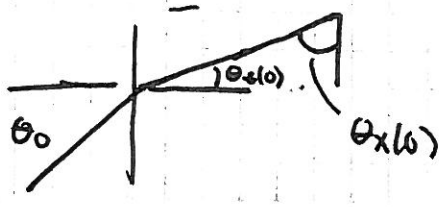
$$\sin\theta_0 > \sqrt{n(0)^2 - 1} = \sqrt{1.3^2 - 1} = 0.83$$

[3p] $\theta_0 > 56.1^\circ$

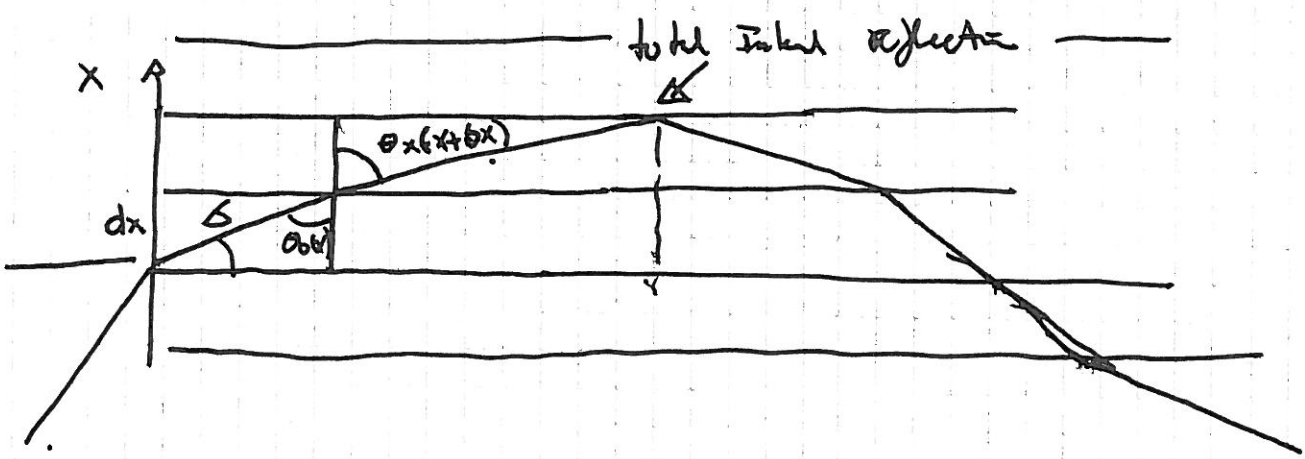
May give some credit for wrong answer 1p

A1 c

The ray will just reflect at interface air



Method 1) Subdivide into several discrete layers approximate smaller for ever larger.



$n(x) \sin\theta_x(x) = n(x+dx) \sin\theta_x(x+dx)$

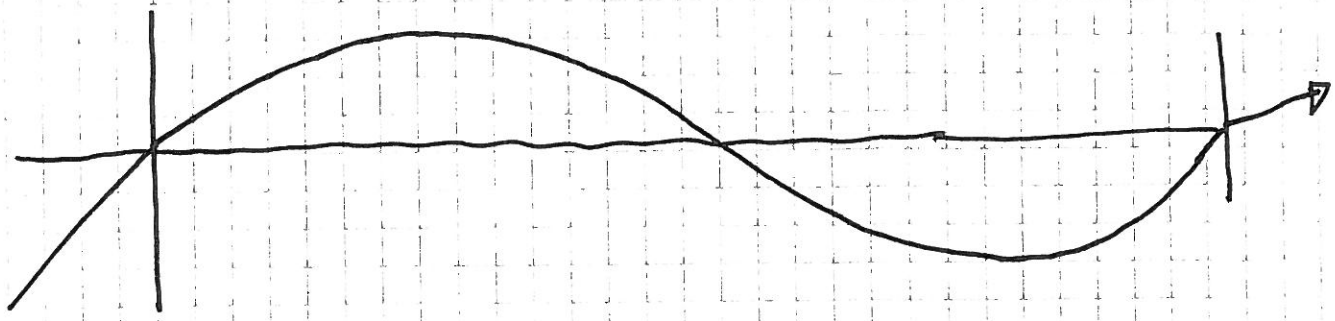
as $n(x+dx) < n(x)$ ray will reflect on the x-axis

~~As the ray bends~~

If ~~the~~ $\sin\theta_x(x+dx) = \dfrac{n(x)}{n(x+dx)} \sin\theta_x > 1$

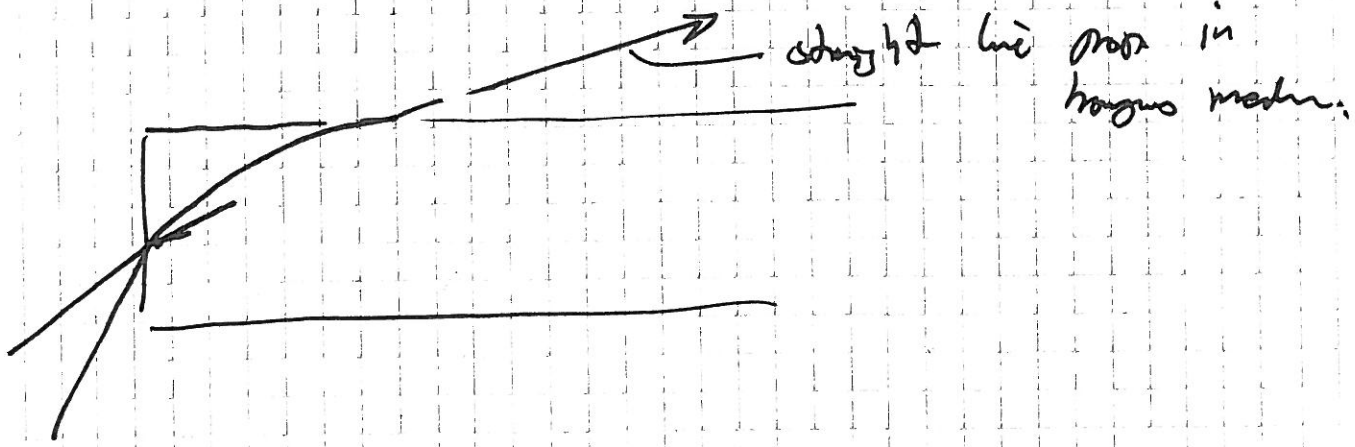
then total internal reflection takes place. The ray will thus turn.

The process is identical on each side ⇒



If ray is bound

If the ray is angle θ_0 it becomes too large, it is expected that the ray may not turn where $|x| < 0.3$ ⇒ exits at is lost



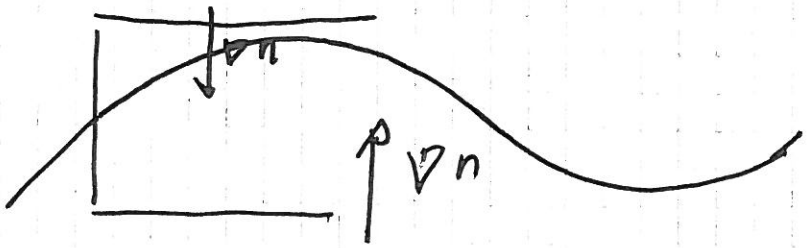
This is identical to not reaching the total internal reflection condition

Method (2)

The key eqn. gives

$$\nabla n = \frac{d}{d\ell}(n\vec{s})$$

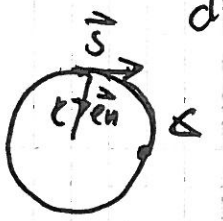
From experience with mirrors etc., the answer will turn in the direction of ∇n



"Proof"

$$\nabla n = \frac{d}{d\ell} n\vec{s} + n\frac{d\vec{s}}{d\ell}$$

Let $\frac{d\vec{s}}{d\ell} \triangleq \frac{\vec{e}_n}{R}$



R is radius of curvature

$$\vec{e}_n \cdot \nabla n = \frac{n}{R}$$

$R > 0$ always, therefore ∇n is in direction of $\vec{e}_n$

$$\Rightarrow \quad \frac{d}{dx}(n \sin\theta_x) = \frac{dn}{dx} = \omega \quad (= -1)$$

$$\frac{d}{dx}(n \sin\theta_x) = 0$$

$$n \sin\theta_x = \text{const} = n \cos\theta_x$$

$$n(0) \sin\theta_x(0) = n(0) \cos\theta_z(0)$$

$$= \sqrt{n^2(\omega) - \sin^2\theta_a}$$

$$\Rightarrow \quad n(x) \sin\theta_x(x) = \sqrt{n^2(\omega) - \sin^2\theta_a}$$

$$\sin\theta_x(x) = \frac{\sqrt{n^2(\omega) - \sin^2\theta_a}}{n(x)}$$

i.e. in accordance with the Snell's law!

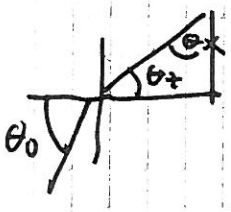
Find expression for $\sin\theta_x(x)$ from my eqn.

From first argument

$$n(x+dx)\sin\theta_x(x+dx) = n(x)\sin\theta_x(x) \cdots n(0)\sin\theta(0)$$

$$\sin\theta_x(0) = \cos\theta_z(0) = \sqrt{1-\frac{1}{n(0)}\sin^2\theta_0}$$

$$= \frac{1}{n(0)}\sqrt{n(0)-n^2\theta_0}$$



$$\Rightarrow n(x)\sin\theta_x(x) = \frac{1}{n(0)}\sqrt{n(0)-\sin^2\theta_0}$$

$$\sin\theta_x(x) = \frac{\sqrt{n(0)-\sin^2\theta_0}}{n(x)}$$

$$= \frac{\sqrt{n(0)-\sin^2\theta_0}}{1.3-|x|}$$

* My program, based on the eqn. eqn., then

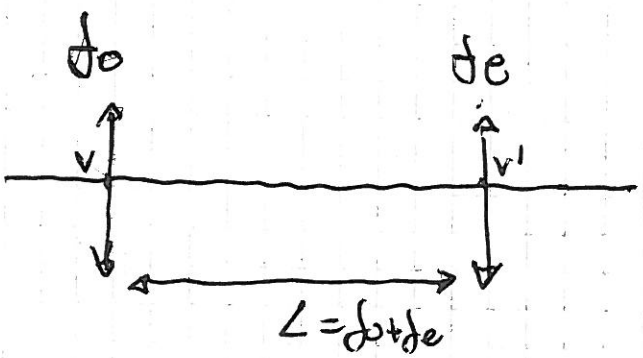
Let $\frac{dn}{dx}\hat{x} = \frac{d}{d\ell}(n s_x \hat{x} + n s_z \hat{z})$

$$= \frac{d}{d\ell}(n\sin\theta_x\hat{x} + n\cos\theta_z\hat{z})$$

$$\frac{dn}{dx} \Rightarrow = \frac{d}{d\ell}(n\cos\theta_x\hat{x} + n\sin\theta_x\hat{z})$$

A2

a)



δ min

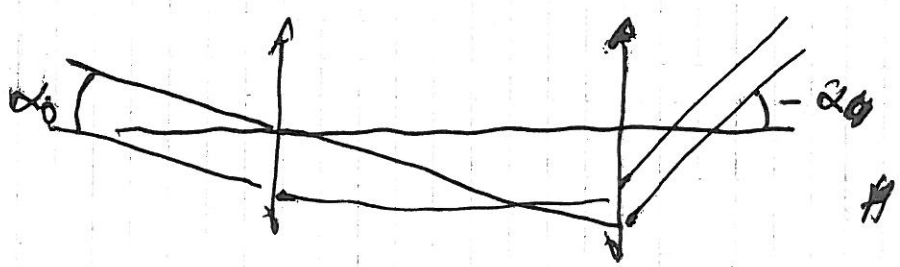
$$M_{vv'} = \begin{bmatrix} 1 & 0 \\ -\frac{1}{fe} & 1 \end{bmatrix} \begin{bmatrix} 1 & fe+fo \\ 0 & 1 \end{bmatrix} \begin{bmatrix} 1 & 0 \\ -\frac{1}{fo} & 1 \end{bmatrix}$$

$$= \begin{bmatrix} 1 & 0 \\ -\frac{1}{fe} & 1 \end{bmatrix} \begin{bmatrix} -fe/fo & fe+fo \\ -\frac{1}{fo} & 1 \end{bmatrix}$$

$$= \begin{bmatrix} -fe/fo & fe+fo \\ +\frac{1}{fe} + \frac{1}{fo} & -\frac{fe}{fe} \end{bmatrix} = \begin{bmatrix} A & B \\ C & D \end{bmatrix}$$

1p the total refractive power $P = -C = -\frac{1}{fe} + \frac{1}{fo} = \frac{fe+fo}{fo\,fe}$

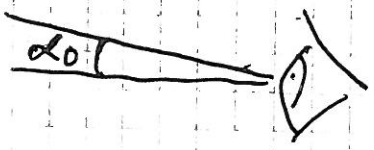
4p $$\begin{bmatrix} y \\ \alpha \end{bmatrix} = \begin{bmatrix} -fe/fo & fe+fo \\ \text{(illegible)} & -\frac{fo}{fe} \end{bmatrix} \begin{bmatrix} y_0 \\ \alpha_0 \end{bmatrix}$$



b) Angular magnification is $\frac{\alpha}{\alpha_0} = -\frac{fo}{fe}$

2p

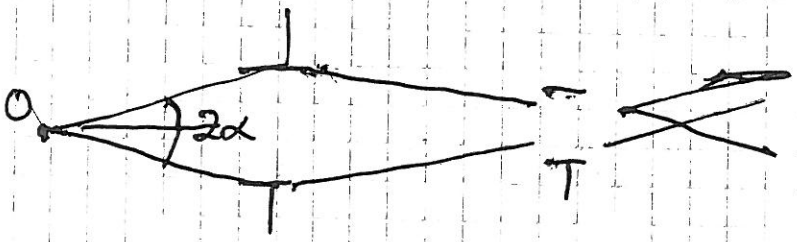
b) contd. This is relative to the eye which observes without the telescope.



c)

Aperture stop:

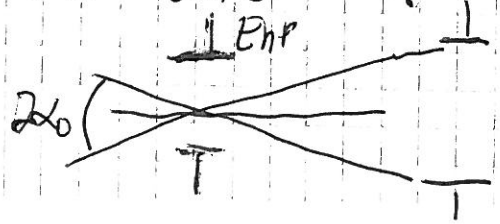
2p The Aperture that limits double of rays from object point through the system



2p The entrance pupil is the image of the AS on the object side

2p The exit pupil is the image of the AS on the image side

2p The field stop is the Aperture that limits the angular field of view, seen from the center of the Ent p



$2\alpha_0$ — Angular field of view.

c) contd.

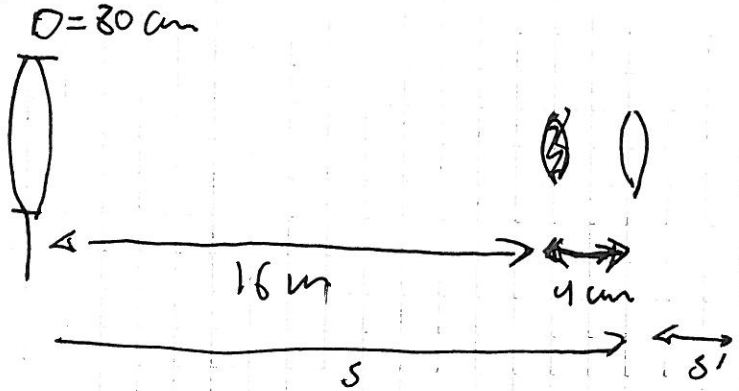


Image of Eyep = Eyep to image side

$$\frac{1}{s} + \frac{1}{s'} = \frac{1}{f_e}$$

$$\frac{1}{s'} = \frac{1}{f_e} - \left(\frac{1}{f_o + f_e}\right)$$

$$= \frac{f_o + f_e - f_e}{(f_o + f_e) f_e} = \frac{f_o}{(f_o + f_e) f_e}$$

$$s' = f_e \frac{(f_e + f_o)}{f_o} = f_e + \frac{f_e^2}{f_o} = 4\text{ cm} + \frac{16}{1600}\text{ cm}$$

2p

$$\approx 4{,}01\text{ cm}$$

magnification $m = -\frac{s'}{s} \approx -\frac{4}{1600}$

2p

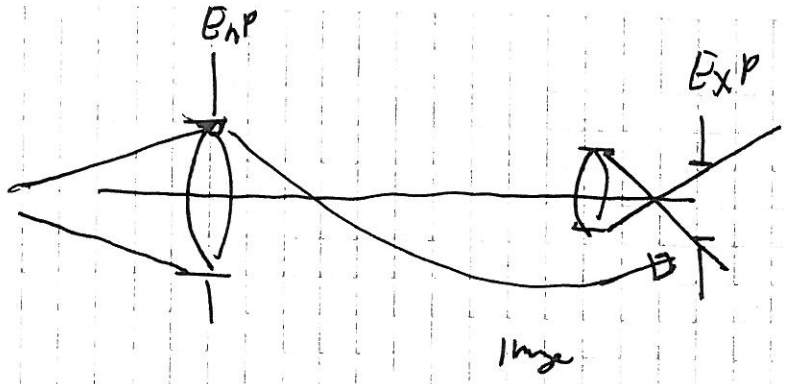
$$\approx -2.5 \times 10^{-2}$$

$$\frac{1}{s'} = \frac{1}{f_e} - \left(\frac{1}{f_o + f_e}\right)$$

$$= \frac{f_o}{f_o + f_e}$$

size of Exit Pupil = $m \cdot D = 0.2$ cm

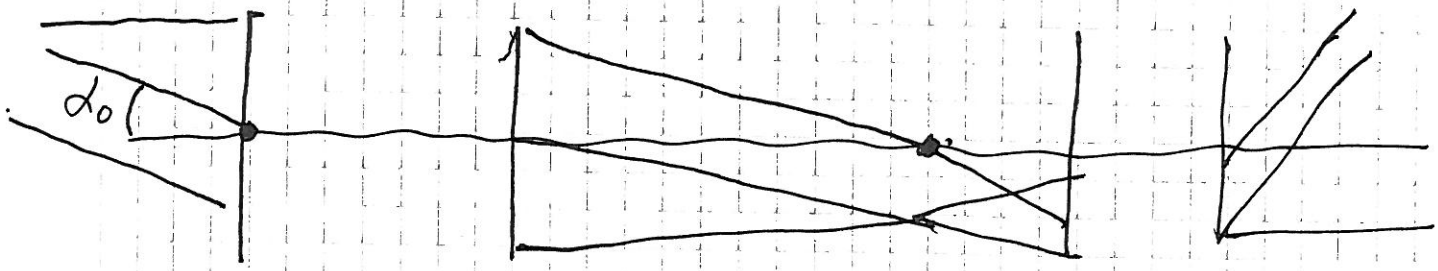
$= \underline{2\text{ mm}}$



The eye should be placed at the Exit Pupil of the system. The pupil of the eye

2p which is 2 mm diameter in daytime and ~ 6 mm at night time, will thus allow all light to reach the eye, i.e. maximize luminosity.

c)



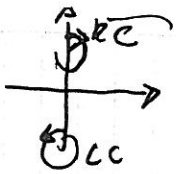
2p The modal power is identical to the power fed into submerged in air.

2p Parallel rays must at the focal point

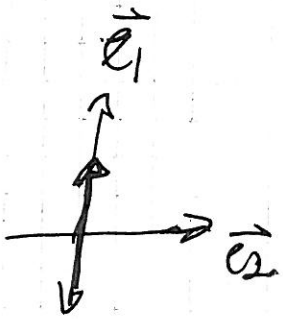
2p Rays from focal point → parallel

a) 2p

$$J = \begin{bmatrix} E_{01} \\ E_{02} e^{i\delta} \end{bmatrix} e^{ikz} e^{-i\omega t}$$

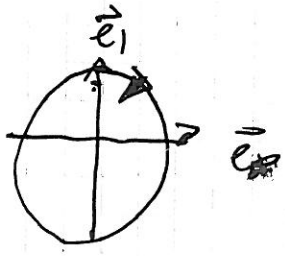


1p $\begin{bmatrix} 1 \\ 0 \end{bmatrix}$ — Linearly polarized light



1p $\begin{bmatrix} 1 \\ i \end{bmatrix}$ — Left circular pol. light (Pedrotti)

direction of pol.



1p $\begin{bmatrix} 1 \\ -i \end{bmatrix}$ circ. pol. light

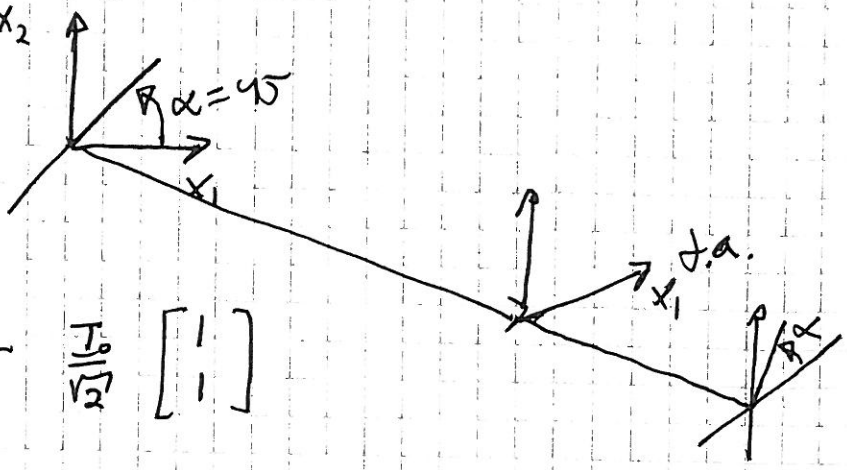
opposite ~~rotation~~ rotation (Light - Pedrotti)

b) 3p

The Jones vector describes only ideal fully polarized light, such as after a perfect polarizer or from a laser source.

The Stokes vector may describe partially polarized light or natural unpolarized light (such as from the sun or a light bulb.

c)



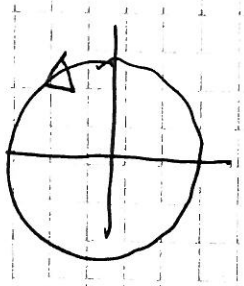
2p After Polarizer $\frac{I_0}{\sqrt{2}} \begin{bmatrix} 1 \\ 1 \end{bmatrix}$

QW plate $\begin{bmatrix} 1 & 0 \\ 0 & e^{i\delta} \end{bmatrix} \xrightarrow{\delta = \frac{\pi}{2}} \begin{bmatrix} 1 & 0 \\ 0 & i \end{bmatrix}$ along slow axis

1p $\boxed{\delta = \frac{\pi}{2}}$

HW plate $\delta = \pi$

2p $\begin{bmatrix} 1 & 0 \\ 0 & i \end{bmatrix} \begin{bmatrix} 1 \\ 1 \end{bmatrix} = \begin{bmatrix} 1 \\ i \end{bmatrix}$ – Left Circular Light



2p $E(\alpha) = R(-\alpha) \begin{bmatrix} 1 & 0 \\ 0 & 0 \end{bmatrix} R(\alpha) \begin{bmatrix} 1 \\ i \end{bmatrix}$

2p $I = E^{\dagger}(\alpha) E(\alpha) = \frac{I_0}{\sqrt{2}}$ (3) (barely one counts)

calc or statement 3p However, it is not necessary to do the calculus because Intensity will be independent of the rotation of the polarizer

B2 Interference from thin film:

b) $\delta = 0$ for $d \neq 0$ (no thickness of film)

$\Rightarrow r = r_{01} + r_{12} t_1 e^{i\delta} + t_2 e^{2i\delta}$ } 2p

$\Rightarrow r = r_{01} + t_1 + t_2 \ldots$

must be identical to the reflected without film.

The layer

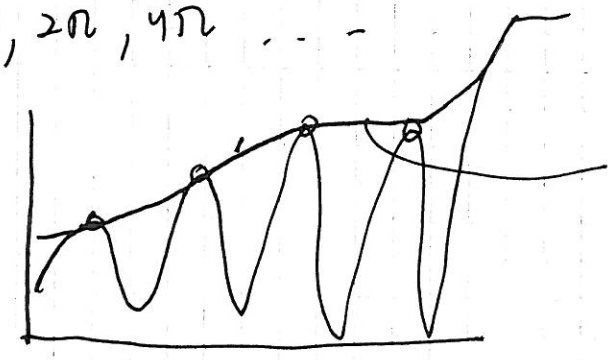
If there film $\delta \neq 0$, hence

$r_{12} e^{i\delta} = (\cos\delta + i \sin\delta)\, |r_{12}^{s/p}| e^{i\delta_{B/P}}$

Must assume that the reflection is lower with film ~ the proposition

The maxima in the figure must thus correspond to

$\delta = 0, 2\pi, 4\pi \ldots$



2p

d) Hãy xác định $\delta = 0, 2\pi, 4\pi$ tương ứng với maxima

$\Rightarrow$ độ lệch giữa hai maxima liên tiếp

là bằng 2π

$$\Rightarrow \frac{2\pi d_1}{\lambda}\sqrt{n_1^2 - n_0^2 n^2 \varphi_{i+1}} - \frac{4\pi d_1}{\lambda}\sqrt{n_1^2 - n_0^2 n^2 \varphi_i} = 2\pi$$

1đ
$$d_1 = \frac{\lambda}{2}\left(\frac{1}{\sqrt{n_1^2 - n_0^2 n^2 \varphi_{i+1}} - \sqrt{n_1^2 - n_0^2 n^2 \varphi_i}}\right)$$

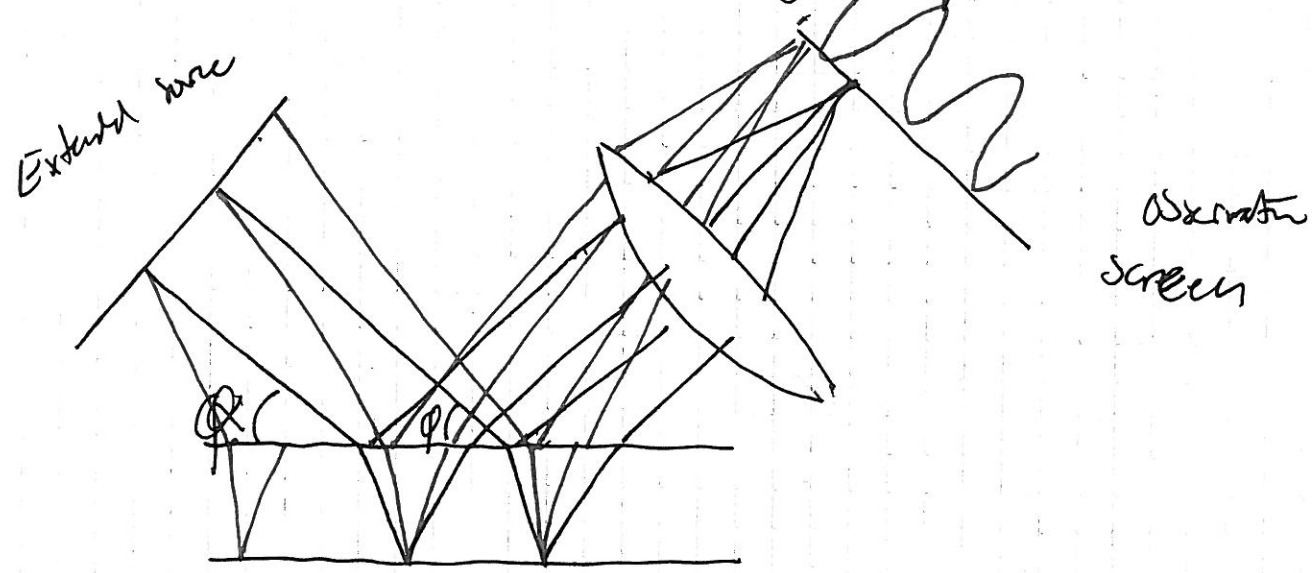
$$n(600, SiO_2) = 1.448 + \frac{3642}{(600)^2} = \underline{1.4581}$$

1đ
$$d_1 = 360 \cdot \frac{16,3}{} = 4989 \text{ nm} \approx \underline{5 \mu m}$$

$$\sqrt{n_1^2 - \sin^2 47.25} = 1.2597 12$$

$$\sqrt{n_1^2 - \sin^2 38,3} = 1.31984$$

c) This must be the Huygens experiment:



Fringes are observed as a function of φ on the screen!

a)

$$r = \sqrt{(X-x)^2 + (Y-y)^2 + z^2}$$

expand

$$X^2 + x^2 - 2Xx + Y^2 + y^2 - 2Yy + z^2$$

choose to expand as $R = X^2 + Y^2 + z^2$

25p $$r = R\left(1 - \frac{(2Xx + 2Yy)}{R^2} + \frac{(x^2 + y^2)}{R^2}\right)^{\frac{1}{2}}$$

(small)

perform binomial expansion $(1+b)^{\frac{1}{2}} \approx 1 + \frac{1}{2}b$

$$r = R\left(R - \frac{(Xx + Yy)}{R} + \frac{1}{2R}(X^2 + Y^2)\right)$$

The Fresnel diffraction can thus be written as:

25p $$U(x,y,z) = \frac{1}{i\lambda} e^{ikR} \frac{z}{R^2} \iint U(x,y,0)\, e^{\frac{ik}{2R}(x^2+y^2)} \approx z \times e^{-ik(Xx+Yy)}\, dx\, dy$$

$z \approx R$ $\qquad \frac{1}{R} \sim \frac{1}{z}$

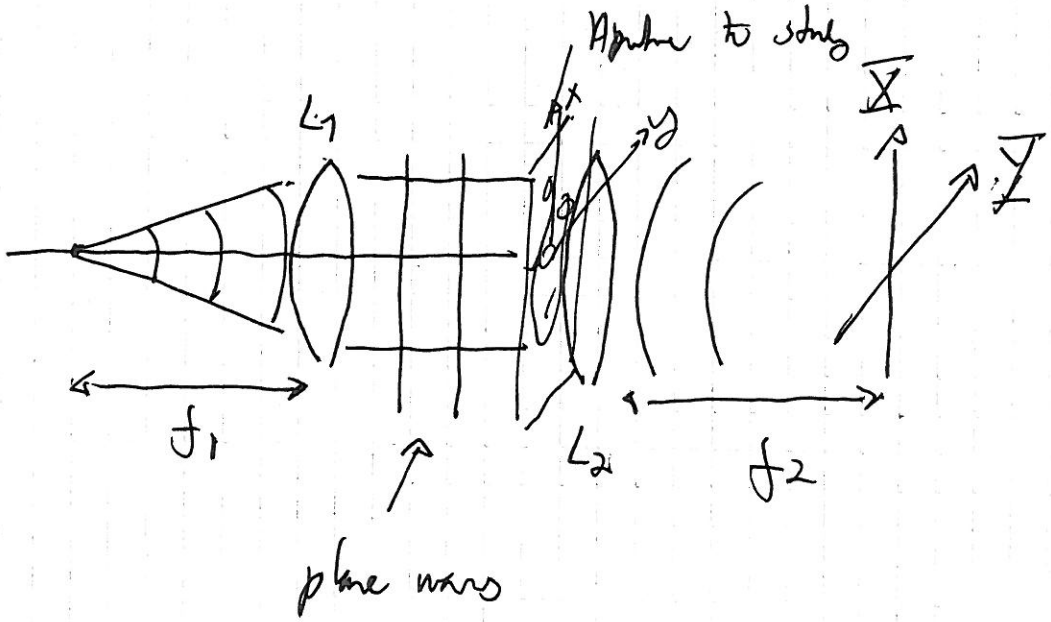
The phase function of a lens

3f when inserted just after the aperture will cancel the term : $e^{\frac{ik}{2R}(X^2+Y^2)}$ in the

paraxial approx ($R = f$)

$$\Rightarrow U(x,y,z) = \frac{1}{i\lambda z} e^{ikz} \iint U(x,y,0)\, e^{\frac{ik}{2f}(x^2+y^2)}\, t_{lens}(x,y)$$

$\Rightarrow$ Fraunhofer diffraction formula



3f+

c)

In this type of problem one uses the convolution theorem in order to get an easy light to handle

1p $t(x,y) \triangleq \frac{U(x,y,0)}{\bar{U}(x,y,0)}$ ← Bright thing dark

1p Let each hole be represented by a Dirac delta fn

~~$\delta(x-a,y-b)$~~

$\delta(x+a, y+b)$ o $\delta(x-a, y-b)$

o o $\delta(x+a, y-b)$

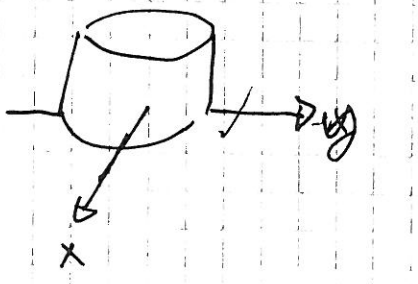
$\delta(x+a, y+b)$

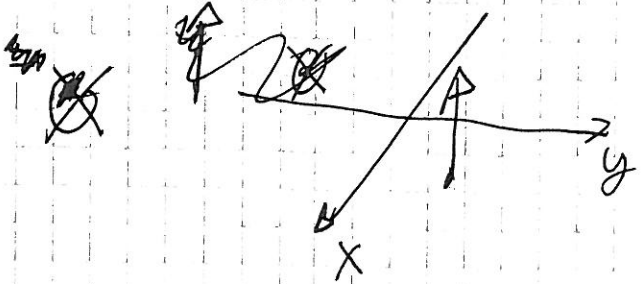
Each hole may be written as a convolution

7p such as

$$\delta(x-a, y-b) \otimes \mathrm{circ}\left(\frac{s}{b/2}\right),$$

where $s = \sqrt{x^2+y^2}$





The Final transmittance is then

1p $t(x,y) = circ\left(\frac{s}{D/2}\right) \otimes \{ \delta(x-a, y-b) + \delta(x+a, y-b) + \delta(x-a, y+b) + \delta(x+a, y+b) \}$

d)

The scalar electric field

3p $U(x,y,z) = \frac{e^{ikz}}{i\lambda z} \cdot \mathcal{F}\{t(x,y)\}$

$\mathcal{F}\{t(x,y)\} = \mathcal{F}\{circ\left(\frac{s}{D/2}\right)\}$

$\times \{ e^{ik_x a} e^{ik_y b} + e^{-ik_x a} e^{ik_y b} + e^{ik_x a} e^{-ik_y b} + e^{-ik_x a} e^{-ik_y b} \}$

$e^{ik_x a}\left(e^{ik_y b} + e^{-ik_y b}\right) + e^{-ik_x a}\left(e^{ik_y b} + e^{-ik_y b}\right)$

$= 2\cos k_x a \; 2\cos k_y b$

The Fourier transform of

$$\mathcal{F}\{\mathrm{circ}(r/D/2)\} = 2\pi\left(\frac{D}{2}\right)^2 \frac{J_1(Kd)}{Kd}$$

$$U(x,y,z) = \frac{e^{ikz}}{i\lambda z} \mathcal{F}\{t(x,y)\}$$

15p

$$= \frac{e^{ikz}}{i\lambda z} \left\{ 2\pi\left(\frac{D}{2}\right)^2 \frac{J_1(Kd)}{Kd} \right\} \cdot 4\cos(k_X a)\cos(k_Y$$

---

observable intensity

15p

$$\langle I \rangle = U(x,y,z)\, U^*(x,y,z)$$

$$= \frac{\langle I_0 \rangle}{\lambda^2 z^2} \pi^2 D^4 \left(\frac{2J_1(Kd)}{Kd}\right)^2 \cdot 16\cos^2(k_x a)\cos$$

where $k_X = k\frac{X}{z}$, $k_Y = k\frac{Y}{z}$

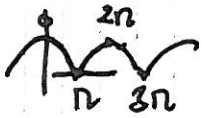
e)

Maxima in intensity given by

2p $\qquad k_X a = 2\pi m \quad \wedge \quad k_Y b = 2\pi n$

$$k_X a = \frac{2\pi}{\lambda}\frac{X}{z}, \qquad k_Y b = \frac{2\pi}{\lambda}\frac{Y}{z}$$

$$X = \frac{\lambda z}{a} m \quad \wedge \quad Y = \frac{\lambda z}{b} n$$



Minima

$$k_X a = \pi + m\pi = 2\pi\pi\left(m+\tfrac{1}{2}\right)$$

$$k_Y b = 2\pi\left(m+\tfrac{1}{2}\right)$$

$$X = \frac{\lambda z}{a}\left(m+\tfrac{1}{2}\right)$$

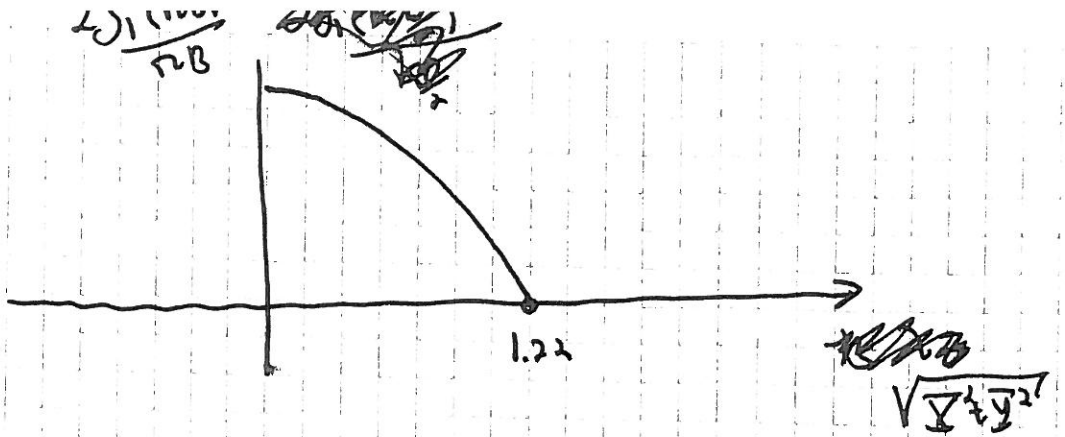
$$Y = \frac{\lambda z}{b}\left(n+\tfrac{1}{2}\right)$$

If the holes became smaller, then

$$\frac{2J_1(kR)}{kR} \doteq \frac{2J_1\left(\pi\left(\frac{2}{\lambda z}\sqrt{X^2+Y^2}\right)R\right)}{\pi\left(\frac{2}{\lambda z}\sqrt{X^2+Y^2}\right)R}$$

$$k = \sqrt{k_X^2 + k_Y^2} = k\sqrt{\left(\frac{X}{z}\right)^2 + \left(\frac{Y}{z}\right)^2}.$$

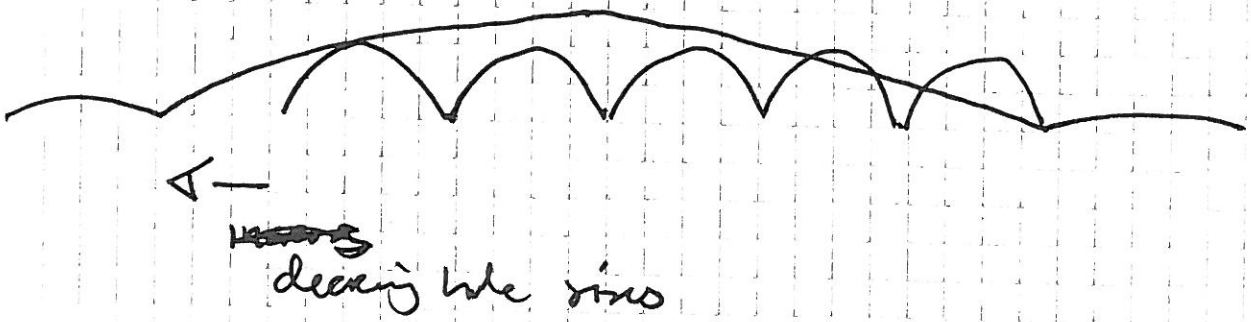
$$= \frac{2\pi}{\lambda z}\sqrt{X^2+Y^2}$$



$$b = \frac{\sqrt{X^2+Y^2}\, D}{\lambda z}$$

If D becomes smaller, then $\sqrt{X^2+Y^2}$ becomes a factor larger to reach 1.22 (first minimum) 2p

Hence more interference fringes should be observed. ▷



decreasing hole size

identical D becomes larger

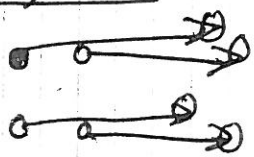
If $\lambda \to 0$, then the geometric optics condition is reached, 2p and rays follow straight lines

What happens when $\lambda \to 0$

$$\cos^2\left(k_x \frac{a}{2}\right) \cos^2\left(k_y \frac{b}{2}\right) \times \frac{2J_1\left(k\frac{D}{2}\right)}{k\frac{D}{2}}$$

Very sharp peaks are in X-RAYS's

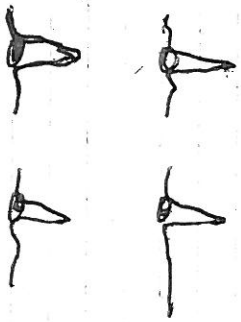
However, it is known that $\lambda \to 0$, then geometric optics is recovered $\Rightarrow$



Must then introduce coherence of sources

spatial coherence $\Rightarrow$

$\Rightarrow$ Add intensities, from each hole.



Very Difficult!!