7.1 Section (A)

(i) The ray eqns $\frac{d(n\hat{s})}{d\sigma} = \nabla n$ is the

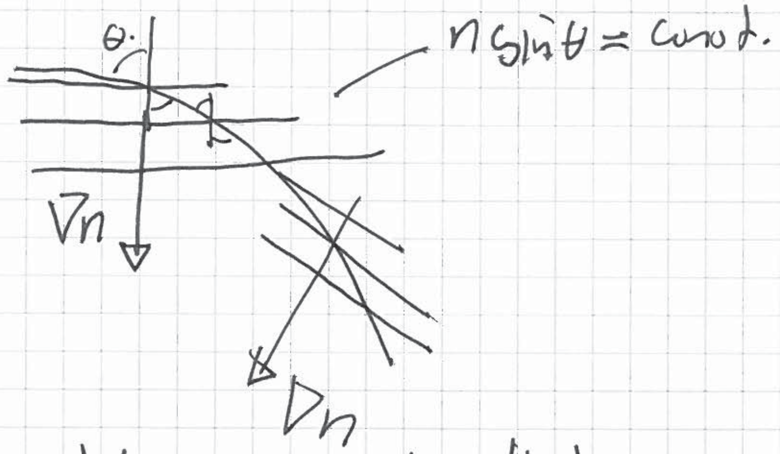
3p

differential eqn. corresponding to the variational problem

$$\delta S = \delta \int_A^B n \, d\sigma = 0 \iff$$ Fermat's principle of least

time. The ray eqn basically gives the generalised Snell's law, which can be used to determine that the ray always curves towards ∇n.

Example

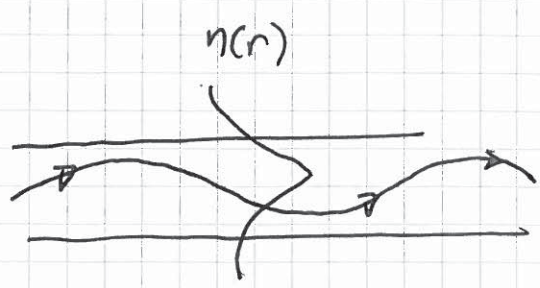


The ray will take the path between A & B that makes $\delta \int n = 0 \iff$ takes least time, i.e. follows curve given by soln of $\frac{d(n\hat{s})}{d\sigma} = \nabla n$

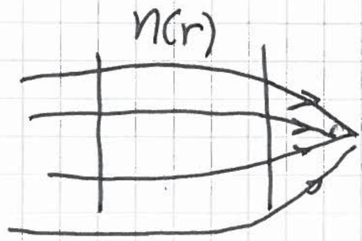
(ii) 3p

Device/application

* gradient index fibre
opto communications

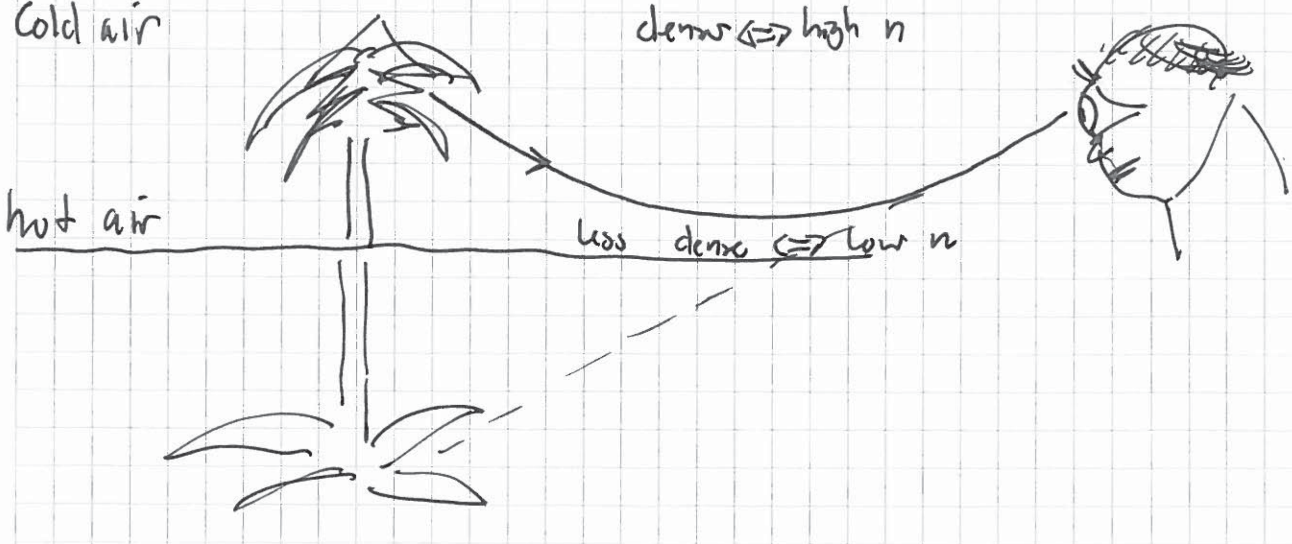


* Inhomogeneous lens
e.g. endoscopes



Nature:

Mirage or "luftspeilings"



A.2

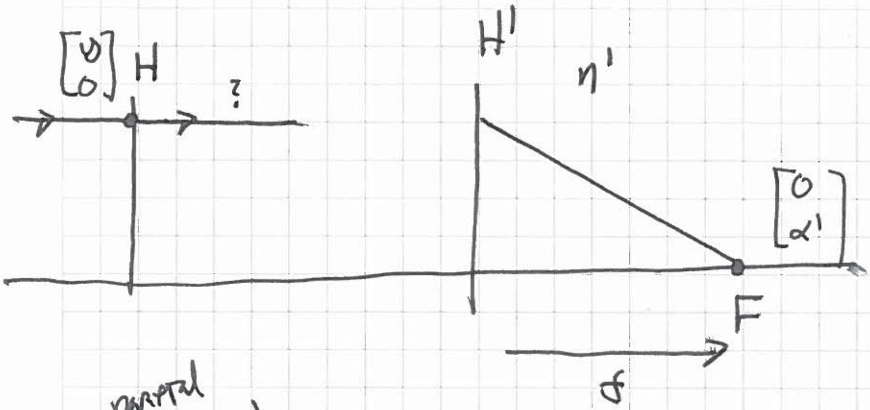
(a)

$$M_{HH'} = \begin{bmatrix} 1 & 0 \\ C & \frac{n}{n'} \end{bmatrix} = \begin{bmatrix} A' & B' \\ C' & D' \end{bmatrix}$$

For $B' = 0 \Leftrightarrow$ imaging

2p] $M_t = 1 = A'$.

$M_\alpha = \frac{n}{n'} = D'$

p] 

Def. of focal point:

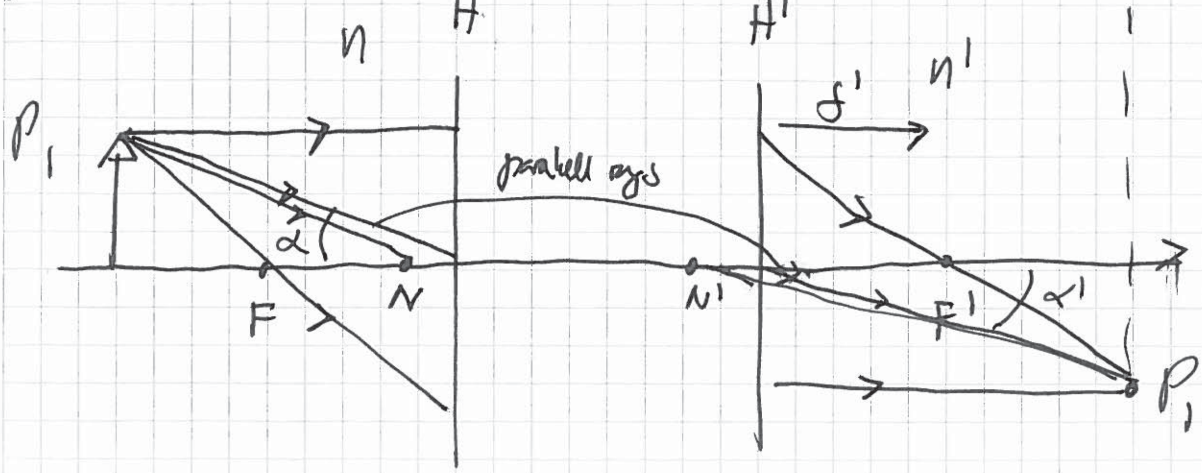
consider paraxial ray vector from H $\begin{bmatrix} y \\ \alpha \end{bmatrix} = \begin{bmatrix} y \\ 0 \end{bmatrix}$, want at F ray vector $\begin{bmatrix} 0 \\ \alpha' \end{bmatrix}$

$$\Rightarrow \begin{bmatrix} 0 \\ \alpha' \end{bmatrix} = \underbrace{\begin{bmatrix} 1 & f \\ 0 & 1 \end{bmatrix} \underbrace{\begin{bmatrix} 1 & 0 \\ C & \frac{n}{n'} \end{bmatrix} \begin{bmatrix} y \\ 0 \end{bmatrix}}_{\begin{bmatrix} y \\ Cy \end{bmatrix}}}$$

$$\begin{bmatrix} 0 \\ \alpha' \end{bmatrix} = \begin{bmatrix} y + fCy \\ Cy \end{bmatrix} \Rightarrow y = -fCy$$

$$\Leftrightarrow \underline{C = -\frac{1}{f}}$$ as required

(b)
[3p]



$$\frac{\alpha'}{\alpha} = \frac{n}{n'}$$

(c)

$$\overline{\overline{M}}_{OI} = \begin{bmatrix} 1 & s' \\ 0 & 1 \end{bmatrix} \underbrace{\begin{bmatrix} 1 & 0 \\ -\frac{1}{f'} & \frac{n}{n'} \end{bmatrix} \begin{bmatrix} 1 & s \\ 0 & 1 \end{bmatrix}}$$

$$\begin{bmatrix} 1 & s \\ -\frac{1}{f'} & -\frac{s}{f'} + \frac{n}{n'} \end{bmatrix}$$

$$\begin{bmatrix} 1 - s'/f' & s - \frac{s's}{f'} + \frac{s'n}{n'} \\ -\frac{1}{f'} & -\frac{s}{f'} + \frac{n}{n'} \end{bmatrix} = \begin{bmatrix} A' & B' \\ C' & D' \end{bmatrix}$$

For image : $B' = 0 \Rightarrow s + \frac{s'n}{n'} = \frac{s's}{f'} \quad / s's$

$$\Rightarrow \underline{\frac{n'}{s'} + \frac{n}{s} = \frac{n'}{f'}}$$

Further $A' = M_t =$

$$A_t = M_t = \frac{h'}{h} = 1 - s'/f'$$

$$= 1 - \frac{s'}{n'}\left[\frac{n'}{s'} + \frac{n}{s}\right] = \underline{-\frac{s'}{s}\frac{n}{n'}}$$

A3

(a) Show that $M_{OI} = \begin{bmatrix} -1 & 0 \\ 0 & -1 \end{bmatrix} = \begin{bmatrix} M_t & 0 \\ -\frac{1}{f} & M_\alpha \end{bmatrix}$

afocal $\Longleftrightarrow$ $f = \infty$ like telescope.

Imaging system

$$M_{OI} = \begin{bmatrix} 1 & f \\ 0 & 1 \end{bmatrix} \begin{bmatrix} 1 & 0 \\ -\frac{1}{f} & 1 \end{bmatrix} \begin{bmatrix} 1 & 2f \\ 0 & 1 \end{bmatrix} \begin{bmatrix} 1 & 0 \\ -\frac{1}{f} & 1 \end{bmatrix} \begin{bmatrix} 1 & f \\ 0 & 1 \end{bmatrix}$$

$$\begin{bmatrix} 1 & f \\ -\frac{1}{f} & 0 \end{bmatrix}$$

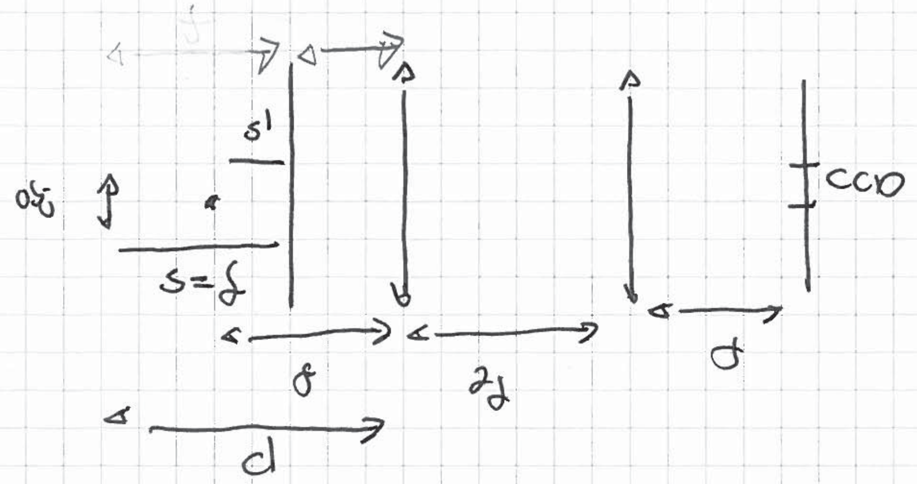
$$\begin{bmatrix} -1 & f \\ -\frac{1}{f} & 0 \end{bmatrix}$$

$$\begin{bmatrix} -1 & f \\ 0 & -1 \end{bmatrix}$$

$$M_{OI} = \begin{bmatrix} -1 & 0 \\ 0 & -1 \end{bmatrix}$$

QED

(b)



Find virtual image of object through interface liquid-air

This VI must be the object for the 4f system.

$\Rightarrow$

$$\frac{n_{Liq}}{s} + \frac{1}{s'} = \frac{1}{f'} = 0 \qquad f = \infty \text{ for plane interface,}$$

$\Rightarrow$

$$s' = -\frac{s}{n_{Liq}} = -\frac{s}{1.5} = -\frac{f}{n'}$$

can be seen from paraxial matrix

$$\begin{bmatrix} 1 & 0 \\ \frac{n-n'}{Rn'} & \frac{n}{n'} \end{bmatrix} \quad n \quad n' \quad R$$

$= -\frac{1}{f'} = 0 \qquad R \to \infty$

$$d = f - \frac{s}{1.5} + f$$

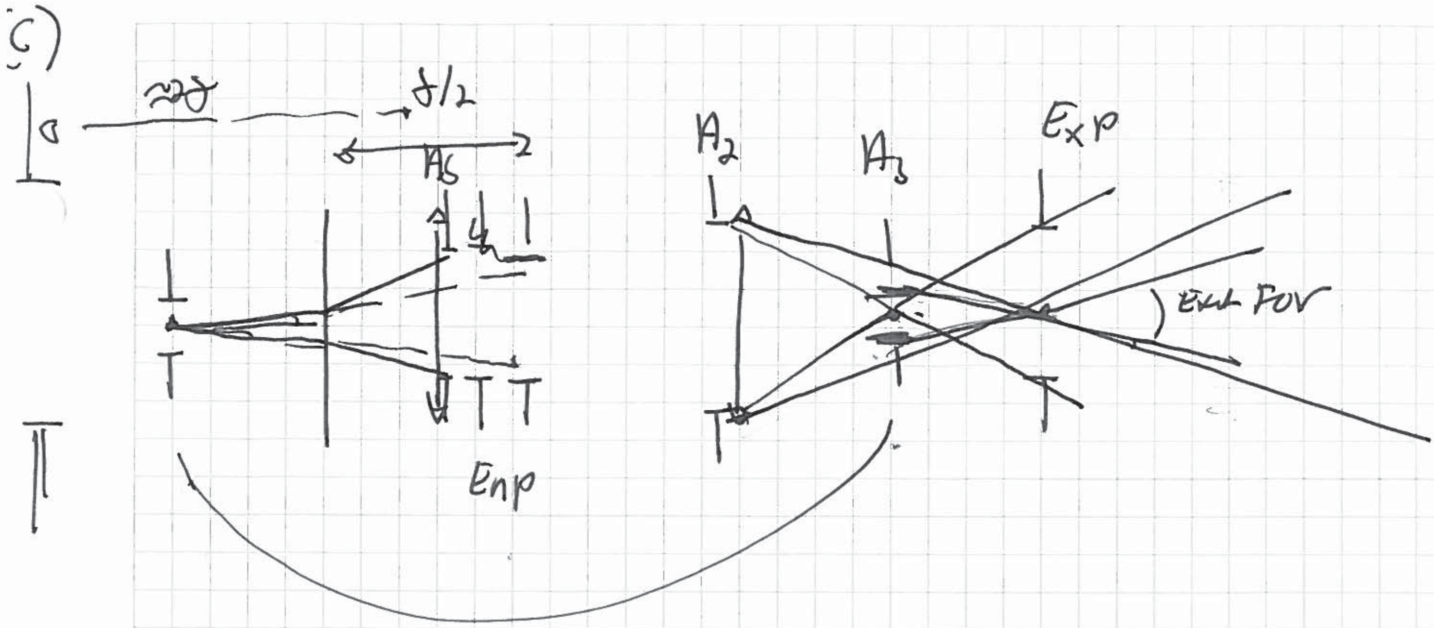
$$= 2f - \frac{f}{n'} \quad , \; n' = 1.5$$

$$= f\left(2 - \frac{1}{3/2}\right) = f\left(\frac{6-2}{3}\right) = f\frac{4}{3} = 233 \text{ mm}$$

obj.–VI

$$M_t = -\frac{n}{n'}\frac{s'}{s} = -\frac{n_L}{1}\frac{(-f/n_L)}{f} = 1$$

c)

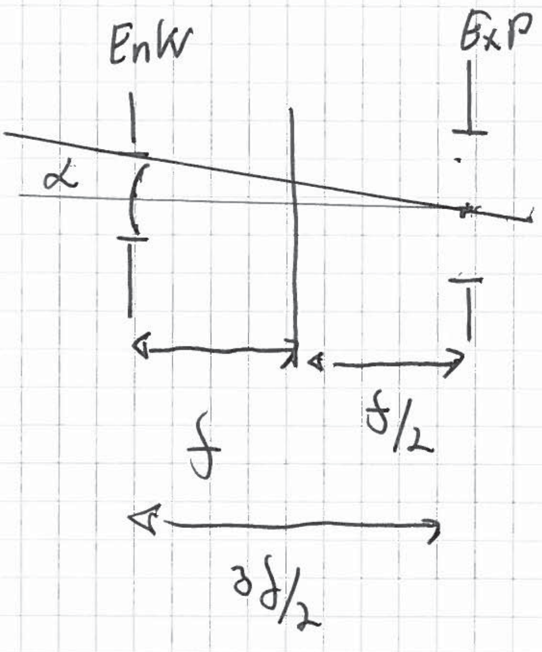


It is seen that A_3 must be Exit Window and thus FS. ⇒ Image of A3 onto object plane is EnW.

since $M_t = M_{0-VI} \cdot M_{ey} = -1$

⇒ EnW is located d in front of lens L_1 and has size 1 cm, as the CCD chip.

FOV:



$$\tan(2\alpha) = \frac{10\text{ mm}}{3f/2} =$$

$$2\alpha = 2.18^\circ$$

$$\alpha = 1.09^\circ$$

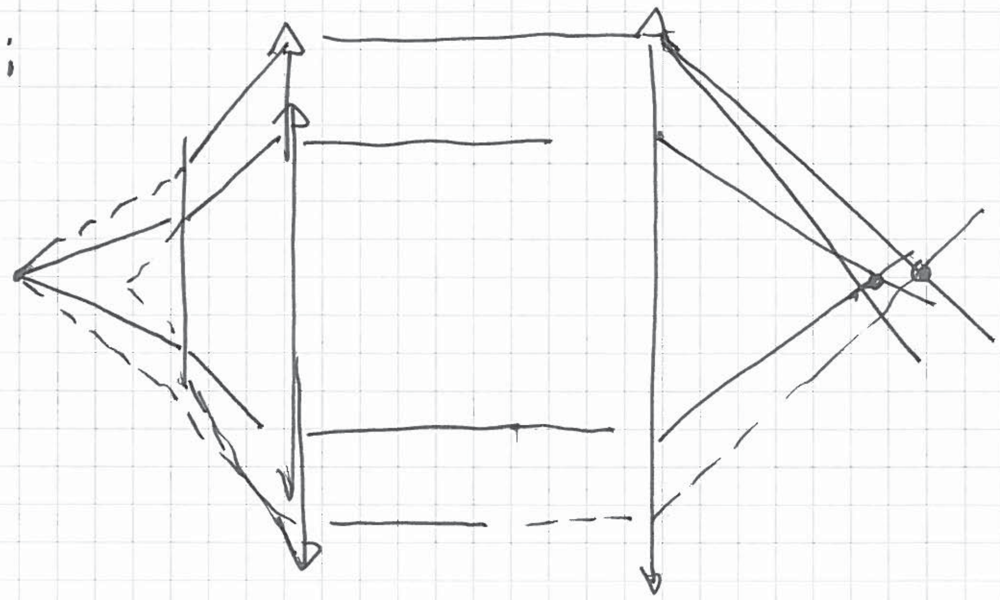
$$\left(\tan\alpha = \frac{5\text{ mm}}{\frac{3}{2}\cdot 175}\right)$$

d) The paraxial ray matrix used to calculate the image location is derived w/ the paraxial form of Snell's law:

$$n'\theta' = n\theta$$

Rays, that are non paraxial will thus form a different image point.

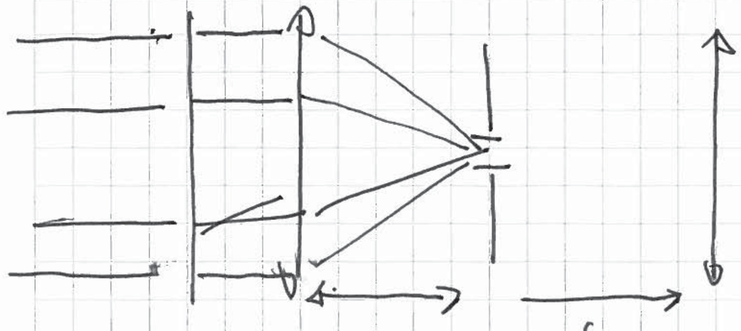
Example:



The AS reduces the n° of nonparaxial rays ~~and~~ ~~this~~ should therefore improve the image quality.

[

(d) If the AS is located at f between lens L_1 & L_2:
we have a telecentric system in both object & image space



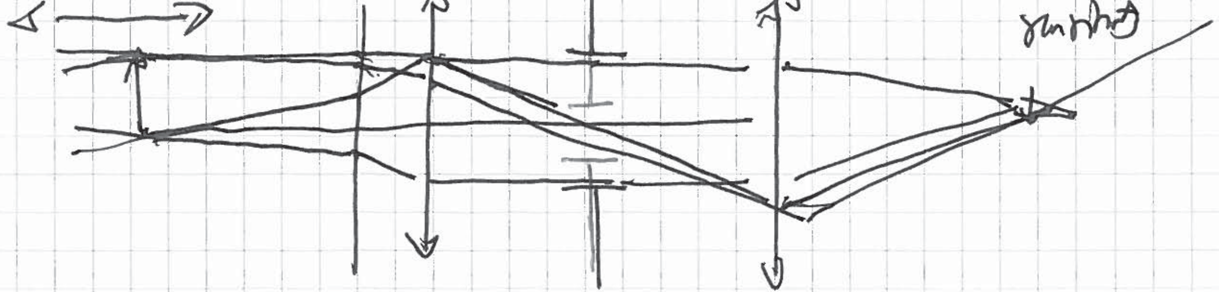
$$\frac{1}{s'} + \frac{1}{f} = \frac{1}{f}$$

$$\frac{1}{s'} = 0 \quad \Rightarrow s' = \infty \quad , \; -\frac{s'}{s} = \infty$$

Both the EnP & the ExP's are located in ∞.

This is a telecentric system. Its advantage is that it only allows highly parallel rays from a certain object point to pass

Example



It's prime advantage is therefore that

we are (i) little sensitive to focusing errors

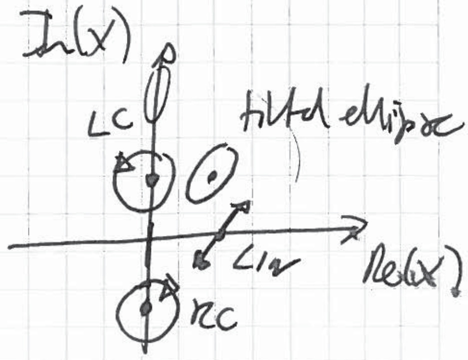
(ii) also the magnification is little sensitive to depth, hence details both in front and at the back of the ~~focal~~ object plane will have same size (loss of perspective)

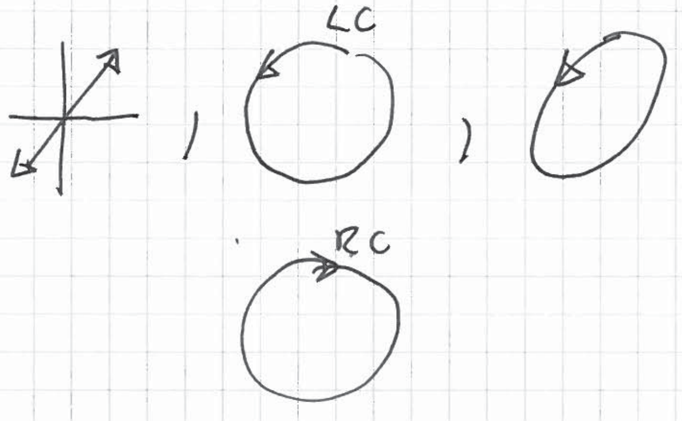
(iii)

B.1.

(a) $\chi = 1, \pm i, 1+i$

$$\begin{bmatrix} 1 \\ 1 \end{bmatrix}, \begin{bmatrix} 1 \\ \pm i \end{bmatrix}, \begin{bmatrix} 1 \\ 1+i \end{bmatrix}$$

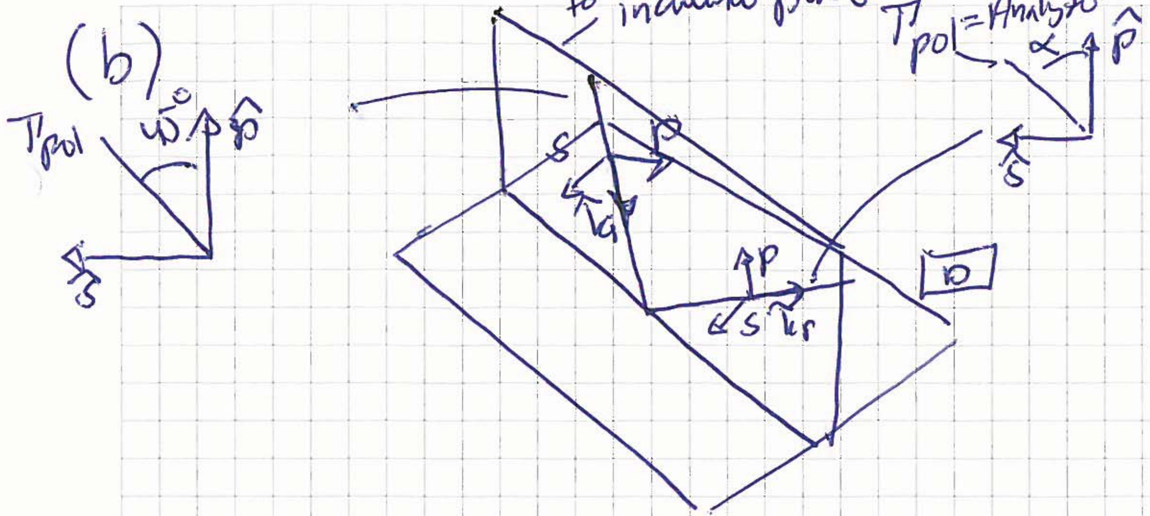




$\chi = 1$: $\vec{E}(\vec{r}, t) = E_0 \cos(kz - \omega t)\,\hat{x} + E_0 \cos(kz - \omega t)\,\hat{y}$

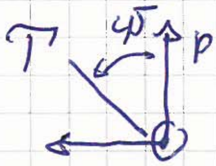
$\chi = i$: $\vec{E}(\vec{r}, t) = \underbrace{E_0 \cos(kz - \omega t)}_{E_{0x} = \cos \omega t}\,\hat{x} + \underbrace{E_0 \cos(kz - \omega t + \frac{\pi}{2})}_{E_{0y} = \sin \omega t}\,\hat{y}$

(b)



(i) $$E_D = \bar{\bar{R}}(-\alpha)\begin{bmatrix}1 & 0\\ 0 & 0\end{bmatrix}\bar{\bar{R}}(\alpha)\begin{bmatrix}r_p & 0\\ 0 & r_s\end{bmatrix}\begin{bmatrix}1\\ 1\end{bmatrix}\sqrt{\frac{I_0}{2}}$$

It is understood that the Jones vector must be given in terms of $\begin{bmatrix}E_p\\ E_s\end{bmatrix}$, and that the linear polarizer ~~orient~~ oriented 45° as follows



gives a lin. pol. state $\begin{bmatrix}1\\ 1\end{bmatrix}\frac{1}{\sqrt{2}}$

$$\bar{E}_D = \begin{bmatrix}\cos\alpha & -\sin\alpha\\ \sin\alpha & \cos\alpha\end{bmatrix}\underbrace{\begin{bmatrix}1 & 0\\ 0 & 0\end{bmatrix}\begin{bmatrix}\cos\alpha & \sin\alpha\\ -\sin\alpha & \cos\alpha\end{bmatrix}}_{\begin{bmatrix}\cos\alpha & \sin\alpha\\ 0 & 0\end{bmatrix}}\begin{bmatrix}r_p\\ r_s\end{bmatrix}\sqrt{\frac{I_0}{2}}$$

$$\begin{bmatrix}\cos^2\alpha & \cos\alpha\sin\alpha\\ \sin\alpha\cos\alpha & \sin^2\alpha\end{bmatrix}\begin{bmatrix}r_p\\ r_s\end{bmatrix}\sqrt{\frac{I_0}{2}} = \sqrt{\frac{I_0}{2}}\underline{\underline{\begin{bmatrix}\cos^2\alpha\, r_p + \cos\alpha\sin\alpha\, r_s\\ \sin\alpha\cos\alpha\, r_p + \sin^2\alpha\, r_s\end{bmatrix}}}$$

(ii) $\alpha = 45^\circ \Rightarrow \quad \bar{E}_p = \sqrt{\frac{I_0}{2}} \begin{bmatrix} \frac{r_p}{2} + \frac{r_s}{2} \\ \frac{r_p}{2} + \frac{r_s}{2} \end{bmatrix} = \frac{1}{2}\sqrt{\frac{I_0}{2}}\,(r_p + r_s) \begin{bmatrix} 1 \\ 1 \end{bmatrix}$

$$I_D = \frac{1}{4} \cdot \frac{1}{2} \cdot I_0 \, |r_p + r_s|^2 \underbrace{\begin{bmatrix} 1 & 1 \end{bmatrix} \begin{bmatrix} 1 \\ 1 \end{bmatrix}}_{2}$$

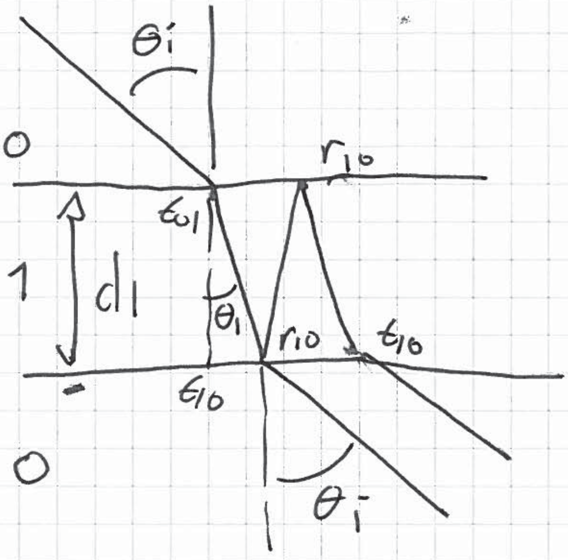
$$= \frac{1}{4} I_0 \, |r_p + r_s|^2$$

$$= \frac{1}{4} I_0 \left(|r_p|^2 + |r_s|^2 + r_p r_s^* + r_s r_p^* \right)$$

$$= \frac{1}{4} I_0 \left(|r_p|^2 + |r_s|^2 \right) \left(1 + \frac{2\,\mathrm{Re}(r_s r_p^*)}{|r_p|^2 + |r_s|^2} \right)$$

B2.

(a)



$n_1 \approx 3.42$

$$\beta = \frac{2\pi}{\lambda} n_1 \cos\theta_1 d_1$$

$$t^{s/p} = t_{10}^{s/p} t_{01}^{s/p} e^{i\beta}\left[1 + (r_{10}^{s/p})^2 e^{i2\beta}\right]$$

for s-pol:

$|t|^2 = T$ since both incident medium and exit medium are identical.

$$\Rightarrow T = |t_{10}|^2 |t_{01}|^2 \Big(\underbrace{\left[1 + (r_{10})^{2*} e^{-i2\beta}\right]\left[1 + (r_{10})^2 e^{+2i\beta}\right]}\Big)$$

$$1 + |r_{10}|^4 + 2\mathrm{Re}\left((r_{10})^{2*} r_{10}^2 e^{-i2\beta}\right)$$

$$= |t_{10}|^2 |t_{01}|^2 \left(1 + |r_{10}|^4 + 2|r_{10}|^4 \cos 2\beta\right)$$

2 contd.

(b) $T = a + b\cos 2\beta$

$\Rightarrow$ one period of oscillation $= 2\pi$

neglect dispersion, and read off 10 periods

$\lambda_1 \approx 13\,\mu m$

$10 \times 2\pi$

$\lambda_{10} \approx 17.2\,\mu m$

$$2\beta_1 - 2\beta_{10} = 10 \cdot 2\pi$$

$$\Rightarrow \frac{4\pi}{\lambda_1} n_1 \cos\theta_1 d_1 - \frac{4\pi}{\lambda_{10}} n_1 \cos\theta_1 d_1 = 20\pi$$

$$d_1 \underbrace{\left(\frac{\lambda_{10}}{\lambda_1 \lambda_{10}} - \frac{\lambda_1}{\lambda_{10}\lambda_1}\right)}_{} = \frac{20}{4} = \frac{5}{n_1 \cos\theta_1}$$

$$d_1 \left(\frac{\lambda_{10} - \lambda_1}{\lambda_1 \lambda_{10}}\right) = 5/n_1 \cos\theta_1$$

$$d_1 = \boxed{\left(\frac{\lambda_1 \lambda_{10}}{\lambda_{10} - \lambda_1}\right)} \frac{5}{n_1 \cos\theta_1}$$

tc

$$n_1 \cos\theta_1 = 3.42 \cdot \sqrt{}$$

$$\cancel{n_0} \sin^2\theta_1 = \left(\frac{n_0 \sin\theta_0}{n_1}\right)^2$$

$$n_1 \cos\theta_1 = n_1\sqrt{1 - \sin^2\theta} = n_1\sqrt{1 - \frac{\sin^2\theta_0}{n_1^2}} = \sqrt{n_1^2 - \sin^2\theta_0}$$

$$= \sqrt{(3.42)^2 - \sin^2 30^\circ} = 3{,}3832$$

$$d_1 = \left(\frac{17.2 \cdot 13}{17.2 - 13}\right) \cdot \frac{5}{3{,}3832}$$

$$= 78.67\ \mu m$$ [more accurate is 80 μm]

exact reading $\lambda_1 = 11.4\ \mu m$, $\lambda_{10} = 14.44\ \mu m$

$$d = \frac{10\,\lambda_1 \lambda_2}{(\lambda_1 - \lambda_2)}\ \frac{1}{2\sqrt{n_1^2 - \sin^2\theta_0}}$$

$n_1 = 3.42$, $\theta_0 = 30^\circ$

$$\Rightarrow d = 80\ \mu m$$

B3(c) revised

from (a) $t = t_{01} \, t_{10} \, e^{i\beta} \left[1 + (r_{10})^2 \, e^{2i\beta} \right]$, $T = |t|^2$

If $d = 0$, then $2\beta = 0$, $\beta = 0$

$\Rightarrow \quad t = t_{01} \, t_{10} \, (-1) \left[1 + (r_{10})^2 \right] = 1$ no film

$\Rightarrow$ For every $2\beta = 0, 2\pi, 4\pi, 6\pi, \ldots$

$t = 1 \Rightarrow T = 1$ & $R = (1-T) = 0$

! This is a result of constructive interference in the forward direction.

For the maximum reflectivity $R \approx 1 - T_{min} \approx 1 - 0.23 \approx 0.77$

where $T \approx 0.23$ has been read in the figure.

conclusion, one film may not create a perfect ~~Anti~~ reflective coating, which requires a stack of layers.

33 a)

It is clear that the lens brings us to the Fraunhofer diffraction approx.

$$V(\bar{X},\bar{Y},f) = \mathfrak{Z}(f) \iint t_A(x,y)\, \underbrace{t_{lens}\, e^{\frac{ik}{2z}(x^2+y^2)}}_{= e^{i\frac{2\pi}{\lambda} n \Delta_0}} e^{-i(k_X x + k_Y y)}\, dx\,dy$$

1)

where $t_A(x,y) = \dfrac{U^+(0,x,y)}{U^-(0,x,y)}$.

$\approx$ plane wave illumin. $\Rightarrow V_0 e^{ikz} \Rightarrow U^-(0,x,y) e^{ikz}$

2) $V(\bar{X},\bar{Y},f) = \mathfrak{Z}(f)\, e^{i\frac{2\pi}{\lambda} n \Delta_0}\, \mathcal{F}\left\{ \mathrm{circ}\left(\frac{\rho}{(D_A/2)}\right)\right\}$, $\rho = \sqrt{x^2+y^2}$

3p) $\mathcal{F}\left\{\mathrm{circ}\left(\rho/_{D_A/2}\right)\right\} = \pi \left(\frac{D_A}{2}\right)^2 \dfrac{2 J_1(K D_A/2)}{K D_A/2}$ from appendix

$K = \sqrt{k_{\bar{X}}^2 + k_{\bar{Y}}^2}$, $k_j \approx \frac{2\pi}{\lambda} \frac{\cancel{j}}{z}$, $j = \bar{X},\bar{Y}$

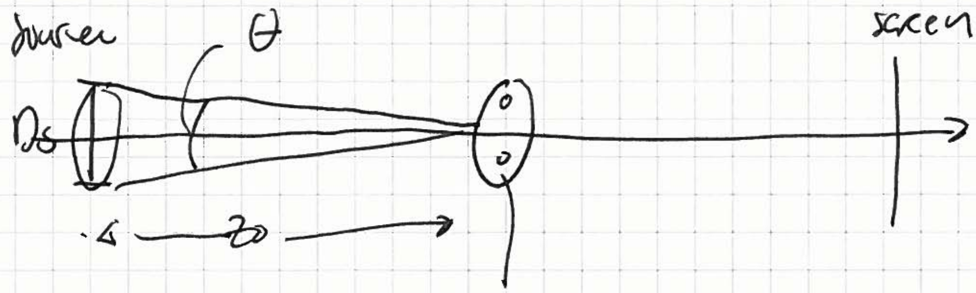
$$U(\bar{X}, \bar{Y}, f) = \frac{1}{i\lambda z} e^{ikz} e^{ik\frac{(\bar{X}^2+\bar{Y}^2)}{2z}} e^{i\frac{2\pi}{\lambda} n\Delta_0} \mathcal{F}\{circ(\frac{g}{D_A/2})\}$$

2p $$I = \frac{I_0}{\lambda^2 z^2} \pi^2 (D_{A/2})^4 4 \left| \frac{J_1(k D_{A/2})}{k D_{A/2}} \right|^2,$$

where $I_0 = \frac{1}{2}\epsilon_0 c |U_0^-(x, y, 0)|^2$

b)

3p



coherence area of which within two sources may give rise to interference/ diffraction effects to be observed on the screen

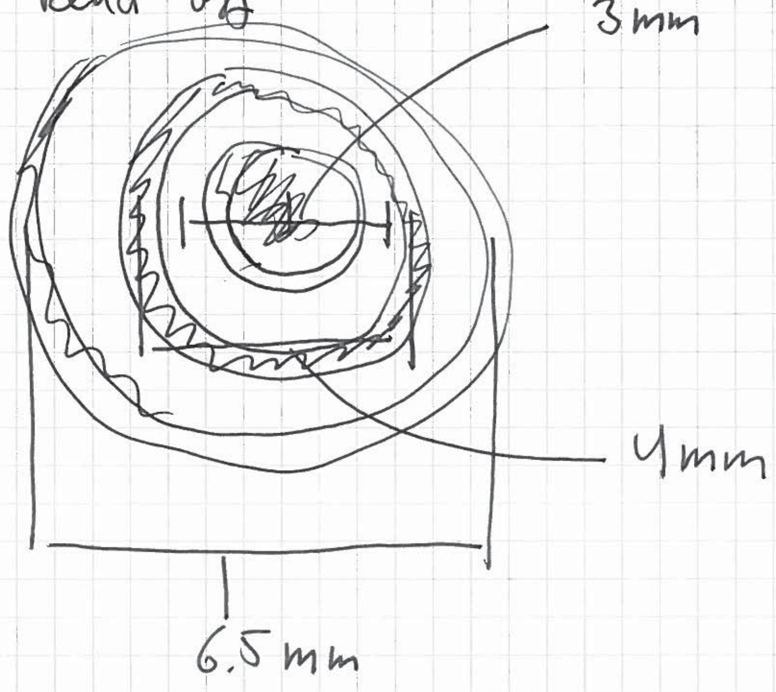
It is reasonable to assume that $l_t > D_A$ to observe diffraction effects from the full aperture, otherwise ~~the~~ several intensity patterns will be added "incoherently" on the screen, ~~washing~~ washing out any diffraction effects.

$$\Rightarrow \frac{1.22 \lambda z_0}{D_S} > D_A$$

$$\underline{z_0 > \frac{D_A D_S}{1.22 \lambda}}$$ or, ~~$\frac{z_0}{D_S}$~~

or $$\underline{\theta = \frac{D_S}{z_0} < \frac{1.22 \lambda}{D_A}}$$

c) Read off



Use centre to first minimum $\Rightarrow$ 5 μm

From (a) $\left| \frac{J_1(k D_A/2)}{k D_A/2} \right|$ & appendix,

$$\Rightarrow \frac{\cancel{2}\cancel{\pi}}{\lambda} \frac{D_A/\cancel{2}}{f} \underbrace{\sqrt{X^2+Y^2}}_{5\mu m} = \cancel{\pi} \cdot 1.22$$

$\lambda = 632$ nm, $z = f = 20$ cm

$$\Rightarrow D_A = \frac{1.22 \lambda f}{5 \mu m} = 0.03 \text{ m} = \underline{\underline{3 \text{ cm}}}$$

Alternatively:

- Centre to 1st maximum: $2\,mm \times \frac{10\,\mu m}{3\,mm} = 6.67\,\mu m$

$$D_A = \frac{1.63\,\lambda f}{6.67\,\mu m} = 3\,cm \quad \text{O.K.}$$

- between 1^{st} max and 2^{nd} max etc. . . .