

Optics TFY4195

Solutions to problems from the exam.

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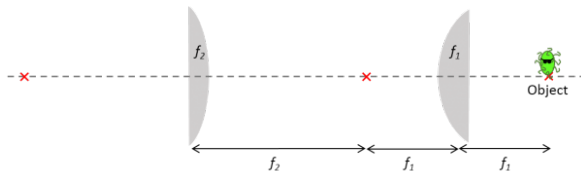
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Geometrical optics

There is a cool bacterium you want to study in closer detail so you want to build a microscope to see it. You are told that you can build a microscope by placing two positive lenses (having focal lengths f_1 and f_2) at a separation distance equal to the sum of their focal lengths as shown below.



- Trace the rays to show how your microscope works and where the image is formed. Is the image real or virtual? upright or inverted? Note that this time the object is on the right and the image will form at the left.
- Calculate how much magnification you get from this optical system.
- Will your microscope still work if we change the distance between the lenses?
- Does changing the distance between the lenses affect the amount of magnification you achieve?

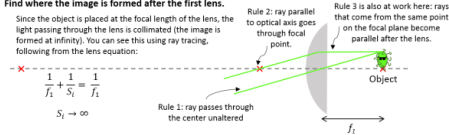
A) Trace the rays to show how your microscope works and where the image is formed. We will see that the image is real and inverted.

Find where the image is formed after the first lens.

Since the object is placed at the focal length of the lens, the light passing through the lens is collimated (the image is formed at infinity). You can see this using ray tracing, following from the lens equation:

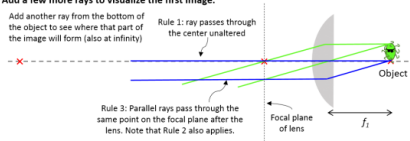
$$\frac{1}{f_1} + \frac{1}{S_i} = \frac{1}{f_1}$$

$$S_i \rightarrow \infty$$



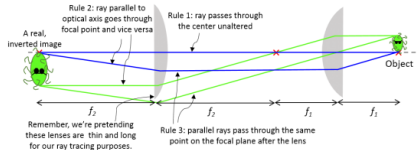
Add a few more rays to visualize the first image.

Add another ray from the bottom of the object to see where that part of the image will form (also at infinity)



Find where the image will form after the second lens.

Since the first image was formed at infinity, it is difficult to draw the image of the first lens to be the object of the second lens (like we did in previous examples). Instead, let's trace the collimated rays from the first lens through the second lens to see where the final image forms.



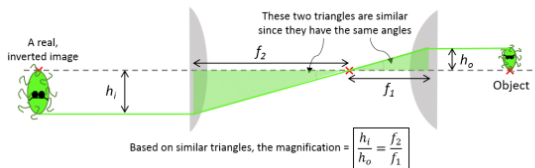
Note: Although an image at infinity is hard to draw, we can still put it into the lens equation to see where the image forms

$$\frac{1}{\infty} + \frac{1}{S_i} = \frac{1}{f_2} \quad S_i \rightarrow f_2$$

- By definition, the rays stay parallel and go on to infinity when light from an object originates exactly at the focal point.
- The rays technically shouldn't focus, but practically moving the lens slightly away so we can get an image focused on a far away screen the way a projector does.
- Depending how precise your placement is, you could get a clear, fuzzy, or no image on a screen far away.
- As the screen gets farther away, your image gets larger and dimmer, so it couldn't form a real image at a very far distance anyway let alone infinity.
- It has just become an expression to say an image is "formed" even though an image is never formed since you can never reach infinity.
- It would similar to saying that parallel lines "meet" at infinity, but that would contradict the meaning of "parallel".
- A better description would be to say that an image is formed very far away as an object approaches the focal point.

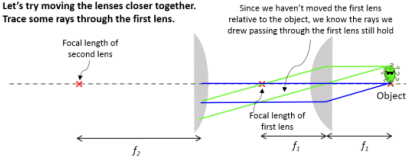
B) Calculate how much magnification you get from this optical system.

For the magnification, find a ray that relates the height of the object to the height of the final image so you can calculate their ratio.

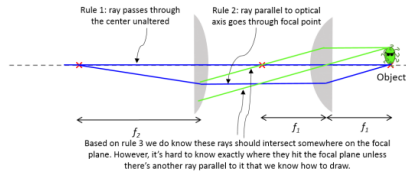


C) Will your microscope still work if we change the distance between the lenses?

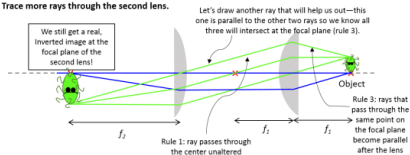
Let's try moving the lenses closer together.
Trace some rays through the first lens.



Trace some rays through the second lens.

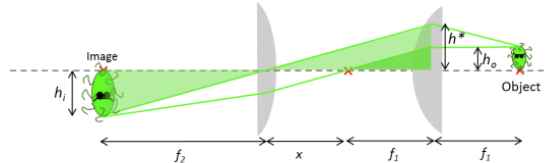


Trace more rays through the second lens.



D) Does changing the distance between the lenses affect the amount of magnification you achieve?

Find rays that relate the height of the object to the height of the final image.



Based on similar triangles:

$$\frac{h_i}{h^*} = \frac{f_2}{x + f_1} \text{ and } \frac{h^*}{h_o} = \frac{x + f_1}{f_1}$$

Thus, the magnification is:

$$\boxed{\frac{h_i}{h_o} = \frac{f_2}{f_1}}$$

This is the same result we had when the lenses were $f_1 + f_2$ apart!

Principle of superposition

5-7. At the indicated position,

$$\psi_1 = A_1 \cos(8\pi/3 - \omega t), \quad A_1 = 4 \text{ cm}, \omega = 20/\text{s}$$

$$\psi_2 = A_2 \cos(3\pi/2 - \omega t), \quad A_2 = 2 \text{ cm}, \omega = 20/\text{s}$$

Using Eqs (5.9) and (5.10),

$$A = \sqrt{A_1^2 + A_2^2 + 2 A_1 A_2 \cos(\alpha_2 - \alpha_1)} = \sqrt{20 + 16 \cos(3\pi/2 - 8\pi/3)} \text{ cm} = 2.48 \text{ cm}$$

$$\tan \alpha = \frac{4 \sin(8\pi/3) + 2 \sin(3\pi/2)}{4 \cos(8\pi/3) + 2 \sin(3\pi/2)} \Rightarrow \alpha = 2.51$$

$$\psi_R = (2.48 \text{ cm}) \cos(2.51 - (20/\text{s})t)$$

Interference

7-9. The fringe separation is,

$$y_{m+1} - y_m \equiv \Delta y = \lambda L / a$$

(a) So the slit to screen distance should be,

$$L = \frac{\Delta y a}{\lambda} = \frac{(0.001)(0.0005)}{6 \times 10^{-7}} \text{ m} = 0.833 \text{ m}$$

(b) The optical path difference can be written in terms of wavelengths as $\Delta = m \lambda$. The with and without the plate of thickness t ,

$$\Delta_2 - \Delta_1 = \Delta m \lambda \Rightarrow \Delta m = \frac{n t - t}{\lambda} = (n - 1) \frac{t}{\lambda} = (1.5 - 1) \frac{10^{-4}}{6 \times 10^{-7}} = 83.3 \text{ fringes}$$

(c) $I = 4 I_0 \cos^2\left(\frac{\pi \Delta}{\lambda}\right)$ where $\Delta = a y / L$. At $\Delta = 0$, $I = I_{\max} = 4 I_0$. Then for $I = 2 I_0 = I_{\max} / 2$:

$$2 I_0 = I = 4 I_0 \cos^2\left(\frac{\pi \Delta}{\lambda}\right) \Rightarrow \Delta = \lambda / 4 = 150 \text{ nm}$$

Thin film interference

7-15. At normal incidence, $(m + 1/2) \lambda = 2 n t$. At 45° , $(m + 1/2) \lambda' = 2 n t \cos \theta_t$.

Here θ_t is the angle the ray makes with the normal in the film which can be found from Snell's law,

$$\sin(45^\circ) = n \sin \theta_t \Rightarrow \sin \theta_t = \frac{1}{\sqrt{2} n} = \frac{1}{\sqrt{2} (1.38)} = 0.5124 \Rightarrow \theta_t = 30.825^\circ = \cos \theta_t = 0.859$$

Then,

$$\lambda' = \frac{2 n t \cos \theta_t}{m + 1/2} = \frac{(m + 1/2) \lambda \cos \theta_t}{m + 1/2} = \lambda \cos \theta_t = (580 \text{ nm}) (0.859) = 498 \text{ nm}$$

Diffraction

11-3. See Figure 11 19 that accompanies the problem in the text.

(a) The diffraction minima are located at angles $\theta_m = y_m/L$ where $L = 2$ m is the slit to screen distance, The positions of the minima are given by $m\lambda = b \sin \theta_m = by_m/L \Rightarrow y_m = m\lambda L/b$. Then,

$$y_3 - y_{-3} = \Delta y = (3 - (-3))\lambda L/b \Rightarrow b = \frac{6\lambda L}{\Delta y} = \frac{6(632.8 \times 10^{-7} \text{ cm})(200 \text{ cm})}{5.625 \text{ cm}} = 0.013 \text{ cm} = 0.13 \text{ mm}$$

(b) $L_{\min} = b^2/2\lambda$, so,

$$\frac{L}{L_{\min}} = \frac{200 \text{ cm}}{(0.0135 \text{ cm})^2/(2 \cdot 632.8 \times 10^{-7} \text{ cm})} = 139$$

The screen is in the far field.

Beam diffraction

Example 2

Imagine a parallel beam of 546-nm light of width $b = 0.5$ mm propagating a distance of 10 m across the laboratory. Estimate the final width W of the beam due to diffraction spreading.

Solution

Using Eq. (15),

$$W = \frac{2L\lambda}{b} = \frac{2(10 \text{ m})(546 \times 10^{-9} \text{ m})}{0.5 \times 10^{-3} \text{ m}} = 0.0218 \text{ m} = 21.8 \text{ mm}$$