

x-hm exams oppgave!

~~3.6.~~ $\vec{n} = (\sin\theta \cos\phi, \sin\theta \sin\phi, \cos\theta)$

①

Average of any function of $\vec{n}$, $f(\vec{n})$ is

$$\langle f(\vec{n}) \rangle = \frac{1}{4\pi} \int_0^{2\pi} d\phi \int_0^\pi \sin\theta \cdot d\theta \cdot f(\vec{n})$$

$$\sin^2\theta = \frac{1}{2}(1 - \cos 2\theta)$$

$$\langle \vec{n} \rangle = \hat{x}: \frac{1}{4\pi} \int_0^{2\pi} d\phi \int_0^\pi \sin\theta \cdot d\theta \cdot \sin\theta \cos\phi$$

$$= \frac{1}{4\pi} \int_0^{2\pi} \cos\phi d\phi \int_0^\pi \frac{1}{2}(1 - \cos 2\theta) d\theta$$

$$= \frac{1}{8\pi} \int_0^{2\pi} \cos\phi d\phi \left[\theta - \frac{1}{2} \sin 2\theta \right]_0^\pi$$

$$= \frac{1}{8} \int_0^{2\pi} \cos\phi d\phi = \frac{1}{8} \left[\sin\phi \right]_0^{2\pi} = 0$$

$$\hat{y}: \frac{1}{4\pi} \dots \text{prv} \cdot \frac{1}{8} \left[-\sin\phi \right]_0^{2\pi} = 0$$

$$\hat{z}: \frac{1}{4\pi} \int_0^{2\pi} d\phi \int_0^\pi \sin\theta d\theta \cdot \cos\theta = \frac{1}{2} \left[\cos^2\theta \right]_0^\pi = \frac{1}{2}(1-1) = 0$$

$$\langle \vec{n} \rangle = 0 !$$

If you are smart you do not need to calculate.
Average of unit vector over all orientations is
of course zero....

2.

$$d = \begin{pmatrix} 0 & 0 & 0 & d_{14} & 0 & 0 \\ 0 & 0 & 0 & 0 & d_{14} & 0 \\ 0 & 0 & 0 & 0 & 0 & d_{36} \end{pmatrix} \Rightarrow$$

$$\begin{pmatrix} P_x \\ P_y \\ P_z \end{pmatrix} = \begin{pmatrix} 2d_{14} E_y E_z \\ 2d_{14} E_z E_x \\ 2d_{36} E_x E_y \end{pmatrix} \quad \text{unit } \hat{o}: \begin{pmatrix} -\sin\phi \\ \cos\phi \\ 0 \end{pmatrix} \quad \hat{e}: \begin{pmatrix} \cos\phi \sin\theta \\ \sin\phi \sin\theta \\ -\cos\theta \end{pmatrix}$$

we get for

$$d_{\text{oe}}: \left[2d_{14} \cdot 0 \hat{x} + 2d_{14} \cdot 0 \hat{y} + 2d_{36} \cdot (-\sin\phi \cos\phi) \hat{z} \right] \cdot \hat{e}$$

$$= 2d_{36} \sin\phi \cos\phi \sin\theta = \underline{d_{36} \sin 2\phi \sin\theta}$$

$$d_{\text{eo}}: \left[-2d_{14} \sin\phi \sin\theta \cos\theta \hat{x} - 2d_{14} \cos\phi \sin\theta \cos\theta \hat{y} + 2d_{36} \sin\phi \cos\phi \sin\theta \cos\theta \hat{z} \right] \cdot \hat{o}$$

$$= 2d_{14} \sin^2\phi \sin\theta \cos\theta - 2d_{14} \cos^2\phi \sin\theta \cos\theta$$

$$= -2d_{14} \cos 2\phi \sin\theta \cos\theta = \underline{-d_{14} \cos 2\phi \sin 2\theta}$$

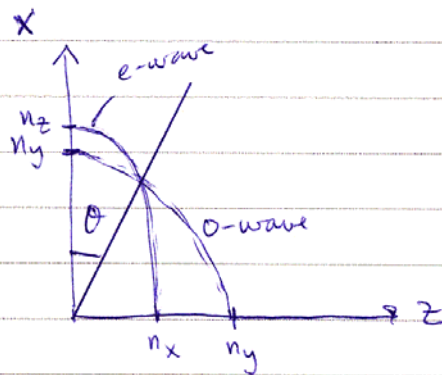
+ also ok due to crystal symmetry

$$\cos 2\alpha = \cos^2\alpha - \sin^2\alpha$$

$$2\sin^2\alpha - 1 = -\cos 2\alpha$$

$$\sin 2\alpha = 2\sin\alpha \cos\alpha$$

3. Optics axis will occur in the plane of the "highest and lowest" refractive index. In this case xz -plane



o-wave θ independent (not along $\hat{y}$) $n_x = n_y$

e-wave θ dependent $\frac{1}{n_{eff}^2(\theta)} = \frac{\sin^2 \theta}{n_x^2} + \frac{\cos^2 \theta}{n_z^2}$

along opt^r axis o- & e-wave same speed $\Rightarrow$

$$\frac{1}{n_y^2} = \frac{\sin^2 \theta}{n_x^2} + \frac{\cos^2 \theta}{n_z^2} \quad ; \quad \text{a little algebra yields}$$

$$\sin^2 \theta = \frac{n_y^2 n_x^2 - n_z^2 n_x^2}{n_y^2 n_x^2 - n_y^2 n_z^2} = \frac{20.25 - 26.01}{20.25 - 104.04} = \frac{5.76}{83.79}$$

$$\sin \theta = 0.2622 \quad \Rightarrow \underline{\underline{+15.2^\circ = \theta}} \quad \text{angle from } x\text{-axis}$$

4. A. first QWP does nothing

second 45° away from polarizer axis

$\Rightarrow$ circ polarized (left)

equivalent Jones

see Hecht 8.6

$$e^{i\pi/4} \begin{pmatrix} 1 & 0 \\ 0 & -i \end{pmatrix} \begin{pmatrix} -1 \\ 1 \end{pmatrix} \rightarrow -e^{i\pi/4} \begin{pmatrix} 1 \\ i \end{pmatrix}$$

QWP vert circ in

B. first QWP $\rightarrow$ circ pol

2nd QWP circ $\rightarrow$ linear, it

cannot be horizontal (C),

therefore vertical LP ($\pm 45^\circ$)

$$\begin{pmatrix} 1 & 0 \\ 0 & -i \end{pmatrix} \begin{pmatrix} 1 \\ i \end{pmatrix} = \begin{pmatrix} 1 \\ 1 \end{pmatrix} \quad +45^\circ$$

C. $2 \times$ QWP $\Rightarrow$ HWP

HWP 45° from LP $\rightarrow$ LP $\perp$

i.e. LP horizontal

5. Symmetry must be retained.

FT. Therefore:

$$1 - E$$

$$2 - 0$$

$$3 - V$$

6.

$$\Delta y_{ij} = \begin{pmatrix} 0 & 0 & r_{13} \\ 0 & 0 & r_{23} \\ 0 & 0 & r_{33} \\ \sqrt{E} & r_{41} & r_{42} & 0 \\ \sqrt{E} & r_{51} & r_{52} & 0 \\ \sqrt{E} & 0 & 0 & r_{62} \end{pmatrix} E_0 \begin{pmatrix} 1 \\ 0 \\ \sqrt{5} \end{pmatrix} =$$

$$= \begin{pmatrix} r_{13}\sqrt{5}E_0 & r_{62}\sqrt{5}E_0 & r_{51}E_0 \\ r_{62}\sqrt{5}E_0 & r_{23}\sqrt{5}E_0 & r_{41}E_0 \\ r_{51}E_0 & r_{41}E_0 & r_{33}\sqrt{5}E_0 \end{pmatrix} \leftarrow \text{ans}$$

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$$(2) \quad \epsilon_{abc} \frac{\partial B_c}{\partial x_b} = -i\omega \mu_0 \epsilon_0 K_{ad} \bar{E}_d$$

change index of (1)

$$\Rightarrow \epsilon_{ckj} \frac{\partial \bar{E}_j}{\partial x_k} = i\omega B_c \quad \text{or} \quad B_c = \frac{1}{i\omega} \epsilon_{ckj} \frac{\partial \bar{E}_j}{\partial x_k}$$

$$\Rightarrow (2) \quad \frac{1}{i\omega} \epsilon_{abc} \frac{\partial}{\partial x_b} \epsilon_{ckj} \frac{\partial \bar{E}_j}{\partial x_k} = -i\omega \mu_0 \epsilon_0 K_{ad} \bar{E}_d$$

ϵ_{ckj} constant

$$\mu_0 \epsilon_0 = \frac{1}{c^2}$$

$$\Rightarrow (2) \quad \epsilon_{abc} \epsilon_{ckj} \frac{\partial}{\partial x_b} \frac{\partial \bar{E}_j}{\partial x_k} = \frac{\omega^2}{c^2} K_{ad} \bar{E}_d$$

↓

$$\epsilon_{cab} \epsilon_{ckj}$$

$$(\delta_{ak} \delta_{bj} - \delta_{aj} \delta_{bk})$$

$$(\delta_{ak} \delta_{bj} - \delta_{aj} \delta_{bk}) \frac{\partial}{\partial x_b} \frac{\partial \bar{E}_j}{\partial x_k} = \frac{\omega^2}{c^2} K_{ad} \bar{E}_d$$

$$\frac{\partial}{\partial x_b} \frac{\partial \bar{E}_b}{\partial x_a} - \frac{\partial}{\partial x_b} \frac{\partial \bar{E}_a}{\partial x_b} = \frac{\omega^2}{c^2} K_{ad} \bar{E}_d$$

$$\frac{\partial}{\partial x_a} \left(\frac{\partial \bar{E}_b}{\partial x_b} \right) - \left(\frac{\partial}{\partial x_b} \right)^2 \bar{E}_a = \frac{\omega^2}{c^2} K_{ad} \bar{E}_d$$

identity: $\vec{\nabla} (\vec{\nabla} \cdot \vec{E}) - \vec{\nabla}^2 \cdot \vec{E} = \frac{\omega^2}{c^2} \vec{K} \vec{E} \quad \text{QED}$

8. A. SHG $\Rightarrow \omega \rightarrow 2\omega$ or $2\lambda \rightarrow \lambda$ ($\omega = \frac{2\pi c}{\lambda}$)

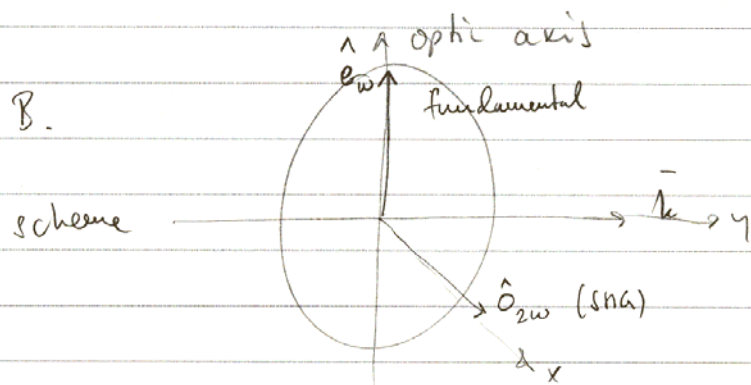
critical phase matching $n_o \rightarrow n_e$ or $n_e \rightarrow n_o$.

From figure we see immediately

that in the λ range 240-250 nm for $n_o =$

n_e at approximately 480-500 $\Rightarrow$ SHG for

$$\frac{n_e}{\omega} \rightarrow \frac{n_o}{2\omega}$$



or

problem 5

Solution:

Snell's law relates θ_i and θ_t ;

Specifically, $n_i \sin \theta_i = n_t \sin \theta_t$

since air, $n_i = 1$; n_t depends on θ_t since

the crystal is uniaxial, hence

$$n_t(\theta_t) = \frac{\sin \theta_i}{\sin \theta_t}$$

what is $n_t(\theta_t)$? introduce help-angle

$\alpha = \theta_c - \theta_t$, specifically

$$\frac{1}{n_{\text{eff}}^2(\theta_t)} = \frac{\sin^2 \alpha}{n_c^2} + \frac{\cos^2 \alpha}{n_o^2}$$

$$n_{\text{eff}}^2(\theta_t) = \frac{1}{\frac{\sin^2 \alpha}{n_c^2} + \frac{\cos^2 \alpha}{n_o^2}} = \frac{\sin^2 \alpha + \cos^2 \alpha}{\frac{\sin^2 \alpha}{n_c^2} + \frac{\cos^2 \alpha}{n_o^2}} =$$

$$= \frac{n_o^2 \cdot \tan^2 \alpha + n_c^2}{\frac{n_o^2}{n_c^2} \tan^2 \alpha + 1}$$

$$\Rightarrow \frac{\sin \theta_i}{\sin \theta_t} = n_t(\theta_t) = \frac{n_o \sqrt{1 + \tan^2(\theta_c - \theta_t)}}{\sqrt{1 + \frac{n_o^2}{n_c^2} \tan^2(\theta_c - \theta_t)}} \quad \text{QED}$$