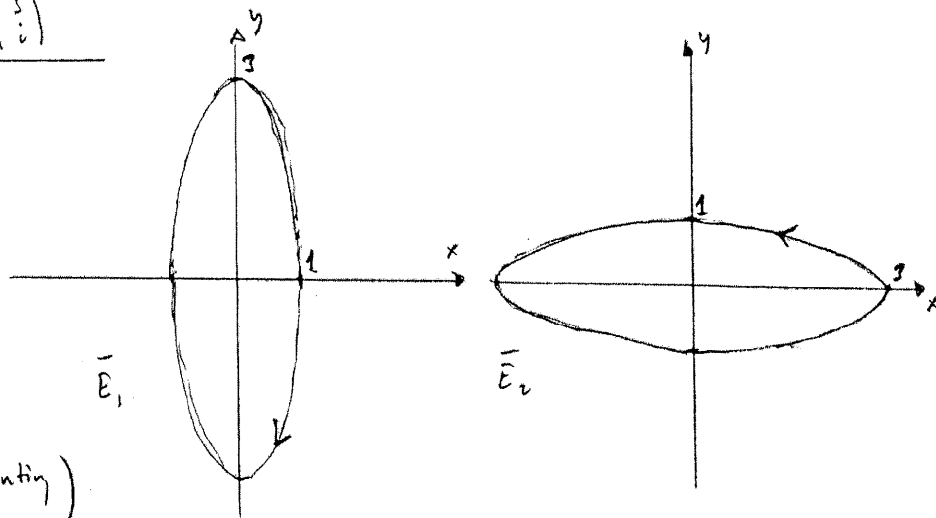


P1:

Ans: $\bar{E}_2 = \begin{pmatrix} 3 \\ i \end{pmatrix}$

Sketch:

$z=0$
(z-axis pointing out)



Solution: $\bar{E}_1^* \cdot \bar{E}_2$ gives $1 \cdot e_{21} + (-3i)^* \cdot e_{22} = e_{21} + 3ie_{22} = 0$
 $\Rightarrow e_{21} = 3 ; e_{22} = i \Rightarrow \bar{E}_2 = \begin{pmatrix} 3 \\ i \end{pmatrix}$

Sketch pol.-ellipse

Def. $\vec{E} = \begin{pmatrix} E_x \\ E_y \end{pmatrix} = \begin{pmatrix} E_{0x} e^{i(kz - \omega t + \phi_x)} \\ E_{0y} e^{i(kz - \omega t + \phi_y)} \end{pmatrix} \Big|_{\substack{\phi_x = \phi_y = 0 \\ z=0}} = \begin{pmatrix} E_{0x} e^{i(-\omega t)} \\ E_{0y} e^{i(-\omega t)} \end{pmatrix}$

$\bar{E}_1 = \begin{bmatrix} 1 \cdot e^{i(-\omega t)} \\ -3i \cdot e^{i(-\omega t)} \end{bmatrix} = \begin{bmatrix} 1 \cdot e^{i(-\omega t)} \\ 3 \cdot e^{i(-\omega t - \frac{\pi}{2})} \end{bmatrix} \Big|_{\text{real part}} = \begin{bmatrix} 1 \cdot \cos(-\omega t) \\ 3 \cdot \cos(-\omega t - \frac{\pi}{2}) \end{bmatrix}$

in the same way

$\bar{E}_2 = \begin{bmatrix} 3 \cdot \cos(-\omega t) \\ 1 \cdot \cos(-\omega t + \frac{\pi}{2}) \end{bmatrix}$

Table

		$\omega t = 0$	$= \frac{\pi}{4}$	$= \frac{\pi}{2}$	$= \frac{3\pi}{4}$
$\bar{E}_1$	x	1	$1/\sqrt{2}$	0	$-1/\sqrt{2}$
	y	0	$-3/\sqrt{2}$	-1	$3/\sqrt{2}$
$\bar{E}_2$	x	3	$3/\sqrt{2}$	0	$-3/\sqrt{2}$
	y	0	$1/\sqrt{2}$	1	$1/\sqrt{2}$

The Stokes vector is given by:

$$S_0 = \langle E_x E_x^* \rangle + \langle E_y E_y^* \rangle = 1 + 9 = 10$$

$$S_1 = \langle E_x E_x^* \rangle - \langle E_y E_y^* \rangle = -8$$

$$S_2 = 2 \langle |E_x| \cdot |E_y| \rangle \cos \Delta = 2 \cdot 1 \cdot 3 \cdot \cos(-\pi/2) \\ = 0$$

$$S_3 = 2 \langle |E_x| |E_y| \rangle \sin \Delta = 2 \cdot 1 \cdot 3 \cdot \sin(-\pi/2) \\ = -6$$

(P21)

Ans. $\frac{w_x}{w_y} \sim 0.44$ (0.4-0.5 range OK)

Detailed solution

At the focal plane the field distribution is the exact FT of the aperture field distribution.

$$U(x, y) = C \cdot \int_{-\frac{w_x}{2}}^{\frac{w_x}{2}} e^{-2\pi i \frac{xy}{\lambda z}} dz \int_{-\frac{w_y}{2}}^{\frac{w_y}{2}} e^{-2\pi i \frac{yz}{\lambda z}} dy$$

↑ observation (FT) plane
↑ aperture plane

Integrals are separable. C complex constant.

Consider x -integral I_x : $\int_{-\frac{w_x}{2}}^{\frac{w_x}{2}} e^{-2\pi i \frac{xy}{\lambda z}} dz = \dots = w_x \cdot \frac{\sin \pi \left(\frac{w_x x}{\lambda z} \right)}{\pi \left(\frac{w_x x}{\lambda z} \right)}$

I_x has zeros for $\frac{w_x x}{\lambda z} = m \cdot 1$ where $m \geq 1$

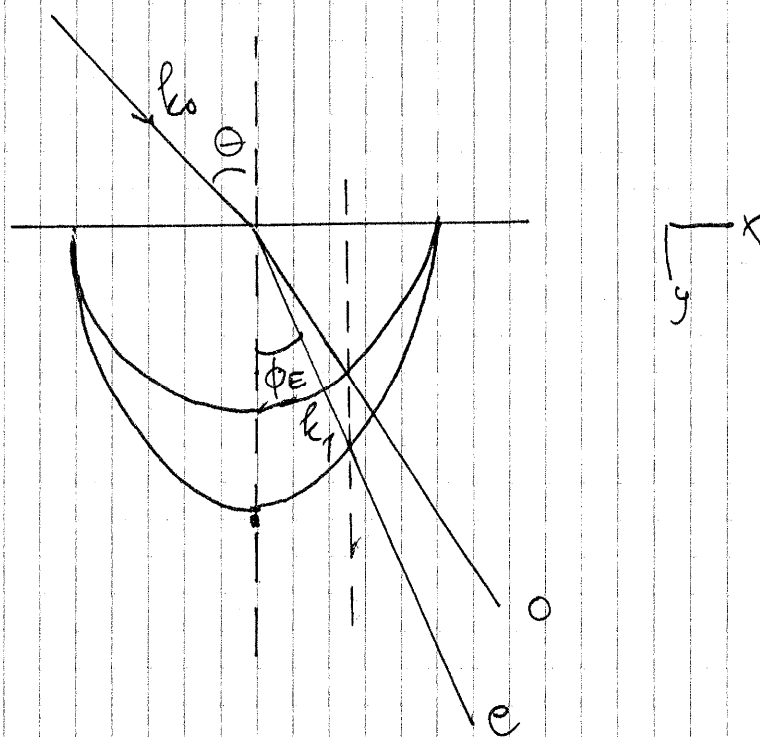
Hence, distance between minima inversely proportional to w_x ($x = \frac{m \cdot \lambda \cdot z}{w_x}$). The same argument for vertical dimension (w_y & y).

$\frac{w_x}{w_y}$ can be directly measured in the figure.

E.g. locate the "seventh" minimum of each (x & y).

$$\frac{w_x}{w_y} \text{ is } \frac{\frac{1}{x} [\text{mm}]}{\frac{1}{y} [\text{mm}]} = \frac{19.5 \text{ mm}}{44 \text{ mm}} \approx 0.443$$

Probl 3.



Momentum conservation along boundary

$$k_0 \sin \theta = k_1 \sin \phi_E$$

Ellipse:
$$\frac{k_x^2}{\left(n_0 \frac{\omega}{c}\right)^2} + \frac{k_y^2}{\left(n_E \frac{\omega}{c}\right)^2} = 1$$

$$\Rightarrow \frac{\left(\frac{\omega}{c} \sin \theta\right)^2}{\left(n_0 \frac{\omega}{c}\right)^2} + \frac{k_y^2}{\left(n_0 \frac{\omega}{c}\right)^2} = 1$$

$$\Rightarrow k_y = n_E \left(\frac{\omega}{c}\right) \sqrt{1 - \frac{\sin^2 \theta}{n_0^2}}$$

$$\tan \phi_E = \frac{k_x}{k_y} = \frac{n_0 \sin \theta}{n_E \sqrt{n_0^2 - \sin^2 \theta}}$$

4: The rotation matrix is given by

$$\bar{R} = \begin{pmatrix} \cos \phi & 0 & -\sin \phi \\ 0 & 1 & 0 \\ \sin \phi & 0 & \cos \phi \end{pmatrix}$$

With $\phi = -60^\circ$ we get:

$$\bar{R} = \begin{pmatrix} \frac{1}{2} & 0 & \frac{1}{2}\sqrt{3} \\ 0 & 1 & 0 \\ -\frac{1}{2}\sqrt{3} & 0 & \frac{1}{2} \end{pmatrix}$$

And:

$$\bar{E}' = \bar{R} \bar{E} \bar{R}^{-1}$$

$$= \begin{pmatrix} \frac{1}{2} & 0 & \frac{1}{2}\sqrt{3} \\ 0 & 1 & 0 \\ -\frac{1}{2}\sqrt{3} & 0 & \frac{1}{2} \end{pmatrix} \begin{pmatrix} 2.25 & 0 & 0 \\ 0 & 2.25 & 0 \\ 0 & 0 & 2.56 \end{pmatrix} \begin{pmatrix} \frac{1}{2} & 0 & -\frac{1}{2}\sqrt{3} \\ 0 & 1 & 0 \\ \frac{1}{2}\sqrt{3} & 0 & \frac{1}{2} \end{pmatrix}$$

$$= \begin{pmatrix} 2.4825 & 0 & 0.1342 \\ 0 & 2.25 & 0 \\ 0.1342 & 0 & 2.3275 \end{pmatrix}$$

5:

We start with the
The "wavevectors" ; see Mikael's solution Ch. 7.

$$\hat{e}_0 = \begin{pmatrix} -\sin\phi \\ \cos\phi \\ 0 \end{pmatrix}$$

$$\hat{e}_\theta = \begin{pmatrix} \cos\phi \cos\theta \\ \sin\phi \cos\theta \\ -\sin\theta \end{pmatrix}$$

$$\Rightarrow \begin{pmatrix} P_x \\ P_y \\ P_z \end{pmatrix} = \begin{pmatrix} d_{31}(E_x^{\omega_1} E_z^{\omega_2} + E_z^{\omega_1} E_x^{\omega_2}) - d_{22}(E_x^{\omega_1} E_y^{\omega_2} + E_y^{\omega_1} E_x^{\omega_2}) \\ -d_{22}(E_x^{\omega_1} E_x^{\omega_2}) + d_{22} E_y^{\omega_1} E_y^{\omega_2} + d_{31}(E_y^{\omega_1} E_z^{\omega_2} + E_z^{\omega_1} E_y^{\omega_2}) \\ d_{31}(E_x^{\omega_1} E_x^{\omega_2} + E_y^{\omega_1} E_y^{\omega_2}) + d_{33} E_z^{\omega_1} E_z^{\omega_2} \end{pmatrix}$$

Insert E^{ω_1} and E^{ω_2} as e-waves $\Rightarrow$

$$= \begin{pmatrix} d_{31}(-2 \cos\phi \sin\theta \cos\theta) - d_{22}(2 \sin\phi \cos\phi \cos^2\theta) \\ -d_{22}(\cos^2\phi \cos^2\theta - \sin^2\phi \cos^2\theta) + d_{31}(-2 \sin\phi \sin\theta \cos\theta) \\ d_{31}(\cos^2\phi \cos^2\theta + \sin^2\phi \cos^2\theta) + d_{33} \sin^2\theta \end{pmatrix}$$

Project out the o-wave $\Rightarrow e \cdot e_0$

$$\begin{aligned}
 d_{\text{eff}} &= d_{31}(2\sin\phi\cos\phi\sin\theta\cos\theta) + d_{22}(2\sin^2\phi\cos\phi\cos^2\theta) \\
 &\quad - d_{22}(\cos^3\phi\cos^2\theta - \sin^2\phi\cos\phi\cos^2\theta) + d_{31}(-2\sin\phi\cos\phi\sin\theta\cos\theta) \\
 &= d_{31}(0) - d_{22}(\cos^3\phi\cos^2\theta - 3\sin^2\phi\cos\phi\cos^2\theta) \\
 &= -d_{22}(4\cos^2\phi - 3\cos\phi)\cos^2\theta \\
 &= -d_{22}\cos^2\theta\cos 3\phi
 \end{aligned}$$

e_0 :

$$\begin{pmatrix} P_x \\ P_y \\ P_z \end{pmatrix} = \begin{pmatrix} d_{31}(\sin\theta\sin\phi - d_{22}(\cos^2\phi\cos\theta - \sin^2\phi\cos\theta)) \\ -d_{22}(-\sin\phi\cos\phi\cos\theta) + d_{22}\sin\phi\cos\phi\cos\theta + d_{31}(-\sin\theta\cos\phi) \\ d_{31}(-\cos\phi\sin\phi\cos\theta + \sin\phi\cos\phi\cos\theta) \quad d_{33} \cdot 0 \\ = 0 \end{pmatrix}$$

Project out the e-wave

$$\begin{aligned} \Rightarrow & d_{31} (\cancel{\sin \theta \sin \phi \cos \phi \cos \theta} - \cancel{\sin \theta / \cos \phi}) \\ & - d_{22} (\cos^3 \phi \cos \theta - \sin^2 \phi \cos \theta) \cos \phi \cos \theta \\ & - d_{22} (-\cos \phi \sin \phi \cos \theta - \sin \phi \cos \phi \cos \theta) \sin \phi \cos \theta \\ & - d_{22} (\cos^3 \phi - \sin^2 \phi \cos \phi) \cos^2 \theta \\ & + d_{22} (2 \cos \phi \sin^2 \phi) \cos^2 \theta \\ = & - d_{22} (\cos^3 \phi - 3 \cos \phi \sin^2 \phi) \cos^2 \theta \\ = & - d_{22} (4 \cos^3 \phi - 3 \cos \phi) \cos^2 \theta \\ = & - d_{22} \cos 3\phi \cdot \cos^2 \theta \end{aligned}$$

6a

Start with eq 6.97 in handout materials

$$\begin{vmatrix} (n_x \frac{\omega}{c})^2 - k_y^2 - k_z^2 & k_x k_y & k_x k_z \\ k_y k_x & (n_y \frac{\omega}{c})^2 - k_x^2 - k_z^2 & k_y k_z \\ k_z k_x & k_z k_y & (n_z \frac{\omega}{c})^2 - k_x^2 - k_y^2 \end{vmatrix} = 0$$

Crossing with x-axis, $k_x \neq 0$, $k_y = 0$, $k_z = 0$

$$\Rightarrow \begin{vmatrix} (n_x \frac{\omega}{c})^2 & 0 & 0 \\ 0 & (n_y \frac{\omega}{c})^2 - k_x^2 & 0 \\ 0 & 0 & (n_z \frac{\omega}{c})^2 - k_x^2 \end{vmatrix} = 0$$

Solutions:

1: $\frac{k_x}{\omega/c} = n_y$

2: $\frac{k_x}{\omega/c} = n_z$

In the same way you find the crossings with the y - and z -axis.

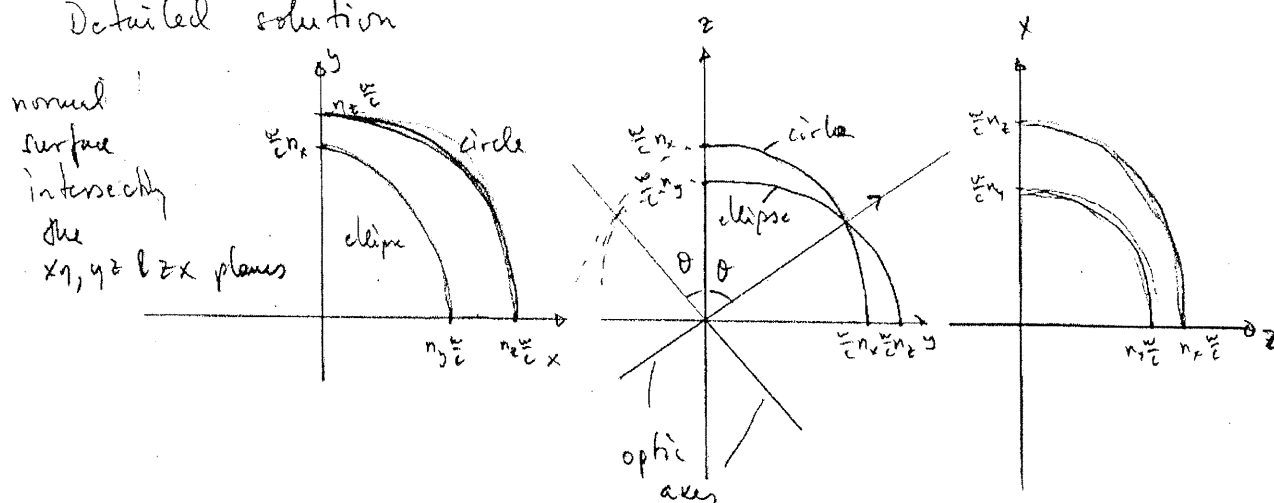
b)

By simply plotting the curves in the $x-y$, $x-z$ and $y-z$ planes you see immediately that the curves will cross in the plane containing the ~~the~~ highest and lowest n . See also Mikael's Compendium p. 89.

P3:

Ans. In the yz -plane $\pm 54^\circ$ from the z -axis.

Detailed solution



The normal surface(s) represent the solution(s) in terms of $\vec{k}$ or $\vec{s}$. It should be well known, easy to see that the optic axes are in the plane spanned by the extreme refractive indices (i.e. n_y & $n_z \rightarrow yz$ -plane)

In this plane the ordinary ray (circle) is independent of θ $n = n_x$. The extraordinary ray follows the usual

$$\frac{1}{n_{eff}^2} = \frac{\cos^2 \theta}{n_y^2} + \frac{\sin^2 \theta}{n_z^2} \quad \text{with } \theta \text{ defined as above.}$$

Along optic axis extraordinary and ordinary ray propagates with the same speed. $\Rightarrow$

$$\frac{1}{n_x^2} = \frac{\cos^2 \theta}{n_y^2} + \frac{\sin^2 \theta}{n_z^2} ; \text{ a little algebra gives } \theta = 54^\circ$$

7.

$$\frac{r_p}{r_s} = \frac{\epsilon \cos \phi - \sqrt{\epsilon - \sin^2 \phi}}{\epsilon \cos \phi + \sqrt{\epsilon - \sin^2 \phi}} \cdot \frac{\cos \phi + \sqrt{\epsilon - \sin^2 \phi}}{\cos \phi - \sqrt{\epsilon - \sin^2 \phi}}$$

$$= \frac{\epsilon \cos^2 \phi - \cos \phi \sqrt{\epsilon - \sin^2 \phi} + \epsilon \cos \phi \sqrt{\epsilon - \sin^2 \phi} - \epsilon + \sin^2 \phi}{\epsilon \cos^2 \phi + \cos \phi \sqrt{\epsilon - \sin^2 \phi} - \epsilon \cos \phi \sqrt{\epsilon - \sin^2 \phi} - \epsilon + \sin^2 \phi}$$

$$= \frac{\sqrt{\epsilon - \sin^2 \phi} \cos \phi (\epsilon - 1) - \sin^2 \phi (\epsilon - 1)}{\sqrt{\epsilon - \sin^2 \phi} \cos \phi (1 - \epsilon) + \sin^2 \phi (1 - \epsilon)}$$

$$= \frac{\sin \phi \operatorname{tg} \phi - \sqrt{\epsilon - \sin^2 \phi}}{\sin \phi \operatorname{tg} \phi + \sqrt{\epsilon - \sin^2 \phi}} = p$$

$$\Rightarrow \frac{\sqrt{\epsilon - \sin^2 \phi}}{\sin \phi \operatorname{tg} \phi} = \frac{1-p}{1+p}$$

$$\Rightarrow \epsilon = \sin^2 \phi + \sin^2 \phi \operatorname{tg}^2 \phi \left(\frac{1-p}{1+p} \right)^2$$

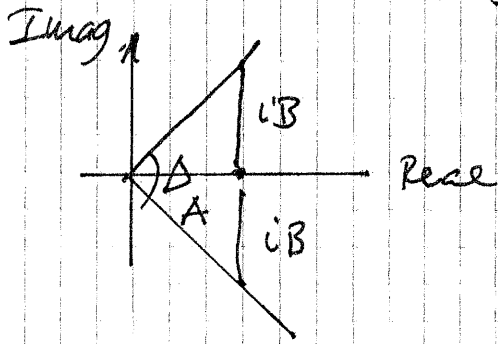
6)

For an inner reflection we replace

$N^2 = \epsilon$ by the inverse

$$\Rightarrow \frac{U}{S} = \frac{\sin\phi \tan\phi - \sqrt{\frac{1}{N^2} - \sin^2\phi}}{\sin\phi \tan\phi + \sqrt{\frac{1}{N^2} - \sin^2\phi}}$$

$$= \frac{N \sin\phi \tan\phi - i \sqrt{N^2 \sin^2\phi - 1}}{N \sin\phi \tan\phi + i \sqrt{N^2 \sin^2\phi - 1}} = \frac{A - iB}{A + iB}$$



We see directly that $\tan \frac{\Delta}{2} = \frac{B}{A} = \frac{\sqrt{N^2 \sin^2 \phi - 1}}{N \sin \phi \tan \phi}$

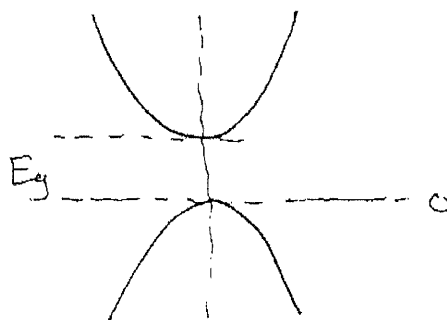
$\Delta = 45^\circ$ for each reflection

Solve for N

$$N^2 = \frac{1}{8 \sin^2 \phi (1 - \tan^2 \frac{\Delta}{2} \tan^2 \phi)} = 2.29$$

$$\underline{\underline{N = 1.514}}$$

Oppgave 3



Energi-båndene er gitt av

$$E_v = -\frac{\hbar^2 k^2}{2m_h} \quad E_c = E_g + \frac{\hbar^2 k^2}{2m_c}$$

$$E_c - E_v = E_g + \frac{\hbar^2 k^2}{2m_r} \quad \frac{1}{m_r} = \frac{1}{m_c} + \frac{1}{m_v}$$

Fra den oppgitte ligningen

$$\begin{aligned} \epsilon_2 &\sim \frac{JDS}{\omega^2} \sim \frac{1}{\omega^2} \int d^3k \delta(E_g + \frac{\hbar^2 k^2}{2m_r} - \hbar\omega) \\ &= \frac{4\pi}{\omega^2} \int k^2 dk \delta(E_g + \frac{\hbar^2 k^2}{2m_r} - \hbar\omega) \end{aligned}$$

$$\text{Fra } \int g(x) \delta(f(x)) dx = \frac{g(x_0)}{\left| \frac{\partial f}{\partial x} \right|_{x_0}} \quad \text{f\u00e5s}$$

$$\epsilon_2 \sim \frac{1}{\omega^2} \frac{k_0^2}{k_0} \quad \text{med } \hbar k_0 = \sqrt{2m_r(\hbar\omega - E_g)}$$

Dette gir til slutt:

$$\epsilon_2 \sim \frac{1}{\omega^2} \sqrt{\hbar\omega - E_g}$$

$$\hbar\omega > E_g$$

$$\epsilon_2 = 0$$

$$\hbar\omega < E_g$$

b) Her får vi

$$\begin{aligned} \epsilon_2 &\sim \frac{1}{\omega^2} \int d^3k \propto k^2 \delta(E_g + \frac{\hbar^2 k^2}{2m_r} - \hbar\omega) \\ &= \frac{4\pi\alpha}{\omega^2} \frac{k_0^4}{k_0/m_r} = 4\pi\alpha m_r \cdot \frac{k_0^3}{\omega^2} \end{aligned}$$

med igjen $\hbar k_0 = \sqrt{2m_r(\hbar\omega - E_g)}$
som gir

$$\epsilon_2 \sim \frac{1}{\omega^2} (\hbar\omega - E_g)^{3/2} \quad \hbar\omega > E_g$$

$$\epsilon_2 = 0$$

$$\hbar\omega < E_g$$