**Norges teknisk–naturvitenskapelige universitet NTNU**

**Institutt for fysikk**
**Fakultet for fysikk, informatikk og matematikk**

Contact during the exam:
Alex Hansen
Telephone: 93649

**Exam in SIF4052 SOLID STATE PHYSICS 1**
Saturday, December 16, 2000
09:00–14:00

Allowed help: Alternativ B
Godkjent lommekalkulator.
K. Rottman: *Matematisk formelsamling* (alle språkutgaver).
O.H. Jahren og K.J. Knudsen: *Formelsamling i matematikk*.

This problem set consists of 2 pages.

## Problem 1

For long wave lengths, the equation of motion for longitudinal waves along the [100] direction of a simple cubic crystal is

$$\frac{d^2u}{dt^2} = \left(\frac{c}{\rho}\right)\frac{d^2u}{dx^2}\ , \tag{1.1}$$

where $c$ is the elastic modulus and $\rho$ is the volume mass density of the crystal. In the same direction, the LA dispersion relation is

$$\omega^2 = \left(\frac{4K}{M}\right)\sin^2\left(\frac{ka}{2}\right)\ , \tag{1.2}$$

where $a$ is the cubic lattice constant, $K$ is the harmonic force constant between atoms and $M$ is the mass of each atom.

**a)** Express $c$ in terms of $a$ and $K$.

## Problem 2

Consider vibrations of a planar square lattice of rows and columns of $N$ identical atoms, mass $M$ spaced a distance $a$ apart and constrained to only move in the 2D plane. $N \gg 1$.

**a)** Using periodic boundary conditions, show that the number of allowed values of $\vec{k}$ per unit area in $\vec{k}$ space is $(Na^2)/(2\pi)^2$.

**b)** What is the radius $k_D$ of the circle in $\vec{k}$-space containing $N$ allowed $\vec{k}$ values?

**c)** Assume that all waves in the plane have the same velocity $v$. The density of states is $g(\omega)$. Show that $g(\omega) = B\omega$ and express the constant $B$ in terms of the given information.

**d)** Using the Debye model, prove that for $T \ll \Theta_D$ (where $\Theta_D$ is the Debye temperature), the heat capacity of this lattice is proportional to $T^2$. Express $\Theta_D$ in terms of the given information.

## Problem 3

A linear monoatomic lattice has lattice constant $a$, and a basis of two identical atoms with mass $m$ at 0 and $a/4$. The potential energy of an electron can be expressed as

$$V(x) = \sum_{j=-\infty}^{+\infty} V_0(x - x_j) \ , \tag{4.1}$$

where $x_j$ is the $x$ coordinate of the $j$th atom.

**a)** Assuming the potential is strong, sketch the electron wave function in light of the Bloch theorem.

**b)** Show that the Fourier coefficient of the potential energy function $\tilde{V}_G = 0$, where $G = l(2\pi/a)$, for $l = 2$. We define

$$\tilde{V}_G = \frac{2\pi}{a} \int_{-a/2}^{+a/2} V(x)\ e^{iGx}\ dx \ . \tag{4.2}$$

**c)** Assume that the potential is not strong enough to bind the electrons. Let the first and the third Fourier coefficients be given as $\tilde{V}_1$ and $\tilde{V}_3$ such that $\tilde{V}_1/\tilde{V}_3 = 3/2$. Make a good sketch of $E(k)$ for the extended zone scheme for $-(3.5\pi)/a < k < +(3.5\pi)/a$. Repeat showing the same bands in the reduced zone scheme on the same $E(k)$ graph.

## Problem 4

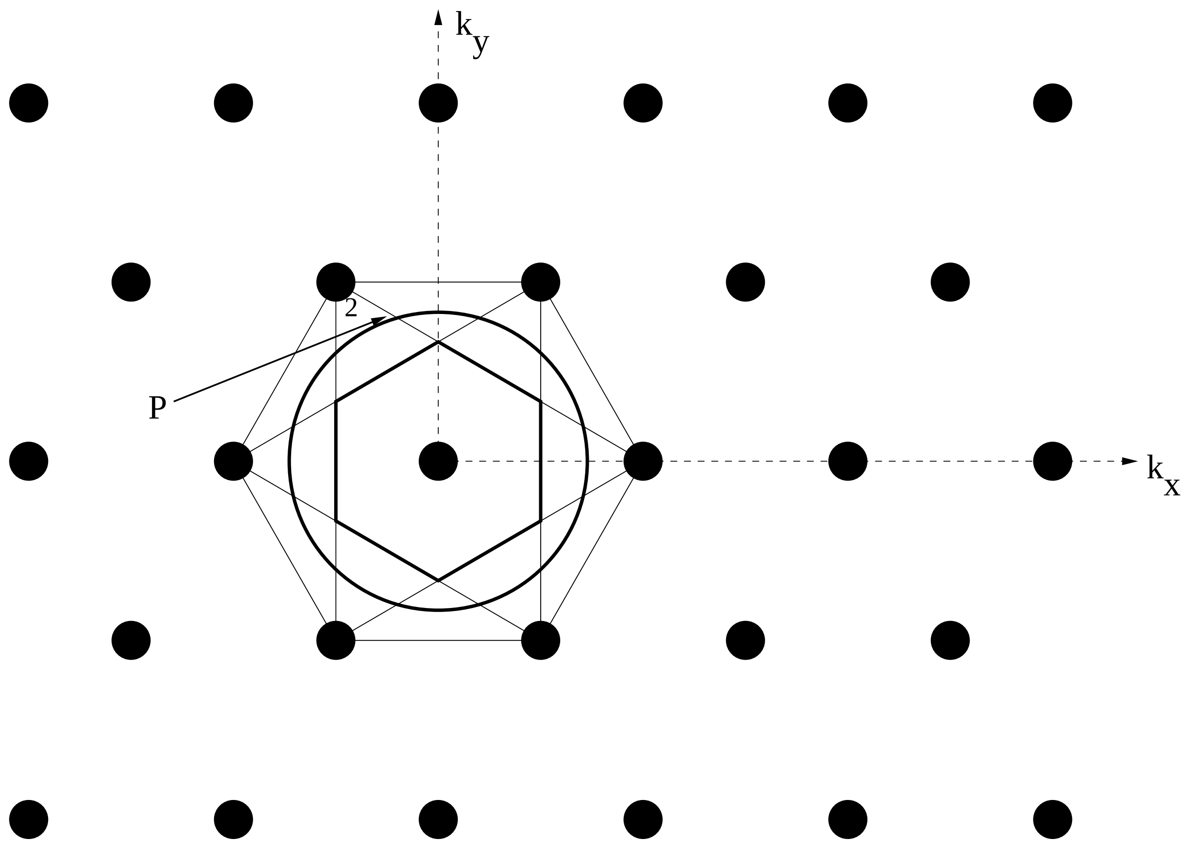
The figure shows the $(k_x, k_y)$ plane for a two-dimensional metal having a hexagonal lattice. Reciprocal lattice points are shown as solid dots. Consider the second Brillouin zone (2BZ). One segment of it is marked "2" in the figure.

**a)** a) We assume that the Fermi-surface lies very close to the incribed circle. Sketch it directly on the figure or on a redrawing of it in the NFL ("Near Free Lattice") approximation.

**b)** What is the direction of the group velocity $\vec{v}_g$ for electrons with $\vec{k}$ vectors in the vicinity of point $P$ just inside 2BZ.

**c)** Draw in all of the $E_F$ contour lines of 2BZ in the reduced zone scheme showing how you determined where one of the elements went. Show point $P$ in the reduced zone scheme.

**d)** Draw in the segments of the Fermi surface of the third Brillouin zone (3BZ) in the periodic zone scheme.



k_y
k_x
P
2