



NTNU – Trondheim
Norwegian University of
Science and Technology

Department of Physics

Examination paper for TFY4225 Nuclear and Radiation Physics

Academic contact during examination: Pål Erik Goa

Phone: 959 08026

Examination date: 18.12.2014

Examination time (from-to): 9.00-13.00

Permitted examination support material (code C):

- Simple specified calculator
- Barnett & Cronin: Mathematical Formulae
- Rottmann: Matematische Formelsammlung

Other information: Each sub-question (1a, 1b, 3 etc) carries equal weight in the evaluation. Exam might be answered in English or Norwegian.

Language: English

Number of pages (front page excluded): 3

Number of pages enclosed: 2

Checked by:

Date

Signature

CONSTANTS

Speed of light	c	$2.99792458 \times 10^8 \text{ m/s}$
Charge of electron	e	$1.602189 \times 10^{-19} \text{ C}$
Boltzmann constant	k	$1.38066 \times 10^{-23} \text{ J/K}$ $8.6174 \times 10^{-5} \text{ eV/K}$
Planck's constant	h	$6.62618 \times 10^{-34} \text{ J} \cdot \text{s}$ $4.13570 \times 10^{-15} \text{ eV} \cdot \text{s}$
	$\hbar = h/2\pi$	$1.054589 \times 10^{-34} \text{ J} \cdot \text{s}$ $6.58217 \times 10^{-16} \text{ eV} \cdot \text{s}$
Gravitational constant	G	$6.6726 \times 10^{-11} \text{ N} \cdot \text{m}^2/\text{kg}^2$
Avogadro's number	N_A	$6.022045 \times 10^{23} \text{ mole}^{-1}$
Universal gas constant	R	$8.3144 \text{ J/mole} \cdot \text{K}$
Stefan-Boltzmann constant	σ	$5.6703 \times 10^{-8} \text{ W/m}^2 \cdot \text{K}^4$
Rydberg constant	R_∞	$1.0973732 \times 10^7 \text{ m}^{-1}$
Hydrogen ionization energy		13.60580 eV
Bohr radius	a_0	$5.291771 \times 10^{-11} \text{ m}$
Bohr magneton	μ_B	$9.27408 \times 10^{-24} \text{ J/T}$ $5.78838 \times 10^{-5} \text{ eV/T}$
Nuclear magneton	μ_N	$5.05084 \times 10^{-27} \text{ J/T}$ $3.15245 \times 10^{-8} \text{ eV/T}$
Fine structure constant	α	$1/137.0360$
	$\hbar c$	$1239.853 \text{ MeV} \cdot \text{fm}$
	$\hbar c$	$197.329 \text{ MeV} \cdot \text{fm}$
	$e^2/4\pi\epsilon_0$	$1.439976 \text{ MeV} \cdot \text{fm}$

PARTICLE REST MASSES

	u	MeV/c ²
Electron	5.485803×10^{-4}	0.511003
Proton	1.00727647	938.280
Neutron	1.00866501	939.573
Deuteron	2.01355321	1875.628
Alpha	4.00150618	3727.409
$\pi^\pm$	0.1498300	139.5669
π^0	0.1448999	134.9745
μ	0.1134292	105.6595

CONVERSION FACTORS

$1 \text{ eV} = 1.602189 \times 10^{-19} \text{ J}$	$1 \text{ b} = 10^{-28} \text{ m}^2$
$1 \text{ u} = 931.502 \text{ MeV}/c^2$ $= 1.660566 \times 10^{-27} \text{ kg}$	$1 \text{ Ci} = 3.7 \times 10^{10} \text{ decays/s}$

Upper region of the periodic table of the elements:

1 H					2 He
3 Li	4 Be				
		5 B	6 C	7 N	8 O
				9 F	10 Ne
11 Na	12 Mg				
		13 Al	14 Si	15 P	16 S
				17 Cl	18 Ar

Atomic masses of some isotopes:

¹⁸ N:	18.014079 u
¹⁸ O:	17.999161 u
¹⁸ F:	18.000938 u
¹⁸ Ne:	18.005708 u
¹⁸ Na:	18.02597 u

Order of the lowest shells in the Woods-Saxon with spin orbit coupling model: $1s_{1/2}$, $1p_{3/2}$, $1p_{1/2}$, $1d_{5/2}$, $2s_{1/2}$, $1d_{3/2}$, $1f_{7/2}$, $2p_{3/2}$, $1f_{5/2}$, $2p_{1/2}$, $1g_{9/2}$

The reduced de-Broglie wavelength:

$$\tilde{\lambda} = 1/k \text{ where } k = p/\hbar$$

Remember:

- **Relevant information that might become useful in solving the problems are stated on page 1 of the exam !!**
- **Each subproblem (1a, 2b, 3 etc) carries equal weight in the evaluation.**

Problem 1

^{18}F is the most used radioisotope for positron emission tomography (PET).

1 a)

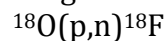
- Write down the full reaction for β^+ decay of ^{18}F and the expression for the reaction energy Q .
- Calculate the value of Q .
- What other decay-process is possible and why?

1 b) Spin-parity state of ^{18}F is 1^+ .

- Discuss this value with relation to the shell model.
- Determine the spin-parity state of the daughter nucleus in the above decay.
- Classify the transition in terms of degree of allowedness/forbiddenness and Fermi vs Gamow-Teller type.

Problem 2

^{18}F is usually produced in the following nuclear reaction:



with protons accelerated in a cyclotron.

2 a)

- Describe how the cyclotron works (at non-relativistic particle velocities).
- Derive the expression for the cyclotron frequency. The centripetal force is given by mv^2/r
- Give an expression for the final proton kinetic energy.
- If the cyclotron has radius 79 cm, what is the required magnetic field to reach 30 MeV for the protons?

2 b)

- Set up the expression for energy conservation of the nuclear reaction stated above, and express it as a function of the reaction energy Q .
- Calculate the value of Q .
- Assume the target is at rest, derive an expression for the kinetic energy available for the reaction. The Center of Mass (CM) reference frame is defined by $\sum p_{i,CM} = 0$

2 c)

- Calculate the semi-classical reaction cross section for the nuclear reaction above, for protons at 30 MeV. Use $R_0 = 1.2 \text{ fm}$ ($=1.2 \cdot 10^{-15} \text{ m}$).
- Calculate the Coulomb potential energy at closest hard sphere distance between the target and projectile, and compare this to the reaction energy Q .
- How much will the Coulomb repulsion change the cross section according to the classical hard sphere model?

Problem 3

In a typical PET-exam the patient is injected with an activity of 400 MBq of ^{18}F .

- Calculate the committed effective dose for the patient. Assume 100 % probability for β^+ decay, uniform distribution of radiation in the body and no physiological clearance. Halflife of ^{18}F is 109 minutes.
- Compare the resulting dose to the recommended maximum annual dose for the general public.

Problem 4

4 a)

- Describe the main principles of PET imaging.
- Discuss possible advantages compared to single photon emission computed tomography (SPECT).

4 b)

- Describe the possible interaction processes between the 511 keV photon and matter.
- The following data of a PET-detector element is given:
 - scintillator:
 - linear attenuation coeff: 50 m^{-1}
 - thickness 2.5 cm.
 - Energy conversion factor: 38000 photons/MeV.
 - PMT:
 - geometric efficiency: 0.2
 - photo-electron efficiency: 0.9
- Calculate the energy resolution of this detector. (FWHM = 2.355σ).

Problem 5

5 a) Analyse the momentum transfer from a moving heavy charged particle to an electron initially at rest, as result of the Coulomb force: $\vec{F} = \frac{q_1 q_2}{4\pi\epsilon_0 r^2} \hat{r}$

- Show that the following expression holds for the energy transferred in the collision (Q), and explain the terms:

$$Q = \left(\frac{Ze^2}{4\pi\epsilon_0 bV} \right)^2 \frac{1}{2m}$$

5 b)

- Use the above equation to explain the so-called Bragg-peak.
- Why is the Bragg-peak relevant with respect to radiation therapy?