



NTNU – Trondheim
Norwegian University of
Science and Technology

Department of Physics

Examination paper for TFY4225 Nuclear and Radiation Physics

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Examination date: 05.12.2016

Examination time (from-to): 9.00-13.00

Permitted examination support material (code C):

- Simple specified calculator
- Barnett & Cronin: Mathematical Formulae
- Rottmann: Matematische Formelsammlung

Other information: Each sub-question (1, 2a, 2b etc) carries equal weight in the evaluation. Exam might be answered in English or Norwegian.

Language: English

Number of pages (front page excluded): 9

Checked by:

Date

Signature

Note: Important information is given in tables, figures and equations after the problem text! Please browse through all sides before you start to work on the problems.

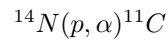
Problem 1

The nuclear shell model was a major step forward in nuclear physics.

- Describe the key ingredient of the model and how this affected the output.
- What were the two main success-criteria for the model?
- Explain what is meant by the "extreme independent particle model".
- Give one example situation where the "extreme independent particle model" does not work.

Problem 2

^{11}C can be used in positron emission tomography and can be produced in a cyclotron via the following reaction:



The mass excess values for the particles involved are: ^{14}N : $3074\mu\text{u}$, ^1H : $7825\mu\text{u}$, ^4He : $2603\mu\text{u}$, ^{11}C : $11434\mu\text{u}$.

Problem 2a

- Calculate the Q-value for the above nuclear reaction.
- Derive the expression for the required minimum (threshold) kinetic energy T_{th} of the proton in order for the reaction to occur.
- Calculate the numeric value of T_{th} .

Problem 2b

- Calculate the Coulomb barrier for the reaction (use $R_0 = 1.6\text{fm}$).
- Relate your results in 2a and 2b to the observed reaction cross-section as shown in figure 1, in particular the range 0-6 MeV.

Problem 2c

Production of ^{11}C :

- Set up the differential equation for the number of ^{11}C product nuclei $N_1(t)$, given a constant reaction rate R .
- Show that the expression for $N_1(t)$ given on the formula sheet is a solution to your equation, and calculate the irradiation time required to reach 90 % of maximum activity.

Problem 2d

The decay scheme of ^{11}C is shown in figure 2.

- In the Fermi theory for beta-decay, explain the approximation behind what is called an *allowed* transition. Show that this approximation is justified.
- Explain/derive the selection rules for angular momentum and parity in the allowed approximation and explain why beta decay of ^{11}C is an allowed transition.
- What is the difference between a Fermi type and a Gamow-Teller type transition? Can you tell which type the ^{11}C beta decay is? Justify your answer.

Problem 2e

A patient is injected with 1 GBq of ^{11}C for a PET exam. Assume even distribution of activity in the body and no biological clearance. Photon attenuation length in soft tissue @ 511 keV is 36 cm.

- Calculate the whole body effective dose to the patient. If you need to make any further simplifying assumptions, please justify/explain them.

Problem 3

In figure 3 an example of a chain of events in a small volume of mass $dm = 1\mu\text{g}$ is shown. A photon of energy $E_\gamma = 662\text{keV}$ is Compton-scattered and an electron is released with initial kinetic energy $T_e = 100\text{keV}$. This electron is scattered and a bremsstrahlung photon with energy $E_b = 20\text{keV}$ is emitted and leaves the volume. Via further interactions, the electron comes to rest within the volume.

- Calculate the energy imparted ε and KERMA K for this example.

Problem 4

In the lab-assignments of this course we used two different detectors: a) NaI scintillator detector and b) HPGe (high-purity Germanium) semi-conductor detector.

- Describe the chain of events in the NaI scintillator detector from the initial interaction between the photon and the scintillator material to a count in the spectrum shown on the computer screen.
- Describe the chain of events in the HPGe detector.
- Describe how the output spectra from the two detectors will look. What is the key physical origin of the difference between the two spectra?

Problem 5

The following 9 topics of "Sources of Radiation" were covered in the first project work of this course:

1. Natural Radioactivity on Earth.
2. Radon Gas: focus on Norway.
3. Cosmic Radiation.
4. Medical Imaging Exams.
5. Chernobyl: Local Consequences and Europe in General.
6. Chernobyl: Consequences in Norway.
7. Fukushima Accident.
8. Low Dose Risk: Linear-No-Threshold Model.
9. Low Dose Risk: Hormesis Model.

Choose one of the above topics and describe what was presented on that topic.

Problem 6

The neutron absorption and scattering cross section for light water (H_2O) and heavy water (D_2O) is given in table 1. With respect to the neutron cycle in a thermal fission reactor, which type of water will give the highest neutron multiplication factor, given all other things equal? Justify your answer.

Table 1: Neutron absorption and scattering cross sections

Material	σ_a (b)	σ_s (b)
H_2O	0.66	49.2
D_2O	0.001	10.6

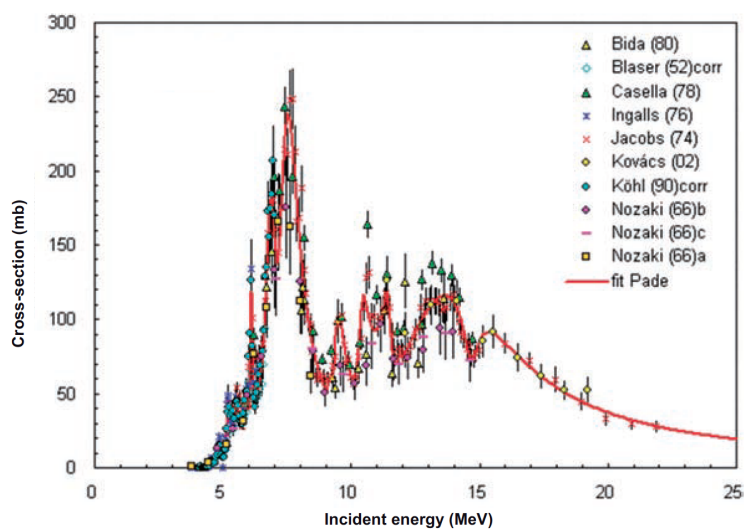


Figure 1: Reaction cross section for $^{14}\text{N}(p, \alpha)^{11}\text{C}$.

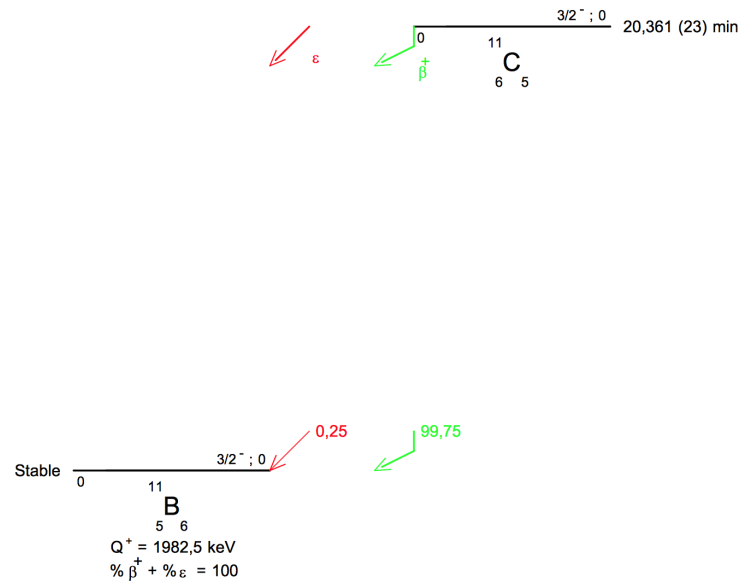


Figure 2: Decay scheme of ^{11}C .

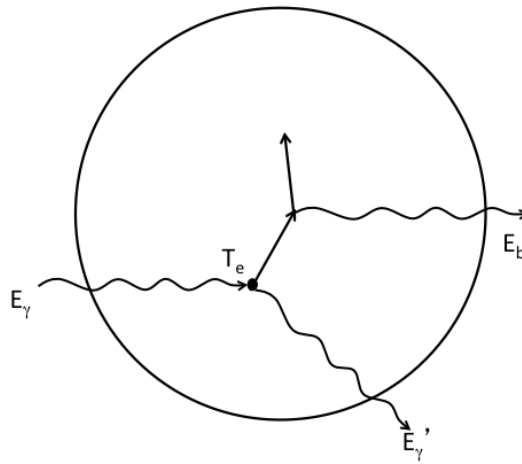


Figure 3: Example radiation interactions in a small volume.

CONSTANTS

Speed of light	c	$2.99792458 \times 10^8 \text{ m/s}$
Charge of electron	e	$1.602189 \times 10^{-19} \text{ C}$
Boltzmann constant	k	$1.38066 \times 10^{-23} \text{ J/K}$ $8.6174 \times 10^{-5} \text{ eV/K}$
Planck's constant	h	$6.62618 \times 10^{-34} \text{ J} \cdot \text{s}$ $4.13570 \times 10^{-15} \text{ eV} \cdot \text{s}$
	$\hbar = h/2\pi$	$1.054589 \times 10^{-34} \text{ J} \cdot \text{s}$ $6.58217 \times 10^{-16} \text{ eV} \cdot \text{s}$
Gravitational constant	G	$6.6726 \times 10^{-11} \text{ N} \cdot \text{m}^2/\text{kg}^2$
Avogadro's number	N_A	$6.022045 \times 10^{23} \text{ mole}^{-1}$
Universal gas constant	R	$8.3144 \text{ J/mole} \cdot \text{K}$
Stefan-Boltzmann constant	σ	$5.6703 \times 10^{-8} \text{ W/m}^2 \cdot \text{K}^4$
Rydberg constant	R_∞	$1.0973732 \times 10^7 \text{ m}^{-1}$
Hydrogen ionization energy		13.60580 eV
Bohr radius	a_0	$5.291771 \times 10^{-11} \text{ m}$
Bohr magneton	μ_B	$9.27408 \times 10^{-24} \text{ J/T}$ $5.78838 \times 10^{-5} \text{ eV/T}$
Nuclear magneton	μ_N	$5.05084 \times 10^{-27} \text{ J/T}$ $3.15245 \times 10^{-8} \text{ eV/T}$
Fine structure constant	α	$1/137.0360$
	hc	$1239.853 \text{ MeV} \cdot \text{fm}$
	$\hbar c$	$197.329 \text{ MeV} \cdot \text{fm}$
	$e^2/4\pi\epsilon_0$	$1.439976 \text{ MeV} \cdot \text{fm}$

PARTICLE REST MASSES

	u	MeV/c^2
Electron	5.485803×10^{-4}	0.511003
Proton	1.00727647	938.280
Neutron	1.00866501	939.573
Deuteron	2.01355321	1875.628
Alpha	4.00150618	3727.409
$\pi^\pm$	0.1498300	139.5669
π^0	0.1448999	134.9745
μ	0.1134292	105.6595

CONVERSION FACTORS

$1 \text{ eV} = 1.602189 \times 10^{-19} \text{ J}$	$1 \text{ b} = 10^{-28} \text{ m}^2$
$1 \text{ u} = 931.502 \text{ MeV}/c^2$ $= 1.660566 \times 10^{-27} \text{ kg}$	$1 \text{ Ci} = 3.7 \times 10^{10} \text{ decays/s}$

Figure 4: Physical constants.

Appendix: Selected expressions

$$\begin{aligned} F &= \frac{|q_1 q_2|}{4\pi\epsilon_0 r^2} \\ P &= \frac{q_1 q_2}{4\pi\epsilon_0 r} \\ \vec{F} &= q\vec{E} + q\vec{v} \times \vec{B} \end{aligned}$$

$$\begin{aligned} \vec{p} &= m\vec{v} \\ \vec{L} &= \vec{r} \times \vec{p} \\ F &= ma \\ F &= \frac{mv^2}{r} \end{aligned}$$

$$\begin{aligned} U(\vec{r}) &= -\int_{\infty}^{\vec{r}} \vec{F} \cdot d\vec{r} \\ \Delta\vec{p}(t) &= \int_0^t \vec{F} d\tau \end{aligned}$$

$$\begin{aligned} T &= \frac{1}{2}mv^2 \\ p &= \frac{mv}{\sqrt{1-v^2/c^2}} \\ E &= \sqrt{p^2c^2 + m^2c^4} \\ T &= E - mc^2 \\ E_0 &= mc^2 \\ \lambda &= \frac{h}{p} \end{aligned}$$

$$\begin{aligned} \Delta p \Delta x &\leq \frac{\hbar}{2} \\ \Delta E \Delta t &\leq \frac{\hbar}{2} \end{aligned}$$

$$\begin{aligned} \text{Plane wave } \psi(x) &= e^{i\vec{p} \cdot \vec{r}/\hbar} \\ \langle l^2 \rangle &= \hbar^2 l(l+1) \\ \langle l_z \rangle &= \hbar m_l \end{aligned}$$

$$\begin{aligned} B &= a_v A - a_s A^{2/3} - a_C \frac{Z(Z-1)}{A^{1/3}} - a_{sym} \frac{(A-2Z)^2}{A} + \delta \\ Q &= \sum_i m_i c^2 - \sum_f m_f c^2 \end{aligned}$$

$$\begin{aligned}
N(t) &= N_0 e^{-\lambda t} \\
A(t) &\equiv \lambda N(t) \\
t_{1/2} &= \frac{\ln 2}{\lambda} = \tau \ln 2 \\
N_1(t) &= \frac{R}{\lambda_1} (1 - e^{-\lambda_1 t}) \\
N_2(t) &= N_0 \frac{\lambda_1}{\lambda_2 - \lambda_1} (e^{-\lambda_1 t} - e^{-\lambda_2 t})
\end{aligned}$$

$$\begin{aligned}
\lambda &= \frac{2\pi}{\hbar} |V_{fi}|^2 \rho(E_f) \\
V_{fi} &= \int \psi_f^* V \psi_i d\nu \\
\lambda &= \frac{g^2 |M_{fi}|}{2\pi \hbar^7 c^3} \int_0^{p_{max}} F(Z', p) p^2 (Q - T_e)^2 dp \\
\pi(ML) &= (-1)^{L+1} \\
\pi(EL) &= (-1)^L
\end{aligned}$$

$$\sqrt{T_b} = \frac{\sqrt{m_a m_b T_a} \cos \theta \pm \sqrt{m_a m_b T_a \cos^2 \theta + (m_Y + m_b) [m_Y Q + (m_Y - m_a) T_a]}}{m_Y + m_b}$$

$$\begin{aligned}
\sigma_{sc} &= \sum_{l=0}^{\infty} \pi \lambda (2l+1) |1 - \eta_l|^2 \\
\sigma_r &= \sum_{l=0}^{\infty} \pi \lambda (2l+1) (1 - |\eta_l|^2)
\end{aligned}$$

$$-\frac{dE}{dx} = \left(\frac{ze^2}{4\pi\epsilon_0} \right)^2 \frac{4\pi Z\rho N_A}{Amv^2} \left[\ln \left(\frac{2mv^2}{I} \right) - \ln(1 - \beta^2) - \beta^2 \right]$$

$$\begin{aligned}
n &= \frac{\rho N_A}{A} \\
N &= N_0 e^{-\mu x} \\
\mu &= n\sigma \\
\sigma &= \sigma_{PE} + Z\sigma_C + \sigma_{PP}
\end{aligned}$$

$$\begin{aligned}
D &\equiv \frac{d\bar{\varepsilon}}{dm} \\
\varepsilon &= R_{in} - R_{out} + \sum Q \\
K &= \frac{dD_{tr}}{dm}
\end{aligned}$$

$$\begin{aligned}
K_C &= E\Phi\left(\frac{\mu_{en}}{\rho}\right) \\
D(r_T, T_D) &= \sum_{r_S} \tilde{A}(r_S, T_D) S(r_T \leftarrow r_S) \\
S(r_T \leftarrow r_S) &= \frac{1}{M(r_T)} \sum_i E_i Y_i \phi(r_T \leftarrow r_S, E_i) \\
\tilde{A}(r_S, T_D) &= \int_0^{T_D} A(r_S, t) dt \\
E &= \sum_T w_T \sum_R w_R D_R(r_T, T_D)
\end{aligned}$$